Chapter 3 Multiples and Factors

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	Review: Division	66	Divide a number by a one-digit number when the quotient is two digits.
1	Multiples	68	Find multiples of a given one-digit number.
2	Common Multiples	70	Find common multiples of two or three given one-digit numbers.
3	Factors	72	Find factors of whole numbers within 120.
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5	Common Factors	75	Find common factors of two or three given numbers within 100.
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Notes

Multiplication and Division

Students learned to multiply and divide a two- or a three-digit number by a one-digit number in Dimensions Math® 3A. While students can add consecutively to find a multiple greater than the tenth multiple, those who have completed Dimensions Math® 3 will be able to multiply instead, often mentally. In order to determine whether a number is a multiple of another, students will have to divide outside of the multiplication table, even when there is a remainder, for example, 64 ÷ 5. If your student cannot multiply and divide all two-digit numbers, and some tens (such as 120 ÷ 6), consider doing chapters 5 and 6 of Dimensions Math® 3A. There are also a few other topics in Dimensions Math® 3A that will also be useful if your student is new to this curriculum, such as using bar models.

Alternatively, this guide includes a brief introduction to multiplication of two-digit numbers with products 120 or less, and to division of a two- or three-digit number when the quotient is a two-digit number. For multiplication in particular, the focus will be on mental math, since students do need to know how to multiply a two-digit number by a one-digit number mentally for the next chapter. The algorithm will be briefly introduced. For division, students do have to recognize the division algorithm in Lesson 1 of this chapter, and so will have to understand it. The most efficient mental math strategy for division is simply based on the algorithm.

Multiples

A multiple of a number is the product when that number is multiplied by an integer. 0 is a multiple of all whole numbers. However, when finding the least multiple, or the first ten multiples, the implication at this level will be that only positive whole numbers greater than 0 are meant. Therefore, the first multiple of 6 is 6 (1 × 6), not 0. It should not be difficult for students to understand what the word “multiple” means from the term itself.

A number is a multiple of another number if it is divisible by that number; that is, there is no remainder when it is divided by that number. 64 is a multiple of 4 since 64 ÷ 4 is 16 with no remainder. 62 is not a multiple of 4, since 62 ÷ 4 is 15 with a remainder of 2. There is no whole number that when multiplied by 4 gives a product of 62.

Students will learn some rules of divisibility in Lesson 1, most of which they have seen before when learning the multiplication and division facts for products within the multiplication table (within 10 times that number). The same rules apply to numbers outside of the multiplication table. Knowing how to easily recognize whether a number is a multiple of another will help them in Lesson 3, when they will find the factors of a number.

- A number is divisible by 2 if it is even (the ones digit is 0, 2, 4, 6, or 8).
- A number is divisible by 3 if the sum of the digits is a multiple of 3.
- A number is divisible by 5 if the ones digit is 0 or 5.
- A number is divisible by 6 if it is divisible by both 2 or 3.
- A number is divisible by 9 if the sum of the digits is a multiple of 9.
- A number is divisible by 10 if the ones digit is 0.

For numbers that are divisible by 3 or 9, if the sum of the digits is a two-digit number, we can continue to test if that sum is a multiple of 3 or 9. The sum of the digits in 69 is 15, and since it is a multiple of 3, the sum of its digits (6) will also be a multiple of 3. This will be useful to students later. In this chapter, they will not have to determine if a number greater than 120 is a multiple of 3 or 9.

In Lesson 2, students will learn how to find common multiples of two more numbers. This is one of the primary reasons to learn about multiples. In particular, it will be useful to know how to find the least common multiple, which will be used when working with computations involving fractions.

Factors

A factor of a given number is a number that can divide the given number with no remainder. 8 is a factor of 24, since $24 \div 8$ is 6 with no remainder. Students have learned the term factor to mean the numbers in a multiplication expression. In $6 \times 8 = 24$, 6 and 8 are factors, and 24 is the product. The term factor is used only for positive or negative whole numbers. $\frac{1}{2}$ is not considered a factor of 24, even though $\frac{1}{2} \times 48 = 24$.

- Every whole number is a factor of itself.
- 1 is a factor of every whole number.
- Every number other than 1 has at least two factors, 1 and itself.
- 0 is not a factor of any non-zero number, but every number is a factor of 0. (Students will not need to be concerned about 0 as a factor.)

In Lesson 3, students will learn how to systematically find all the factors of whole numbers within 120.

In Lesson 4, students will learn about prime numbers and composite numbers. A prime number is a whole number greater than 1 that has only two factors, 1 and itself. 7 is a prime number, since it has only 1 and 7 as its factors. A composite number is a number that has more than two factors. 8 is a composite number, since it has 1, 2, 4, and 8 as its factors.

- 2 is the least and only even prime number.
- 4 is the least composite number.
- 1 is neither prime nor composite (it has only one factor).
- 0 is neither prime nor composite (it has an infinite number of factors).
- Every even number can be expressed as the sum of two prime numbers. (Students do not have to know this at this level.)

To find all prime numbers within 100, Students will use a process called “Sieve of Eratosthenes,” in which they iteratively mark as composite the multiples of each prime number, starting with 2. There are other prime sieves, such as the “Sieve of Sundaram,” but they are more complicated.

In later levels, students will learn how to use prime factorization (find all the prime factors of a number) in order to find the least common multiple or greatest common factor of numbers greater than 120. Prime numbers are of central importance to number theory and have many applications in mathematics. Patterns involving prime numbers can even be found in biology.

In Lesson 5, students will find common factors of two or more numbers within 120. Knowing how to find common factors will be useful in calculations with fractions.

Additional Notes: Divisibility Rules

For your information only, if you are interested, some informal explanations are given below for why the divisibility rules apply even to numbers greater than 100, as well as some divisibility rules for 4, 7, and 8. You may want to bring up the divisibility rule for 4 later with your student, when they divide greater numbers than in this chapter, since knowing divisibility rules is one way to check if the answer to a division problem makes sense.

2, 4, 6, and 8 have 2 as a factor.

$356 = (3 \times 100) + (5 \times 10) + 6.$

Each of the parts is divisible by 2, since multiples of 10 are divisible by 2, so 356 is divisible by 2.

$357 = (3 \times 100) + (5 \times 10) + 7$

The 7 is not divisible by 2, even though the other parts are, so 357 is not divisible by 2. So we only have to look at the last digit to determine if a number is divisible by 2.

Multiples of 10 are divisible by 5.

$455 = (4 \times 100) + (5 \times 10) + 5$

Each of the parts, including 5, is divisible by 5, so the whole number is.

$456 = (4 \times 100) + (5 \times 10) + 6$

The last part, the 6, is not divisible by 5, so 456 is not divisible by 5. We only need to look at the last digit to determine if a number is divisible by 5.

Activity

- After the division review lesson
Materials: Number cards 11–100
Purpose: Practice mental division of two-digit numbers.
Procedure: Tell your student which one-digit number they will divide by. Shuffle the cards and then show them to your student one at a time. Your student needs to give the quotient and the remainder, if there is one. If they get the answer correct, set that card aside. If they get the answer incorrect, return the card to the deck in your hand.

Games

- After Lesson 1
Materials: Hundred chart, game tokens for each player (such as counters, with each player having a different color counter), ten-sided die or 4 sets of number cards 1—10, shuffled
Purpose: Find multiples of a number.
Goal: Get three in a row.
Procedure: Players take turns rolling the die (or drawing a card) and placing one of their tokens on an unoccupied square of the hundred chart with a number that is a multiple of the number rolled. (If a player rolls a 1, they can put their marker on any number.) The first player to get three in a row wins.
Variation: Players can put their token on an occupied square, replacing the token that is there. The first player to get 5 tokens in a row wins.

- After Lesson 2
Materials: Hundred chart, game tokens for each player, two ten-sided dice or 4 sets of number cards 1—10, shuffled
Purpose: Find common multiples of a number.
Goal: Get three in a row.
Procedure: Players take turns rolling the dice (or drawing two cards) and placing one of their tokens on an unoccupied square of the hundred chart with a number that is a common multiple of the numbers rolled. The first player to get 3 in a row wins.
Variation: Players can put their token on an occupied square, replacing the marker that is there. The first player to get 5 in a row wins.

- After Lesson 4
Materials: Two number cubes, one labeled with 0–5 and the other with 4–9, game tokens for each player, **Factor Game Board** printout
Purpose: Find factors of a number.
Goal: Get three in a row.
Procedure: Players take turns rolling the number cubes to form a two-digit number. The player then puts one of their tokens on an unoccupied number on the game board with a number that is a factor of the two-digit number. If the number is a prime number, they put a token on a star. The first player with 3 tokens in a row wins.

Lesson 3 Factors (pp. 65–67)

Think (p. 65)

Give your student 12 squares and ask them to form different rectangles and then write the length and width of each. You can either use square tiles, if you have some, or cut squares from the **Inch Graph Paper** printout. Alternately, tell your student to draw rectangles on graph paper with areas of 12 unit squares. (They may recall doing the same thing at the end of Dimensions Math® 3B when learning about area and perimeter.)

Learn (p. 65)

The side lengths of the rectangles are the factors of 12. The word **factors** is defined here by example. A factor of a given number is a number that divides the given number with no remainder. Point out that if we know that 3 is a factor of 12, then we also know that 12 is a multiple of 3.

Students are introduced the word **divisible**. A number is divisible by its factor.

You may want to tell your student that we can call 1 and 12 , 2 and 6, and 3 and 4 "factor pairs." A factor pair is a set of two factors, which, when multiplied together, give a particular product. This term will facilitate discussion later.

Do (pp. 66–67)

2 Students are shown a systematic way to find the factors of a number. We can test each number successively (1, 2, 3, etc.) until we get to a number that we have already found as part of a factor pair. In this case, we don't have to check 6 since we already know from 3 × 6 that 6 is a factor. Your student can use any divisibility rules and math facts they know before trying to divide to see if there is a remainder. For example, they would not try 5 as a factor of 18, since 18 does not end in 0.

5 First point out Mei's thought. Your student may remember from Dimensions Math® 3A that the only way to have an odd product of two numbers is if both numbers are odd. If not, they can try a few facts or look at the numbers in the **Multiplication Chart** printout to verify this by inductive reasoning. They do

Answers

1 **2** × **4** = 8
1, **2**, **4**, and 8

2 1 × **18** = 18
2 × **9** = 18
3 × **6** = 18
1, 2, 3, **6**, **9**, and **18**

3 (a) Yes 14 is divisible by 7.
(b) No 14 is not divisible by 3.

4 6, 36, 54, 60

5 1 × **75** = 75
3 × **25** = 75
5 × **15** = 76
1, 3, **5**, **15**, **25**, and **75**

(Continued next page.)

know, however, that an odd number is not a multiple of 2, and since all even numbers are multiples of 2, an odd number cannot be a multiple of any other even number.

Discuss ways they can test each odd number to determine if it is a factor of 75 and find the factor pair. Your student can use either the division algorithm or a mental math type of strategy. For example:

- 3: 7 + 5 = 12, so 3 is a factor. Split 75 into 60 and 15, divide each by 3, and add the quotients to get 25.
- 5: The ones digit of 75 is 5, so 5 is a factor. Split 75 into 50 and 25. 10 + 5 = 15.
- 7: Split 75 into 70 and 5. 5 is going to be a remainder, so 7 is not a factor.
- 9: 7 + 5 = 12, which is not a multiple of 9. 9 is not a factor of 75.
- 11: Students should know that within 100 multiples of 11 less than 100 have the same digits in the tens and ones place.
- 13: Students have not been taught to divide by a two-digit number, but the only possibility would be 13 × 5 in order to get a 5 in the ones place, and we have already determined that 5 and 15 is a factor pair.

6 You may want to assign only some of these; there will be plenty of practice in the workbook. Discuss at least one of these, such as (g):

- 1: **1 × 72**
- 2: **2 × 36**; split 72 into 60 and 12.
- 3: **3 x 24**; 3 is a factor since 7 + 2 = 9, split 72 into 60 and 12.
- 4: **4 × 18**; split 72 into 40 and 32.
- 5: 5 is not a factor.
- 6: **6 × 12**; 6 is a factor since 2 and 3 both are. Split 72 into 60 and 12.
- 7: Split 72 into 70 and 2. There will be a remainder of 2, so 7 is not a factor.
- 8: **8 × 9**; this is a multiplication fact.
- 9: We already know 9 is a factor, we do not have to test any more numbers since we have found all the factor pairs.

7 To find the missing factors, we simply divide. Students can use the division algorithm or mental math strategies. For (b), students could split 105 into 70 and 35. For (c), they may recall that 4 quarters make a dollar, or split 100 into 80 and 20.

Answers (continued)

6 (a) 1, 3, 5, 15
(b) 1, 3, 7, 21
(c) 1, 2, 3, 4, 6, 9, 12, 18, 36
(d) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
(e) 1, 2, 3, 6, 9, 18, 27, 54
(f) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
(g) 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
(h) 1, 2, 4, 5, 10, 20, 25, 50,100

7 (a) 12 (b) 25
(c) 25 (d) 16

Chapter 3 Workbook Answers

3 (a) 11 R 6
No
(b) 12
Yes

4

Number	Is 2 a factor?	Is 3 a factor?	Is 5 a factor?
5	No	No	Yes
15	No	Yes	Yes
36	Yes	Yes	No
60	Yes	Yes	Yes
73	No	No	No
84	Yes	Yes	No
100	Yes	No	Yes
114	Yes	Yes	No
120	Yes	Yes	Yes

5 (a) 1, 2, 3, 6
(b) 3, 6, 8, 48
(c) 6, 30, 72

6 (a) 1, 2, 4, 7, 8, 14, 28, 56
(b) 1, 2, 4, 8, 16, 32, 64
(c) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
(d) 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
(e) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
You can omit (d) and (e) if you think it will take your student a long time.

7 1, 2, 4, 8, 16, or 32

8 **6 or 18 vases**
Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54
6 and 18 are even factors between 5 and 20.

9 **9, 25, 49, 81, 121**
Some students should realize after testing the first four numbers that there is an odd number of factors when both numbers in a factor pair are the same.
9 = 3 × 3; 25 = 5 × 5; 49 = 7 × 7; 81 = 9 × 9; 121 = 11 × 11. They may have difficulty with 121, since they have not formally learned how to multiply or divide by a two-digit number, but they can apply logical thinking. 121 is obviously greater than 10 × 10, so to see if it is equal to 11 × 11 they should know that 10 × 11 = 110, and can add on another 11. Or, they can start with 99 and add 11 twice.

10 **4, 9, 25, 49**
Students should realize from the previous problem that numbers with an odd number of factors are those where two factors are the same: 1 × 1, 2 × 2, 3 × 3, etc. They only need to check 1, 4, 9, 16, 25, 36, and 49. The ones with 3 factors are 4, 9, 25, and 49.

Exercise 4 pp. 57–59

1 (a) 19: 1, 19
20: 1, 2, 4, 5, 10, 20
21: 1, 3, 7, 21
23: 1, 23
27: 1, 3, 9, 27
29: 1, 29
(b) 19, 23, 29
(c) 20, 21, 27

Lesson 5 Multiplying by a Multiple of 10 (pp. 94–97)

Think (p. 94)

Have your student write an expression and solve it any way they want. Based on previous experience, they should think to split one of the two factors and add partial products.

$$\begin{array}{l} 34 \times 20 = 600 + 80 = 680 \\ \;/\;\;\backslash \\ 30\;\;\;4 \end{array}$$

$$\begin{array}{l} 34 \times 20 = 340 + 340 = 680 \\ \qquad\;\;/\;\;\backslash \\ \qquad\;10\;\;10 \end{array}$$

For the first method above, they would have to know that 3 tens × 2 tens = 6 hundreds. Your student might instead simply know they can multiply 34 by 2, and the answer is tens.

Alternately, you can give your student twelve place-value discs each for ones, tens, and hundreds, ask them to show you 34 with the discs, and also the product of 34 and 20. They do not have enough discs to make 20 groups of 34. Suggest that they think of what the product of each disc and 20 is. If they struggle, simply proceed to Learn.

Learn (pp. 94–95)

The first two methods are actually different ways of explaining concretely with place value discs the third "method" which is a conclusion, that is, that we can multiply 34 by 2 tens in the same way that we multiply 34 by 2, but the answer is tens.

Answers

680

(Continued next page.)

The first two show that we can treat each ten and each one as a separate unit, and multiply each of these units by 10 and then by 2, or by 2 and then by 10. We are expressing 20 as the product of two factors (10 × 2 or 2 × 10) and then multiplying left to right. The answer is the same, regardless of the order we multiply in (commutative and associative properties of multiplication). Make sure your student does not confuse this with "splitting" a number to show a number bond, and then multiplying the other number by each part and adding the partial products (distributive property of multiplication).

The third method is simply an extension of what students have learned with one-digit numbers, and a conclusion based on the first two methods. At the bottom of the page, Mei is explaining how we can record the answer when using the multiplication algorithm: we are multiplying tens, so we write a 0 in the tens place. Then we find 34 × 2 and record that, writing each digit one place to the left, since each digit is ten times as much as the digit would be if we were actually multiplying by 2 ones.

Do (pp. 96–97)

❶ This problem explains why multiplying two tens together results in hundreds. Another way to look at this is 60 × 70 = 6 × 10 × 7 × 10 = 6 × 7 × 10 × 10 = 42 × 100. Most students will notice a pattern: we can multiply the non-zero digits together, and then append the total number of 0s. It is important that students understand the mathematical reasoning behind this pattern: tens × tens = hundreds.

❹ Students are used to writing the regrouped digit at the top. If we were to strictly adhere to writing the regrouped digit in the correct place according to its value, then the regrouped digit would not be above the digit we are multiplying next. In (a), 8 ones × 9 tens = 72 tens, so the digit 7 is hundreds, and the digit 2 is tens. However, since we are multiplying with the same steps we would multiply ones, we do write the regrouped 7 hundreds above the tens in 68. This makes it easier to remember to add it in after multiplying 6 tens by 9 tens. Your student is likely to do this automatically, without a discussion. Encourage them to write the 0 in the tens place before they do the calculations.

❺ Again, students will likely realize they can find 6 × 7, the answer is thousands (three 0s), and there are three 0s total in the factors: tens × hundreds = thousands.

❼ Sofia is explaining why we write the regrouped digit above the next number, rather than aligned with the place values of the rest of the digits in the problem. This will likely not be of concern to your student.

❽ Students can choose to use mental math or rewrite the problem vertically and use the algorithm with any of these.

You can use the **Mental Math** sheets for more practice. Students will be doing problems such as ❶ and ❺ when estimating an answer and, while estimation can help with determining if your answer is reasonable, the estimated answer also needs to be correct with regard to place value.

Answers (continued)

❶ 4,200

❷ 4
128
1,280

❸ 75
750

❹ (a) 6,120 (b) 5,460

❺ 42,000

❻ 4
600
6,000

❼ 8,580

❽

(a) 7,200	(b) 1,120	(c) 4,500
(d) 1,740	(e) 20,000	(f) 56,000
(g) 32,800	(h) 18,600	(i) 34,320

Lesson 5 Word Problems (pp. 122–125)

Think (p. 122)

Relate the bar model to the information in the word problem. If your student is struggling, it might be useful to draw one as you discuss the problem. You may want to remind them that it is always a good idea to read the entire word problem before attempting to model it, since it is not always helpful to start a model based on the order the information is given (though we can do so in this problem). If your student is not struggling, you can write the word problem on a separate sheet of paper and have them draw a model without first looking at the one in the textbook.

Dion is pointing out one important strategy once a model is drawn, which is to look for a way to make equal units. Once there are equal units, the next step is often to find the value of one unit.

Learn (p. 123)

Discuss the two methods and compare them to your student's solution in Think (if they were able to solve the problem).

Method 1: Make Alex's bar (the number of pieces of trash Alex collected) the unit since that is the value we want to find. If we add 230 to Emma's bar and subtract 110 from Sofia's bar, we will have 3 equal units. Adding or subtracting to the bars means the total changes by the same amount, so we need to add to or subtract from the total.

Answers

1,055
1,055
1,055

(Continued next page.)

Once we know what 3 units are, we can use division to find the value of 1 unit, which is the answer to the problem.

The textbook shows the calculations in one expression to save space, expecting students to do the operations in order from left to right. Students have not learned about order of operations yet, and your student will likely write two equations, one for each step: 3,045 + 230 = 3,275 and 3,275 − 110 = 3,165. They might also have subtracted 110 from 3,045 first, and then added 230. Do not require them to write a single expression like the one shown in the textbook.

Method 2: Make Emma's bar the unit.
Many students may use this method, simply because Emma's bar is the shortest one. To have 3 equal units, we need to subtract the difference between Emma's bar and Alex's bar and the difference between Emma's bar and Sofia's bar from the total. The textbook expression shows the subtraction in one expression. Your student does not need to write a single expression and may have used different steps, such as first adding 230 and 110 before subtracting that sum.

With the second method, once we find the value of 1 unit, we then need to add 230 to find the answer to the problem. A common error is to stop after finding the value of 1 unit. This can be avoided by indicating what we want to find with a question mark. (The question mark was included on the model on page 122, but left out on page 123 to save space.)

Usually there are fewer steps when we make the value we want to find the unit, but it will not always be feasible to do so.

Do (pp. 124–125)

Rather than simply discussing the models in the textbook, it might be more useful if you write at least some of the problems on a separate sheet of paper and have your student attempt to draw a model first. Then they can compare their model to the one in the textbook. Models and methods of solution can vary; the one in the textbook is not a required method.

❸ We could make the cost of the floor lamp the unit instead.

❹ In this case, even though the problem asks for the number of books in the second box, it is not feasible to make that the unit.

❺ In this case, it is also not feasible to make the value we want to find the unit.

❻ Students will need to draw a model. They will need to find the value of 3 units.

Answers (continued)

❶ **168 cards**
5 units ⟶ 1,180 − 340 = 840
1 unit ⟶ 840 ÷ 5 = 168

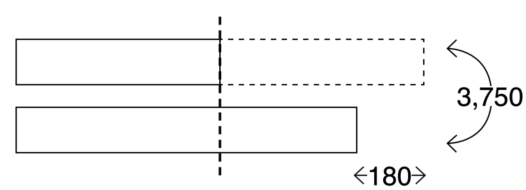
❷ **$249**
7 units ⟶ 1,299 − 284 = 1,015
1 unit ⟶ 1,015 ÷ 7 = 145
145 + 284 = 429

❸ **$387**
70 × 2 = 140
5 units ⟶ 505 + 140 = 645
1 unit ⟶ 645 ÷ 5 = 129
129 × 3 = 387

❹ **162 books**
5 units ⟶ 430 − 25 = 405
1 unit ⟶ 405 ÷ 5 = 81
Second box: 81 × 2 = 162

❺ **543 jars**
5 units ⟶ 980 − 75 = 905
1 unit ⟶ 905 ÷ 5 = 181
181 × 3 = 543

❻ **1,130 and 2,240**



3 units ⟶ 3,750 + 180 = 3,930
1 unit ⟶ 3,930 ÷ 3 = 1,310
1st number: 1,310
2nd number: 1,310 × 2 = 2,620
2,620 − 180 = 2,440

Notes

Multiplying a Fraction by a Whole Number

Students will start by multiplying a fraction by a whole number. This means that the whole number represents the number of equal units, and the fraction the value of the unit. Since students in the U.S. generally learn to interpret the "×" symbol to mean "times", rather than "multiply by," the expression is written with the whole number first. For example, $\frac{5}{9}$ multiplied by 3 will be written as $3 \times \frac{5}{9}$, i.e., 3 "groups of" $\frac{5}{9}$. It is not wrong to write it as $\frac{5}{9} \times 3$, though, in which case the "×" symbol means "multiply by," an interpretation students have already seen when multiplying multi-digit whole numbers.

Most problems where the value of the unit is a fraction will involve measurement. Finding a whole number of groups of a fraction is the same concept as adding equal units, which is how students learned to interpret multiplication previously. For $3 \times \frac{5}{9}$, all the units are quantities of ninths, and since the numerator gives the value of the units of ninths, we can simply multiply the whole number by the numerator. The result is the total number of ninths.

$$3 \times \frac{5}{9} = \frac{5}{9} + \frac{5}{9} + \frac{5}{9} = \frac{15}{9} = 1\frac{6}{9} = 1\frac{2}{3}$$

$$3 \times \frac{5}{9} = \frac{3 \times 5}{9} = \frac{15}{9} = 1\frac{6}{9} = 1\frac{2}{3}$$

Students should realize that the term $\frac{3 \times 5}{9}$ in this example is simply a way to show the calculation process and that fractions should normally be expressed with a single whole number in both the numerator and the denominator.

Initially, students will simplify the product when it is not in simplest form, as in the previous example. They will then learn that they can simplify before they multiply since the whole number becomes a factor of the numerator, and simplifying a fraction involves dividing the numerator and denominator by common factors. We can record the process by crossing out the numerator and denominator and writing the new numerator and denominator next to them. (Do not call this cancellation, as this process is sometimes called, since that term is misleading.)

$$3 \times \frac{5}{9} = \frac{\overset{1}{\cancel{3}} \times 5}{\underset{3}{\cancel{9}}} = \frac{5}{3} = 1\frac{2}{3}$$

We can bypass writing the intermediate expression:

$$\overset{1}{\cancel{3}} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{5}{3} = 1\frac{2}{3}$$

Your student has to thoroughly understand the process and the concepts behind these shortcut methods of keeping track of the calculations first so that they do not make errors. They should always write 1 when the simplification gives a 1 in the numerator, and not simply cross out the number.

Multiplying a Whole Number by a Fraction.

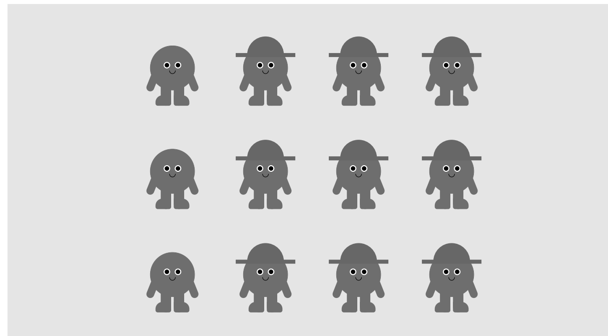
Multiplying a whole number by a fraction means that the fraction represents the number of "groups" and the whole number represents the value of the unit. For example $\frac{3}{4}$ of 1 hour is $\frac{3}{4}$ of 60 minutes, which is 45 minutes. $\frac{3}{4}$ is the number of equal groups (which is less than a whole group since it is a proper fraction) and 60 minutes is the quantity in the group. We can call this finding the fraction of a "set." Generally, we write $\frac{3}{4}$ of 60 as $\frac{3}{4} \times 60$ and the symbol can be interpreted as "times."

Students should not generalize this to thinking that the term "of" always represents multiplication. For example, 9 out of 12 apples, or "9 of 12," is not 9 × 12 apples.

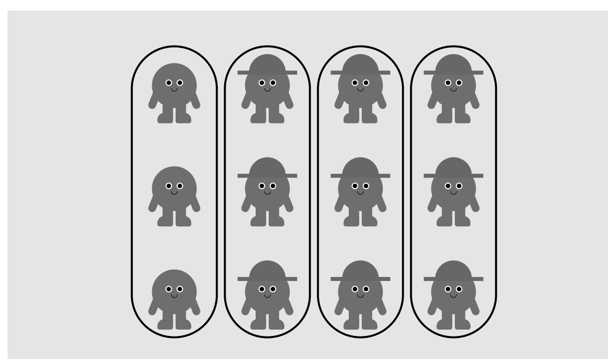
This interpretation of the expression $\frac{3}{4} \times 60$ is not the same as adding $\frac{3}{4}$ sixty times. If the fraction is a proper fraction, the product will be less than the other factor, and division is involved in finding the answer. This will be an important concept with many word problems. With both interpretations, as in the case when both factors are whole numbers, the answer is the same and the general method of calculation can be done in the same way: multiply the whole number and the numerator of the fraction.

Students already have some intuitive grasp of how to find a fraction of a set. For example, they know that a quarter hour ($\frac{1}{4}$ of an hour) is 15 minutes, that $60 \div 4 = 15$, and that $\frac{3}{4}$ of a hour is 3 × 15 minutes.

In Lesson 4, students will first find one quantity as a fraction of the total. For example, in this set of 12 toys, 9 toys have hats.



Since 9 out of 12 toys have hats, the fraction of toys with hats is $\frac{9}{12}$. $\frac{9}{12}$ can be simplified using a common factor of 3. We can group the toys by 3 and have equal groups where all the toys in each group are the same (with or without hats). Taking 3 toys as a unit, we can say that $\frac{3}{4}$ of the toys have hats.



Note that if we simply say, "$\frac{3}{4}$ of the toys have hats," we do not know the actual number of toys that have hats without knowing the total. In Lesson 5, students will learn to calculate the value of the fractional part of a set, given the fraction and the total number in the set. For example: “There are 12 toys. $\frac{3}{4}$ of the toys have hats. How many toys have hats?” We can interpret the denominator of the fraction as the quantity in each equal unit and the numerator as the number of units. So we first divide to find the quantity in one unit and then multiply by the total number of units.

$\frac{1}{4}$ of 12 $\longrightarrow \frac{12}{4}$

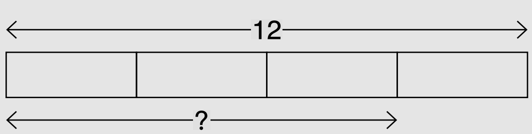
$\frac{3}{4}$ of 12 $\longrightarrow 3 \times \frac{12}{4} = 3 \times 3 = 9$

We get the same answer if we multiply the whole number by the numerator of the fraction first and then divide, simplifying before or after the calculations:

$\frac{3}{4}$ of 12 $\longrightarrow 3 \times \frac{12}{4} = \frac{3 \times 12}{4} = \frac{36}{4} = 9$

$\frac{3}{4}$ of 12 $\longrightarrow 3 \times \frac{12}{4} = \frac{3 \times \cancel{12}^{3}}{\cancel{4}_{1}} = 9$

We can use a bar model to represent the problem, which makes it easy to see that we can divide to find the value of a unit and multiply to find the value of 3 units.

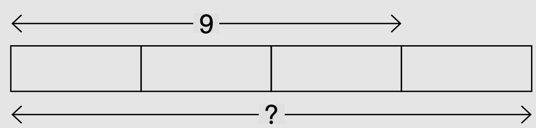


4 units $\longrightarrow$ 12

1 unit $\longrightarrow \frac{12}{4} = 3$

3 units $\longrightarrow 3 \times 3 = 9$

The steps for solving the problem are similar to solving a problem such as, “There are 12 toys in 4 boxes. How many toys are in 3 boxes?” except that the number of units is determined by the denominator of the fraction. Use of bar models for both division and fraction problems further cements the relationship between division and fractions. It also allows students to solve problems that would ordinarily be solved by dividing by a fraction, and will give them an intuitive understanding of why when we divide by a fraction we “invert and multiply,” which they will learn in Grades 5 and 6. For example, “9 of the toys have hats, which is $\frac{3}{4}$ of the toys. How many toys have hats?”



$\frac{3}{4}$ of the toys $\longrightarrow$ 9

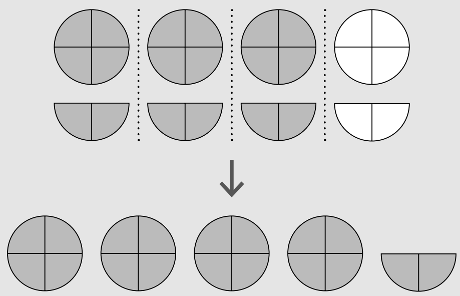
$\frac{1}{4}$ of the toys $\longrightarrow \frac{9}{3} = 3$

$\frac{4}{4}$ of the toys $\longrightarrow 4 \times 3 = 12$

Students will learn later that since we need to find a total, we could solve the problem with division: $9 \div \frac{3}{4} = 9 \times \frac{4}{3}$.
In the above example, we are dividing by 3 and then multiplying by 4, which is the same as multiplying by $\frac{4}{3}$ (the inverse of $\frac{3}{4}$).

We can follow the same reasoning used for finding a fraction of a set when the answer is a whole number in order to find the fraction of a set when the answer is a fraction or mixed number.

For example: “There are 6 pizzas. $\frac{3}{4}$ of the pizzas were eaten. How many pizzas were eaten?”



$\frac{1}{4}$ of 6 $\longrightarrow \frac{6}{4}$

$\frac{3}{4}$ of 6 $\longrightarrow 3 \times \frac{6}{4} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}$

Or:

$$3 \times \frac{6}{4} = \frac{3 \times \overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}} = \frac{9}{2} = 4\frac{1}{2}$$

In the first method, we could simplify the value for $\frac{1}{4}$ of 6 as $\frac{3}{2}$ before multiplying by 3, but it does not make sense to express it as a mixed number. Students did not multiply by a fraction greater than 1 in the lessons on multiplying a fraction by a whole number, but should be able to understand the process with the help of the pictures. The picture will show all the wholes divided into fourths to help students understand that 3 times 6 fourths, or 18 fourths, of the pizzas were eaten.

Games

- After Lesson 4
  Materials: 24 two-color counters (or coins), cup or box
  Purpose: Express a partial quantity as a fraction of the total.
  Goal: Get the least number.
  Procedure: Decide which color (or whether heads or tails) will be the part. Players take turns putting the counters in a cup, shaking them, pouring them out, and determining what fraction of the total is that color. The numerator is their score. For example: The two colors on the counters are yellow and red, and red is the part. A player got 10 red and 14 yellow. The red counters are $\frac{10}{24}$, or $\frac{5}{12}$, of the total. Since the lowest score wins, they want to put the fraction in simplest form when possible, so in this case they should record their score as 5. At the end of the game, players total their score. The one with the lowest score wins.
  Variation: Use 60 counters instead, or 12.
  Variation: This can be an activity for practicing finding the fraction of the set instead of a game.

Lesson 2 Multiplying a Fraction by a Whole Number — Part 1 (pp. 185–187)

Think (p. 185)

Have your student read the problem, write a multiplication expression, and find the answer. Let them use fraction manipulatives if needed. They may realize that as before, they can simply multiply the whole number by the numerator of the fraction.

Learn (p. 185)

The two bars are fraction bars; the total length represents 1 whole. The alternating light and dark colors represent the value being multiplied by 3 ($\frac{2}{7}$) and show why we can multiply the whole number by the numerator to find the answer (which is sevenths), as shown in Method 1. Method 2 relates multiplying a non-unit fraction to multiplying a unit fraction and is just another way to think of the process.

Do (pp. 186–187)

Discuss the first four problems as needed, relating the pictorial representations (fraction bars, fraction circles, number lines and ruler) to the multiplication process.

5 Students are not required to write the intermediate step showing a multiplication expression in the numerator.

Answers

$\frac{6}{7}$

$\frac{6}{7}$

$\frac{6}{7}$

1 $\frac{8}{9}$

2 $= \frac{5 \times 2}{3} = \frac{10}{3} = 3\frac{1}{3}$

3 $= \frac{4 \times 3}{4} = \frac{12}{4} = 3$

4 $= \frac{5 \times 7}{8} = \frac{35}{8} = 4\frac{3}{8}$

5 (a) $\frac{4}{5}$ (b) $\frac{6}{7}$

(c) $\frac{2}{3}$ (d) $3\frac{3}{4}$

(e) 1 (f) $7\frac{1}{2}$

(g) $4\frac{1}{2}$ (h) $2\frac{2}{5}$

6 $2\frac{1}{4}$ lb

$3 \times \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$