## Lesson 3 Factors

## Objectives

- Find factors of whole numbers up to 100.
- Determine if a one-digit number is a factor of another number.


## Lesson Materials

- Graph Paper (BLM)
- Square tiles, 12 for each student


## Think

Provide students with square tiles and Graph Paper (BLM). Have them record the different rectangles they can make with the tiles on the Graph Paper.

Discuss student strategies for solving the problem.

## Learn

Have students compare their solutions from Think with the ones shown in the textbook. Introduce the term "factor": any whole number can be expressed as the product of two or more factors.

When students count how many tiles are on each side, they can multiply the sides together and get the total number of squares. The total is a multiple of both those side numbers.

Students should see that if all of the equations are listed, the factors are also listed. Listing the factors helps students keep their thoughts organized.
$1 \times 12$
$2 \times 6$
$3 \times 4$
Note that factors are typically listed from least to greatest: 1, 2, 3, 4, 6, 12.


Help students relate multiples and factors:

- 12 is a multiple of each factor of 12 . For example, 1 is a factor of 12 , and 12 is a multiple of 1 .
- 4 is a factor of 12 , and 12 is a multiple of 4 .

Discuss Emma's comment on divisibility.

## Do

(2) Students may need to systematically list factors. In this problem, the factors 1,2, and 3 are already given and students find their factor pairs.

Note that here and in (5), factors are found as their pairs and then listed from least to greatest.

Discuss Emma's question. Students should see that all factors greater than 6 have already been found. They only need to check numbers between 3 and 6:

- We know that $4 \times 4$ is 16 , and 18 is only 2 more than 16 , so 4 is a not a factor of 18 .
- The number 18 does not end in 0 or 5 , so we know 5 is not a factor of 18 .

This may be fairly easy to see with the number 18 , however, it will become helpful when finding all the factors of greater numbers, like 60, which has 12 factors.


4 Students can check to see if the numbers are divisible by 6 (i.e. $16 \div 6=2$ remainder 4 ), or they can list the multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60.

Students should know that 6 is a factor of 6.
(5) Mei reminds us that a multiple of an even number is always even. We do not need to check to see if 6,8 , 10,12 , or 14 are factors of 75 . Numbers greater than 15 do not need to be checked since they have already been found.

If this is confusing, have students consider the products of one odd and one even number. For example, $3 \times 2$ is $6,5 \times 2$ is 10 , etc. In each case, even though one of the factors is odd, the product is even.

6 Students may need to list the factors as pairs first, then make a list of the factors from least to greatest.

## Activity

## A Factor Game

Materials: Numbers to 40 Chart - 1 Start (BLM) in dry-erase sleeve, markers

On each turn, players choose and cross off a number on the Numbers to 40 Chart - 1 Start (BLM). That number is the player's score for that round.

After Player One crosses off a number, Player Two then marks all factors of that number and adds them together to get her score for the round.

Example: Player One (red) chooses 21 and records that score. Player Two (blue) finds all of the remaining factors of $21(1,3,7)$ and crosses them off the game board. She adds them to get her score for the round (11).


## ©

List the factors of each number from least to greatest.

(a) $6 \times 12=72$
(b) $15 \times 7=105$
(c) $100=4 \times 25$
(d) $80=16 \times 5$



On the next round, Player Two chooses 29 and adds that to her score of 11 .

Player One finds the factors of 29: 1, 29. Both 1 and 29 are already crossed off, so she scores 0 for this round.

Play ends when there are no more numbers to cross off.

## Exercise 3 • page 53

## Lesson 5 Multiplying by a Multiple of 10

## Objective

- Multiply a two-digit number by a two-digit multiple of 10.


## Lesson Materials

- Place-value discs, ten each of the values 1, 10, 100


## Think

Pose the Think problem. Students should think about how to multiply 34 by 2 tens. Have students think about how they could show $34 \times 20$ with only the discs provided. They do not have enough discs to make 20 groups of 34 .

If students finish quickly, challenge them to solve the problem in more than one way.

Discuss student strategies.

## Learn

Discuss the methods shown in Learn and have students compare their own methods with the methods shown in the textbook.

Sofia thinks of 20 as $10 \times 2$. She multiplies 34 by 10 first and then by 2.

To demonstrate if needed:

- $34 \times 10$ would be 30 ten discs and 40 one discs, which can be regrouped to 3 hundred discs and 4 ten discs.
- 3 hundred discs and 4 ten discs multiplied by 2 is 6 hundred discs and 8 ten discs.


There are 34 ping pong balls in one box. How many ping pong balls are in 20 boxes?

Learn
$34 \times 20$ Method 1

$34 \times 20=34 \times 10 \times 2$
$=340 \times 2$


Alex thinks of 20 as $2 \times 10$. He multiplies 34 first by 2 and then by 10 .

Demonstrate if needed:


Mei thinks that since the number 20 is 2 tens, she can multiply 34 by 2 . She relates that idea to the standard algorithm.

Because 20 is 2 tens, and any single-digit number of tens ends in one zero, a short cut is to multiply 34 by 2 and then append a zero.

Model the language:
" $34 \times 2$ tens is 68 tens, so we can write a 0 in the ones column, and write 8 tens and 6 hundreds in the appropriate columns."

## Do

(1) Students may notice they can just multiply $6 \times 7$ and append two zeros to the product. Explain to them why this works:
$60 \times 70=6 \times 10 \times 7 \times 10$
Since we can multiply in any order, we can change the order of the numbers in the equation:
$6 \times 7 \times 10 \times 10$
Next, multiply the numbers together to make an easier problem:
$42 \times 100=4,200$
4 These problems build on the idea that anytime we multiply a whole number by a whole number with a 0 in the ones place, the product will also have a 0 in the ones place.

We can write a 0 in the ones place in the product and then multiply by the digit in the tens place.

## Lesson 5 Word Problems

## Objective

- Solve multi-step word problems involving multiplicative comparison.


## Think

Pose the Think problem and discuss Alex's questions. Students should think about which of the friends' number of pieces of trash could be used as the unit. If needed, have students redraw the model so that they can mark the model up further.

Prompt students by asking:

- "What information is given on the model?"
- "How can we make the bars for each friend's number of pieces of trash the same length?"
- "Which bar could we consider as the unit?"

Discuss student solutions.


## Learn

Discuss the methods shown in Learn and have students compare their own methods with the methods shown in the textbook. By making all of the bars the same length, each bar is an equal unit, and the value of each unit can be found using division, since the total is known.

## Method 1

If Emma had found 230 more and Sofia had found 110 less, the three friends would all have the same amount, and there would be 3 equal units. One of those units represents the number of Alex's pieces of trash.

If Alex's bar represents the unit:
Emma $=$ Alex -230
Sofia $=$ Alex +110
Alex + Emma + Sofia $=3$ units
3 units $\rightarrow 3,045+230-110=3,165$
By dividing by 3 , solve for 1 unit and find the number of Alex's pieces of trash.

## Method 2

If Alex had found 230 less and Sofia had found $110+230$ less, the three friends would all have the same amount, and there would be 3 equal units.

One of those units represents the number of Emma's pieces of trash.

If Emma's bar represents the unit:
Alex $=$ Emma +230
Sofia $=$ Emma $+230+110$
Emma + Alex + Sofia $=3$ units
3 units $\rightarrow 3,045-230-230-110=2,475$
We subtract to find the value of 3 units, or 2,475. By dividing by 3 , we get the number of Emma's pieces of trash.

Add the number of Emma's pieces of trash (825) and 230 together to get the number of Alex's pieces of trash.

## Do

(1) By first subtracting the amount of cards Dion kept for himself, students can find the value of the remaining 5 equal units and solve for 1 unit.
(2) 1 unit is the cost of one chair.

6 units +1 unit $+284=1,299$
7 units $\longrightarrow 1,299-284=1,015$
(3) If the cost of the table lamp is considered one unit, then the cost of the two floor lamps are each 1 unit

To make the cost of floor lamps equal to a unit, we add 70 to each floor lamp, which adds $\$ 140$ to the total amount Mr. Lopez spent, which is:

3 units $+(1$ unit +70$)+(1$ unit +70$)$
5 units $\rightarrow 505+140=645$
Divide the value of 5 units, or 645 , by 5 to find the value of each table lamp unit:

5 units $\rightarrow 645$
1 unit $\longrightarrow 645 \div 5=129$
To find the total cost of three table lamps:
1 unit $\longrightarrow 129$
3 units $\rightarrow 129 \times 3=387$
Note that students could solve the problem similarly to
Method $\mathbf{2}$ in Learn and use one floor lamp as the unit.
5 units $\rightarrow 505-(3 \times 70)=295$
1 unit $\rightarrow 295 \div 5=59$
One floor lamp costs $\$ 59$.
One table lamp costs \$129 (59 + $70=129$ ).
Three table lamps cost $\$ 387(3 \times 129=387)$.
4. If students are confused about how the units were found, have them redraw the model step by step.

When redrawing the bar model, represent the number of books in the first and second boxes. The first box has 25 more books than the second box.

Add another bar representing the number of books in the third box. Students should think about what "twice as many" means in relation to the third box. That is, the bar for the third box is half as long as the second box:


5 equal units +25 books is 430 books in all. Subtract 25 from the total of 430 to find the value of 5 units.

5 units $\longrightarrow 430-25=405$
1 unit $\longrightarrow 405 \div 5=81$
2 units $\longrightarrow 2 \times 81=162$
This thinking will help students interpret 5 as well.
(5) 5 equal units +75 jars is 980 jars of jam in all. Subtract 75 from 980 to find the value of 5 equal units.

5 units $\longrightarrow 980-75=905$
1 unit $\longrightarrow 905 \div 5=181$
3 units $\longrightarrow 3 \times 181=543$
6 Suggest students draw the second number as twice the length of the first number:



A vendor at the market is selling jam. She has 980 jars in total of huckleberry, blueberry, and peach jam. She has 75 fewer jars of blueberry jam than huckleberry jam. She has three times as many jars of peach jam than blueberry jam. How many jars of peach jam does she have?


$$
181 \times 3=543 ; 543 \text { jars of peach jam }
$$

The sum of two numbers is 3,750 . The second number is 180 less than twice the first number. What are the two numbers?

They can then show the 180 less than twice the first number on the model:


Add 180 to 3,750 to find the value of 3 equal units.
3 units $\longrightarrow 3,750+180=3,930$
1 unit $\longrightarrow 3,930 \div 3=1,310$
2 units $\rightarrow 1,310 \times 2=2,620$
Second number: $2,620-180=2,440$

Exercise 5 • page 109

## Lesson 2 Adding and Subtracting Fractions - Part 2

## Objective

- Add and subtract fractions with related denominators.


## Lesson Materials

- Fraction manipulatives


## Think

Provide students with fraction manipulatives and pose the Think problem. Have students show or write their solutions to the questions.

Students should see that halves and fourths are different sized units. In order to add, we need to make equal fractional units:


## Learn

Discuss Mei's comment.
To add the fractions, we need to use the same units. We can express $\frac{1}{2}$ as an equivalent fraction with a denominator of fourths: $\frac{2}{4}$.

Now that both fractions are expressed in the same unit, fourths, they can be added.

Have students compare their solution from Think with the one in the textbook.


## Do

(1) The two top bars show $\frac{1}{5}$ is the same size as $\frac{2}{10}$. The bottom bar shows that we can now add the equal sized units.

Finally, the answer, $\frac{5}{10}$, is simplified to $\frac{1}{2}$. Students might see that the denominator of the final, simplified answer is not the same as the denominator of either of the original fractions being added.
(2) The top bars show that $\frac{1}{2}$ is the same size as $\frac{3}{6}$. The bottom bar shows the result when $\frac{3}{6}$ is taken away from $\frac{5}{6}$.

Students should understand that the 3 boxes that are crossed out are equivalent to the $\frac{1}{2}$ that is subtracted.

Dion reminds students to make equal units.
(3) - 4) Students first need to find an equivalent fraction. The improper fractions are added or subtracted and then simplified. In (3), the final answer is converted to a mixed number.
(5) Have students share how they solved some of these problems, specifically ( $h$ ) and ( $i$ ), which are new concepts.

Extend by asking students how they would solve a problem with three different denominators such as: $\frac{5}{12}+\frac{1}{3}+\frac{1}{2}$.
(6) Have students draw a bar model, as needed, to compare the numbers and find the difference.


Exercise 2 - page 154

$\frac{5}{6}-\frac{1}{2}=\frac{5}{6}-\frac{13}{6}=\frac{2}{6}=\frac{11}{6} \quad$ To add or subtroct troctions, we need to have equal units.


## $166 \quad$ 7.2 Adding and Subltrating Froctions - Port 2


(5) Add or subtract. Express each answer in its simplest form.
(a) $\frac{1}{9}+\frac{2}{3} \quad \frac{7}{9}$
(b) $\frac{1}{2}-\frac{1}{8}$
(c) $\frac{1}{6}+\frac{1}{2} \quad \frac{2}{3}$
(d) $\frac{3}{4}-\frac{7}{12} \frac{1}{6}$
(e) $\frac{1}{5}+\frac{3}{10} \quad \frac{1}{2}$
(f) $\frac{2}{3}+\frac{7}{12} \quad 1 \frac{1}{4}$
(g) $\frac{9}{8}-\frac{3}{4} \quad \frac{3}{8}$
(h) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8} \frac{7}{8}$
(1) $\frac{5}{12}+\frac{1}{2}+\frac{5}{6} 1 \frac{3}{4}$
(6) Rope $A$ is $\frac{7}{12} m$ long and Rope $B$ is $\frac{2}{3} m$ long. What is the difference in length in meters between the two ropes?
$\frac{2}{3}-\frac{7}{12}=\frac{1}{12} ; \frac{1}{12} \mathrm{~m}$

## Lesson 2 Multiplying a Fraction by a Whole Number - Part 1

## Objective

- Find the value of a whole number times a proper fraction.


## Lesson Materials

- Fraction bars


## Think

Pose the Think problem and have students use the fraction bars to find the amount of water Sofia drank. Students can also draw a model or write an equation to solve the problem. Ask students how this problem is similar to and different from the Think problem in the previous lesson.

Discuss student solutions. Students may have chosen to write either addition or multiplication equations.

## Learn

Have students discuss the comparison model and the two different methods.

Dion thinks of repeated addition for fractions and then recalls that it would be quicker to multiply.

Sofia thinks about unit fractions. She knows $\frac{2}{7}=2 \times \frac{1}{7}$, so $3 \times \frac{2}{7}=3 \times 2 \times \frac{1}{7}$.

Have students compare their methods from Think with the ones in the textbook.


Alex drank $\frac{2}{7}$ L of water. Sofia drank 3 times as much water as Alex. How much water did Sofia drink?

$\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{?}{7}$
$3 \times 2$ sevenths $=$ ? sevenths

Method 2


Sofia drank $\frac{6}{7}$ L of water.

## Do

(1)-3 Ask students to use a different multiplication method to solve these problems. If necessary, remind them of Sofia's method on the previous page.
(4) Have students identify which tick marks show eighths on the ruler (the ruler shows sixteenths). They could also look at a real ruler.

5 Have students share some of their solutions.

## Activity

## A Greatest Product

Materials: Three 10-sided dice
One each turn, players roll the dice and make a proper fraction with the numbers from two of the three dice. They multiply that fraction by the number on the third die.

Players can arrange the numbers in any order. The player with the greatest product is the winner.

## Example:



Player may try the following:
$7 \times \frac{3}{9}$
$9 \times \frac{3}{7}$
$3 \times \frac{7}{9}$
$9 \times \frac{3}{7}$ results in the greatest product: $\frac{27}{7}$ or $3 \frac{6}{7}$.

Exercise 2 page 174


186 8-2 Mutiplying a Fraction by a Whole Number — Part 1
(4) Mei is stapling 5 paper rectangles end-to-end on a bulletin board to make a border. Each rectangle is $\frac{7}{8}$ inches long. How long is the border?

$5 \times \frac{7}{8}=\frac{5 \times 7]}{10}=\frac{35}{18}=4 \frac{3}{18}$
(5) Multiply. Express each answer in its simplest form.
(a) $2 \times \frac{2}{5} \frac{4}{5}$
(b) $2 \times \frac{3}{7} \frac{6}{7}$
(c) $3 \times \frac{2}{9} \frac{2}{3}$
(d) $5 \times \frac{3}{4} \quad 3_{\frac{3}{4}}^{3}$
(o) $2 \times \frac{4}{8} \quad 1$
(f) $3 \times \frac{5}{2} 7 \frac{1}{2}$
(9) $6 \times \frac{3}{4} \quad 4 \frac{1}{2}$
(h) $3 \times \frac{4}{5} \quad 2 \frac{2}{5}$


A bag of broccoli weighs $\frac{3}{4} \mathrm{lb}$. Janice bought 3 bags of broccoli. How many pounds of broccoli did she buy?
$3 \times \frac{3}{4}=\frac{9}{4}=2 \frac{1}{4} ; \quad 2 \frac{1}{4} \mathrm{lbs}$

