## Lesson 7 Divide Decimals by 10, 100, and 1,000

## Objective

- Divide decimals by 10,100 , or 1,000 .

Lesson 7
Divide Decimals by 10,100 , and 1,000

## (7)

## Think

(a) 231 kg of flour is divided equally into 10 containers. How many kilograms of flour are in each container?
(b) 231 kg of coffee is divided equally into 100 bags. How many kilograms of coffee are in each bag?
(c) 231 kg of cinnamon is divided equally into 1,000 bottles. How many kilograms of cinnamon are in each bottle?

## Think

Provide students with place-value discs and pose the Think problems.

Discuss the strategies students used to find their answers.

## Learn

Have students study the discs in (a) and compare values of the discs in the two groups in (a).

Ask students:

- "How do the discs change?" (Each individual disc is replaced by a disc with a value that is one tenth as much.)
- "Whathappens to each digit in the product?" (Each digit now has a value $\frac{1}{10}$ as much as before.)

Looking at the discs in (b), ask students:

- "How does the third group in (b) change compared to the second group?" (Each individual disc is replaced by a disc that is $\frac{1}{10}$ as much and then that disc is replaced by a disc that is $\frac{1}{10}$ as much.)

Looking at the discs in (c), ask students:

- "How does the fourth group in (c) change compared to the third group?" (Each individual disc is replaced three times. The value of each digit is divided by 1,000 .)

Dividing by 1,000 is the same as dividing by 10 three times.

Discuss Mei's comment.
Students should see that each time we divide a number by ten, the value of each digit becomes $\frac{1}{10}$ as much, so each digit, when written in a column on the place-value chart, moves one place to the right. Because the decimal point is always between the ones and tenths, it appears to move one place to the left when the number is divided by 10.

## Do

(1)-3 Discuss the problems with students.
(2) Encourage students to study the table. Ask them:

- "What happens to the digits when we divide the number by 10,100 , or 1,000 ?" (They move to the right on the chart.)
- "Where should the decimal point be in the answer?" (Between the ones and tenths.)
(c) Students can see from the chart that they may have to append zeros to the ones place as well as additional places, depending on how many places the leftmost digit moves to the right (or the decimal point moves to the left).
$65 \div 1,000=0.065$

$231 \div 10 \div 10 \div 10=231 \div 1,000=0.231$
There are 0.231 kg of cinnamon in each bottle.

$231 \div 10 \rightarrow 23.1$
$231 \div 100 \rightarrow 2 \sim^{3} \sim 2.31$
$231 \div 1,000 \rightarrow \sim^{2} \sim^{3} \rightarrow 0.231$

(2)

(a) $65 \div 10=6.5$
(b) $65 \div 100=0.65$
(c) $65 \div 1,000=0.065$
(d) $6.5 \div 10=0.65$
(e) $6.5 \div 100=0.065$
(f) $0.65 \div 10=0.065$
(3) Students should be able to use their knowledge of place value to determine the placement of the decimal point.
(b) It may help students to think of 6.2 as 62 tenths.
$6.2 \div 100=\frac{62}{10} \div 100$
$=\frac{62}{10} \times \frac{1}{100}$
$=\frac{62}{1,000}$
$=0.062$
Sofia shows that if we think of moving the decimal point two places to the left, we will need to include a zero in the tenths place.
(c) It may help students to think of $91 \div 1,000$
$=\frac{91}{1,000}=0.091$.
Dion shows that if we think of moving the decimal point three places to the left, we will need to include a zero in the tenths place.
(4)-5 Students should be able to solve the problems independently.


## Activity

(a) $1.06 \div 10$
d) $63.5 \div 100$
0.635
(g) $2.1 \div 100$ 0.021
(b) $2.3 \div 10$ 0.23
(e) $304.6 \div 100$
3.046
(h) $80 \div 1,000$
0.08
(b) $6.2 \div 100=0.062$

(c) $0.9 \div 100$ 0.009
(f) $7 \div 1,000$
0.007
(i) $300 \div 1,000$
$36 \div 100=\frac{36}{100}=0.36$
(a) Explain why Mei's method works

We can write the division by 100 as a fraction over 100 and then covert to a decimal.
(b) Use this method to divide 36 by 1,000
$36 \div 1,000=\frac{36}{1,000}=0.036$
(c) Use this method to divide 36 by 10 . $36 \div 10=\frac{36}{10}=3 \frac{6}{10}=3.6$

## Exercise 7 - page 21

## Objective

- Multiply a decimal by a decimal with products up to three decimal places.


## Think

Pose the Think problem and discuss the bar model shown. Have students write a multiplication expression, and then try to solve it.

Discuss the methods students used to find their answers.

## Learn

Discuss the three methods shown in the textbook.

## Method 1

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| U |
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## 0

$$
0
$$

Sofia estimates her answer.
Dion thinks about the decimals as fractional units. When we multiply fractions, we multiply the numerators and the denominators together. In this problem, the denominator in the product is 100 , and we can express the answer as a decimal in hundredths.

## Method 2

Emma thinks in terms of units of tenths as well, and multiplies 25 tenths by 9 tenths by first multiplying 25 by 9 .

The answer is in hundredths because 1 tenth of 1 tenth is 1 hundredth. We have 225 hundredths, or 2.25.

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Lesson 4
Multiplying by a Decimal
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## Method 3

Methods 1 and 2 help lead to an understanding of why Method 3 , the standard algorithm, works. Similar to Method 2 , we can multiply the whole numbers first, and then place the decimal point in the product.

Since both numbers are one-place decimals, Mei multiplies both factors by 10 in order to calculate with whole numbers.

The answer to $25 \times 9$, will be $10 \times 10$, or 100 times greater than it should be for $2.5 \times 0.9$. The product has to be divided by 100 to get the correct answer.

The calculation below Mei shows that we can write the algorithm with the decimal places, multiply as with whole numbers, and then place the decimal point two places to the left. (We do not have to rewrite the problem).

Discuss Alex's comment. Ensure that students understand that we are multiplying tenths by tenths to get hundredths, which is a two-place decimal. The product should be a two-place decimal.

Have students compare their solutions from Think with the ones shown in the textbook.

Students may have also solved the word problem using a unitary approach involving division.

1 unit $\rightarrow 2.5 \div 10=0.25 \mathrm{~kg}$
9 units $\underset{\substack{0}}{\underset{0}{2}} 0.25 \times 9=2.25 \mathrm{~kg}$


Method 3

2.5
0.9
$\times 2.25$
WU
0.9 m of pipe weighs 2.25 kg .

Multiply decimals the same way as whole numbers. Place a decimal point in the product according to the number of decimal places being multiplied.

$$
2.5 \leftarrow 1 \text { decimal place }
$$

$\times 0.9 \leftarrow 1$ decimal place
$2.25 \leftarrow 2$ decimal places

## Do

(1)-4 Discuss the problems and the friends' thoughts with students.
(1) In each case, students can see that the placevalue discs show the numerical calculation from the methods in Learn.

We can multiply in the same way that we multiply whole numbers, but in ( $a$ ), one factor is tenths, so the answer is in tenths: 68 tenths. In (b), one factor is hundredths so the answer is in hundredths: 68 hundredths.
(a) $0.2=2 \times 0.1$

To calculate, find $34 \times 2 \times 0.1=68 \times 0.1=6.8$.
(b) $0.02=2 \times 0.01$

To calculate, find $34 \times 2 \times 0.01=68 \times 0.01=0.68$.
(2) Students can relate the bar model and unitary method to the multiplication expression.

10 units $\longrightarrow 1.5$
1 unit $\longrightarrow 1.5 \div 10=0.15$
4 units $\longrightarrow 4 \times 0.15$
$4 \times 0.15=\frac{15}{100} \times 4=\frac{60}{100}$, so this gives the same answer as the method shown in the textbook.

Emma shows a second method. She finds the answer by finding theproduct of multiplying the whole numbers first by the product of the decimal:
$(15 \times 4) \times(0.1 \times 0.1)=(15 \times 4) \times 0.01$
(b) To find 4 hundredths of 1.5 , we could first find 1 hundredth of 1.5 , then multiply that by 4 .

1 unit $\longrightarrow 1.5 \div 100=0.015$
4 units $\longrightarrow 4 \times 0.015$
This gives the same answer as the method shown in the textbook.


## Lesson 2 Equivalent Ratios

## Objectives

- Find equivalent ratios.
- Express ratios in simplest form.


## Lesson Materials

- 20 two-color counters for each student or pair of students


## Think

Discuss the Think question and provide students with two-color counters. They can use one color to represent the cups of lemonade and the other color to represent the cups of unsweetened iced tea.

Students should realize that we need to double each item to double the servings, or halve each ingredient to halve the servings.

Discuss student solutions.

## Learn

Have students compare their solutions from Think with the ones shown in the textbook.

We can write the ratios using the total number of cups. The ratio of $4: 6$ becomes $8: 12$ if we double the recipe.
(b) How can she make 5 servings of this recipe?

## Learn

(a) To make 20 servings she needs to double the amount of each ingredient. The ratio of lemonade to iced tea will be $8: 12$.


What will be the ratio of cups of lemonade to cups of iced tea for each mixture?


Mei is making the Lemon Tea
Lemon Tea
-Lemonade
-Unsweetened iced tea
Mix 4 cups of lemonade with 6 cups
of unsweetened iced tea. Pour over
crushed ice and serve.
Makes 10 one-cup servings.
(a) How can she make 20 servings of this recipe?


The ratio of $4: 6$ becomes $2: 3$ if we halve the recipe. In both cases, there are still 2 units of lemonade for 3 units of iced tea.
$2: 3,4: 6$, and $8: 12$ all represent the same relationship between the two quantities, so they are equivalent ratios.

The ratio that can be expressed with the least number for both quantities is the simplest form of the ratio.

Students should see that the process of finding equivalent ratios is the same as the process of finding equivalent fractions. For ratios, we multiply or divide both terms by the same number, for fractions we multiply or divide the numerator and denominator by the same number.

The simplest form of a ratio is when both terms have no common factor other than 1 . The simplest form of a fraction is when both the numerator and denominator have no common factor other than 1.
(b) To make 5 servings she needs to halve the amount of each ingredient. The ratio of lemonade to iced tea will be $2: 3$.

$2: 3,4: 6$, and $8: 12$ are equivalent ratios.


In each ratio there are 2 units of lemonade for every 3 units of iced tea.

To make equivalent ratios, multiply or divide both terms by the same number. The simplest form of the ratios $8: 12$ and $4: 6$ is $2: 3$.

A ratio is in its simplest form when each term has no common factor other than 1.


Do
(1)-4 Discuss the problems with students. Students can use counters if needed for 1 .
(1) Students may also see that there are 2 red counters for every 3 yellow counters:

(2) (a) We are given both first terms. Since $12=3 \times 4$, both of the terms of the first ratio should be multiplied by 3 to find the equivalent ratio.
(b) We are given both second terms. Since $8=16 \div 2$, both of the terms of the first ratio should be divided by 2 to find the equivalent ratio.

## o

(1) Use 6 red od 9 y yelow counters.
(a) What is the ratio of the total number of red counters to the total number of yellow counters? 6:9
(b) Make groups of 3. Each group should have the same color counters.


What is the ratio of groups of red counters to groups of yellow counters? 2:3
(c) What is the simplest form of the ratio $6: 9$ ? $2: 3$
(2) Find the equivalent ratios.
(a) $3: 4=12: 16$
(b) $12: 16=6: 8$


4 Sofia uses the common factor of 2 to divide the terms and get an equivalent ratio, then she sees that she can also divide the terms of the equivalent ratio by 7 to express the ratio in simplest form.

As with fractions, we can find the simplest form in one step or in more than one step.
(5)-6 Students should be able to solve these problems independently.

## Activity

## How Many?

Materials: Number Cards (BLM) 0-9
Have students draw a personal game board, as shown below, on paper or a dry erase board.


Using each card once in each pair of equivalent ratios, ask students how many pairs of equivalent ratios they can find.

Sample solutions:
$1: 3=8: 24$
$1: 4=7: 28$

Express the ratio 20 : 16 in simplest form.


Express the ratio $56: 42$ in simplest form.


5
Find the missing terms.
(a) $1: 3=7: 21$
(b) $3: 4=18: 24$
(c) $5: 2=15: 6$
(d) $4: 5=36: 45$
(6) Write each ratio in simplest form.
(a) $10: 5 \quad 2: 1$
(b) $40: 8 \quad 5: 1$
(c) $9: 12 \quad 3: 4$
(d) $24: 36 \quad 2: 3$


## Exercise 2•page 137

## Lesson 5 Word Problems

## Objective

- Solve multi-step word problems involving ratios of up to 3 quantities.


## Think

Pose the Think problem and ask students to draw bar models to help solve the problem.

Discuss student solutions.

## Learn

Have students compare their solutions from Think with the one shown in the textbook.

From the model, students can see that the difference between non-fiction and fiction is 3 units, which is 189 books. They need to find the total number of units. To find the total number of units, they can start by finding the value of 1 unit.

## Do

$\stackrel{c}{\stackrel{5}{5}}$
(1)-3 Discuss the problems with students.
(1) There are $4+5+3$ total units.


> Lesson 5
> Word Problems

## (5)



A bookstore sells fiction books and non-fiction books. The ratio of non-fiction books to fiction books is $7: 4$. There are 189 more non-fiction books than fiction books. How many books are there altogether?

Learn


There are 693 books altogether

Do
(1) The ratio of Daniel's to Yusuf's to Liam's savings is $4: 5: 3$. Their total savings is $\$ 540$. How much money did Liam save?

(2)

The ratio of silver-colored coins to copper-colored coins in Esther's coin collection is $4: 1$. There are 54 fewer copper-colored coins than silvercolored coins. How many silver-colored coins are in her collection?


4-7 Students should be able to solve these problems independently.


## Exercise $5^{\circ}$ page 146



The ratio of boys to girls in a swim club is $3: 7$. There are 84 more girls than boys. How many children are in the swim club?


The ratio of the weights of Mom's packed suitcase to Dad's packed suitcase to Junior's packed suitcase is $5: 3: 4$. Mom's suitcase weighs 15 lb more than Dad's suitcase. How much does Junior's suitcase weigh?

(5) The ratio of the lengths of Ribbon $A$ to Ribbon $B$ to Ribbon $C$ is $2: 5: 1$ Ribbons $A$ and $B$ are 84 cm long altogether. What is the total combined length of the three ribbons?

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The ratio of the price of an adult ticket to the price of a child ticket at an amusement park is $4: 3$. Mr. Smith paid $\$ 295.75$ for 1 adult ticket and 3 child tickets. What is the price of each child ticket?


A florist has gerberas and sunflowers in the ratio of $1: 3$. She has $\frac{1}{2}$ as many carnations as sunflowers. If she has 60 carnations, how many gerberas does she have? $\quad 3$ units $\longrightarrow 60$

2 units $\rightarrow 2 \times 20=40$

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                        4 0 \text { gerbero}
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                                13-5 Word Problems
    
## Lesson 2 Rate Problems - Part 1

## Objective

- Find the total amount given a non-unit rate.

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Lesson 2
Rate Problems - Part 1
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Think
Jessica worked for 25 hours one week and was paid $\$ 560$. At this rate, how much would she earn for working 40 hours?


What is her hourly rate of pay?


She would earn \$ 896 .

He can simplify the expression, making the calculations easier.

Students may leave out the middle step when showing their calculations:

$$
\begin{aligned}
& 25 \mathrm{~h} \rightarrow \$ 560 \\
& 40 \mathrm{~h} \rightarrow \$ \frac{560}{25} \times 40=\$ 896
\end{aligned}
$$

## Do

(1)-2 Discuss the problems with students. In each of these problems, the steps in the solution ask students to find the unit rate first. While it is not necessary to calculate the value of the unit rate first, students should understand that this is what the division expression or fraction represents. Students who are struggling may find it easier to find the unit rate first.
(1) The unit rate is 25 words per minute. If students

Do
Dion can type 50 words in 2 minutes. At this rate, how many words can he type in 15 minutes?

(2) Mrs. Chen's hybrid car uses 5 gallons of gas to travel 205 miles. The capacity of the gas tank in her car is 17.4 gallons. How far can she travel on a full tank of gas?


Students could also show their calculations with a combined expression:
$2 \mathrm{~min} \longrightarrow 50$ words
15 min $\longrightarrow \frac{50}{2} \times 15$
(2) The unit rate is 41 miles per gallon.

Students could also show their calculations with a combined expression:

5 gal $\longrightarrow 205$ miles
$17.4 \mathrm{gal} \longrightarrow \frac{205}{5} \times 17.4$
(3) To convert from seconds to minutes, multiply by 60 .

The steps in the solution do not require finding the unit rate first. Students should see that the problem is easy to calculate by simplifying the expression: $60 \times \frac{19}{15}=4 \times 19$.

If we were to find a unit rate first, we would get a repeating decimal, $1.2666 \ldots$... which students do not yet know how to multiply by 60 .

Some students may know that $4 \times 15=60$ and then calculate $4 \times 19$ to find the number of beats in 60 seconds or 1 minute:
$15 \mathrm{~s} \rightarrow 19$ beats
$60 \mathrm{~s} \rightarrow 4 \times 19=76$ beats

(4)-(5) Students should be able to complete these problems independently.

Students may also solve these as follows:
(4) $60 \mathrm{~s} \longrightarrow 420$ caps
$40 \mathrm{~s} \rightarrow \frac{420}{60} \times 40$
(5) $40 \mathrm{~h} \rightarrow \$ 750$
$35 \mathrm{~h} \rightarrow \$ \frac{750}{40} \times 35$
They maysimplify the expression in various ways.

## Exercise2 - page 158

