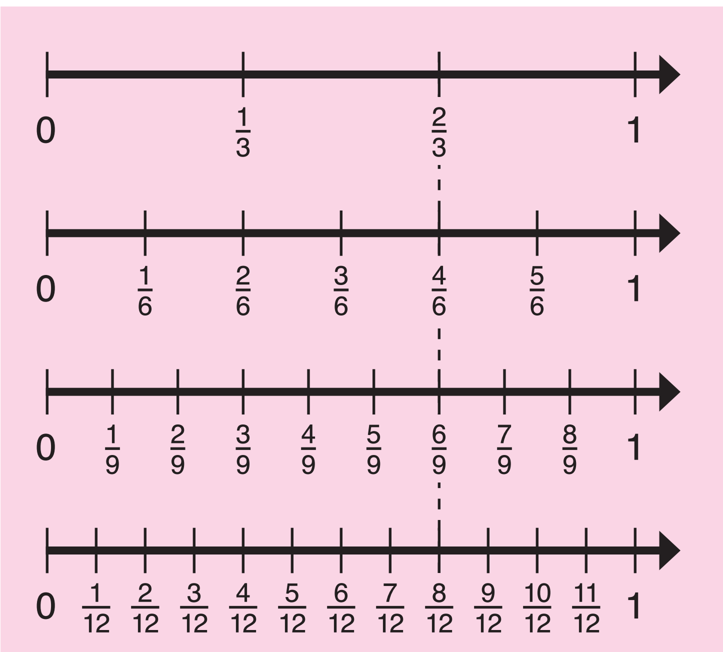
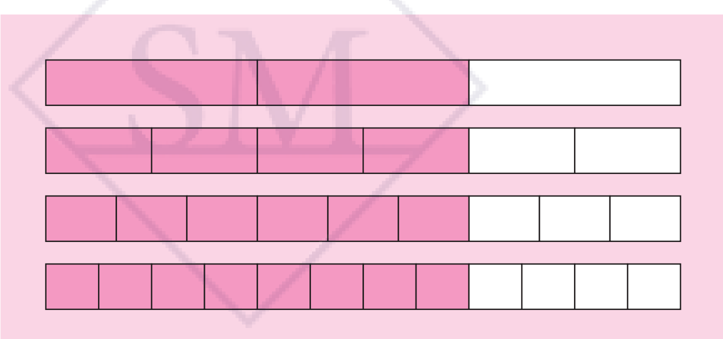
This chapter builds on Chapter 9: Fractions — Part 1 by introducing equivalent fractions. Students will use their understanding of equivalent fractions to simplify and compare fractions. They will then add and subtract fractions with common denominators.

Equivalent Fractions

Fractions are equal if they represent the same number. The same point on a number line can be labeled with the equivalent fractions.



When using bar models to determine whether fractions are equivalent, each bar is the same length, representing the same whole. However, each bar is divided into a different number of equal parts.



Equivalent fractions can be found by:

- Multiplying the numerator and denominator by the same number.

$$\frac{2}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{8}{12}$$

- Dividing the numerator and denominator by the same number.

$$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{2}{3}$$

Ensure that students understand the pictorial representations and number line models before introducing the procedure of multiplying the numerator and denominator by the same number.

A common misrepresentation of the process is to show multiplication by a whole number, for example, $\frac{2}{3} \times 4$ does not equal $\frac{8}{12}$.

When we divide the numerator and denominator by the same number, we are simplifying the fraction. If there is no whole number that both the numerator and denominator can be divided by evenly, the fraction is already in simplest form:

$$\frac{8}{12} \xrightarrow[\div 2]{\div 2} \frac{4}{6} \xrightarrow[\div 2]{\div 2} \frac{2}{3}$$

$\frac{2}{3}$ is the simplest form of $\frac{8}{12}$.

Once students have learned to find equivalent fractions, they have the foundation for fraction operations in future grades.

Comparing Fractions

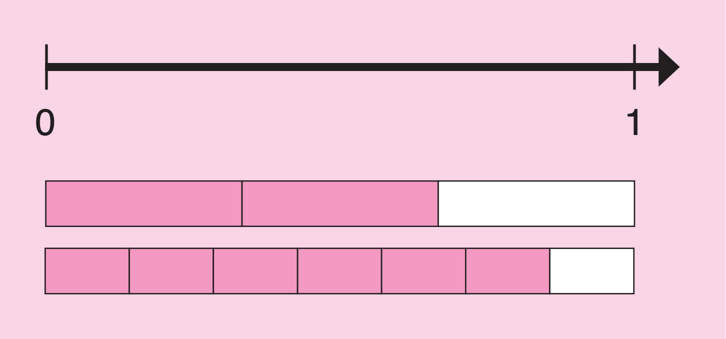
In the previous chapter, students compared fractions with the same numerator or the same denominator. In this chapter, students will compare fractions by finding an equivalent fraction or by comparing to a benchmark fraction. For example:

- Finding an equivalent fraction:

  $\frac{2}{3} > \frac{5}{9}$ because: $\frac{6}{9} > \frac{5}{9}$

- Comparing to 1 whole:

  $\frac{2}{3} < \frac{6}{7}$ because: $\frac{1}{3} > \frac{1}{7}$



Comparing to the benchmark of 1 can be a challenging concept. Although $\frac{2}{3}$ and $\frac{6}{7}$ are each one unit fraction less than 1, $\frac{1}{3}$ is greater than $\frac{1}{7}$. That means that $\frac{2}{3}$ is further from 1 than $\frac{6}{7}$ is from 1.

- Comparing to $\frac{1}{2}$:

  $\frac{3}{5} > \frac{1}{2}$ because $\frac{1}{2} = \frac{3}{6}$ and $\frac{3}{5} > \frac{3}{6}$

Adding and Subtracting Fractions

Students will add and subtract fractions with common denominators. Students will then learn that some of the answers can be simplified.

$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$

$\frac{7}{16} - \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$

Addition and subtraction with related denominators such as $\frac{2}{3} + \frac{1}{6}$ will be taught in **Dimensions Math®** **4A**. However, students might notice that they can find equivalent fractions from related denominators and add them together.

$\frac{2}{3} + \frac{1}{6}$ can be thought of as: $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

$\frac{2}{3} - \frac{1}{6}$ can be thought of as: $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$

Objective

- Find equivalent fractions by multiplying the numerator and denominator by the same number.

Lesson Materials

- Paper strips (same length as in the previous lesson)

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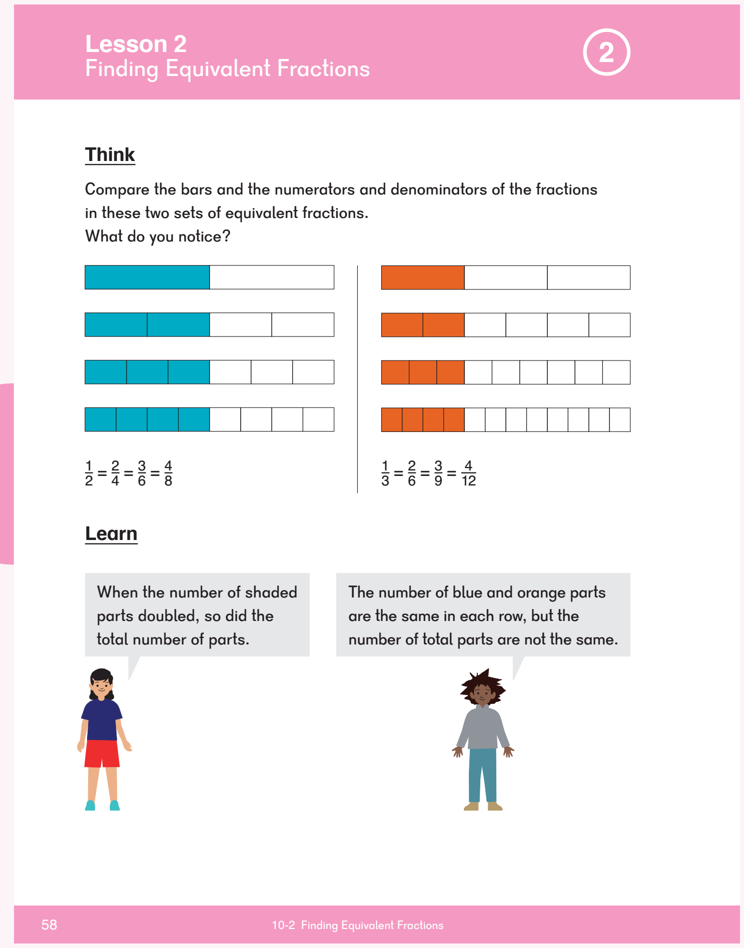

Lesson 2
Finding Equivalent Fractions 2

Think

Compare the bars and the numerators and denominators of the fractions in these two sets of equivalent fractions.
What do you notice?

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$

$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}$

Learn

When the number of shaded parts doubled, so did the total number of parts.

The number of blue and orange parts are the same in each row, but the number of total parts are not the same.

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Think

Have students use the paper strips folded into halves, fourths, and eighths from the previous lesson. Provide students with additional strips of paper and have them fold the strips into thirds, sixths, ninths, and twelfths.

Unfold the strips. Lay them out in a similar way to the image in **Think**.

Have students shade their paper strips similarly to the strips in the **Think** problem.

Discuss what students notice about the sets of strips. Then discuss what students notice about the equivalent fractions listed.

Learn

The friends share what they notice about the sets of bars.

Discuss Mei's and Dion's observations. Students should see that as the bars are divided into more parts, there is a corresponding increase in the number of parts that are colored or shaded.

For example:

- When the strip is folded into thirds, 1 of the thirds is shaded.
- When the strip is folded into sixths, 2 of the sixths are shaded.

Ask students if it makes sense that a paper folded into 6 equal parts will have twice the number of equal parts as a paper folded into thirds.

Ask students, “What does Emma mean when she says the size of the shaded part does not change?” Ensure students understand that the length of the shaded part is the same. As a result, the same fraction of the whole is shaded regardless of the number of folds.

Focus the students’ attention on Alex’s multiplication example and discuss the notation.

We have doubled the total number of parts and the shaded number of parts.

$$\frac{1}{2} \overset{\times 2}{\underset{\times 2}{\longrightarrow}} \frac{2}{4} \overset{\times 2}{\underset{\times 2}{\longrightarrow}} \frac{4}{8}$$

Ask students if the same method will work for $\frac{1}{3}$.

$$\frac{1}{3} \overset{\times 2}{\underset{\times 2}{\longrightarrow}} \frac{2}{6} \overset{\times 2}{\underset{\times 2}{\longrightarrow}} \frac{4}{12}$$

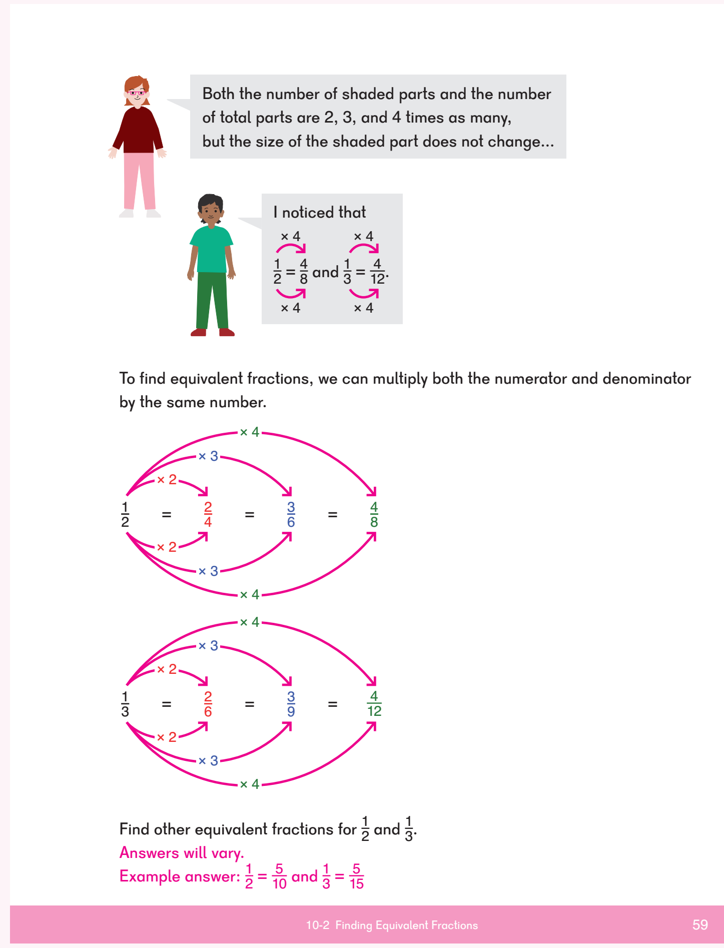
Ensure that students understand that finding equivalent fractions is not adding fractions.

A common student misconception is to add both the numerator and denominator together:

$$\frac{1 + 1 = 2}{2 + 2 = 4}$$

Students should understand that one-half + one-half is two halves: $\frac{2}{2}$, which equals 1.

This understanding becomes even more critical when students begin to add fractions with unlike denominators.


Both the number of shaded parts and the number of total parts are 2, 3, and 4 times as many, but the size of the shaded part does not change...

I noticed that $\frac{1}{2} \overset{\times 4}{\underset{\times 4}{=}} \frac{4}{8}$ and $\frac{1}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{4}{12}$.

To find equivalent fractions, we can multiply both the numerator and denominator by the same number.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

Find other equivalent fractions for $\frac{1}{2}$ and $\frac{1}{3}$.

Answers will vary.

Example answer: $\frac{1}{2} = \frac{5}{10}$ and $\frac{1}{3} = \frac{5}{15}$

10-2 Finding Equivalent Fractions 59

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Do

❸ (a) As an example, students could first think, "What number can 4 be multiplied by to get a product of 12?" (3)

1 × 3 is 3, so the missing numerator is 3.

(d) Students could first think, "What number can 2 be multiplied by to get a product of 6?" (3)

6 × 3 = 18, so the missing denominator is 18.

Activity

▲ Investigate Equivalent Fractions on the Multiplication Chart

Materials: Multiplication Chart (BLM) in a dry erase sleeve, light colored dry erase marker or ruler

A multiplication chart is a good way to further investigate equivalent fractions.

Select two consecutive rows on the chart and have students highlight these or underline them with a ruler.

The first highlighted row represents the numerator and the second highlighted row represents the denominator. ($\frac{2}{3}$) If they move one column to the right (the column that corresponds with "× 2") they will find the equivalent fraction to that found in the first column. ($\frac{4}{6}$)

$$\frac{2}{3} \underset{\times 6}{\overset{\times 6}{=}} \frac{12}{18}$$

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	8	12	15	18	21	24	27	30

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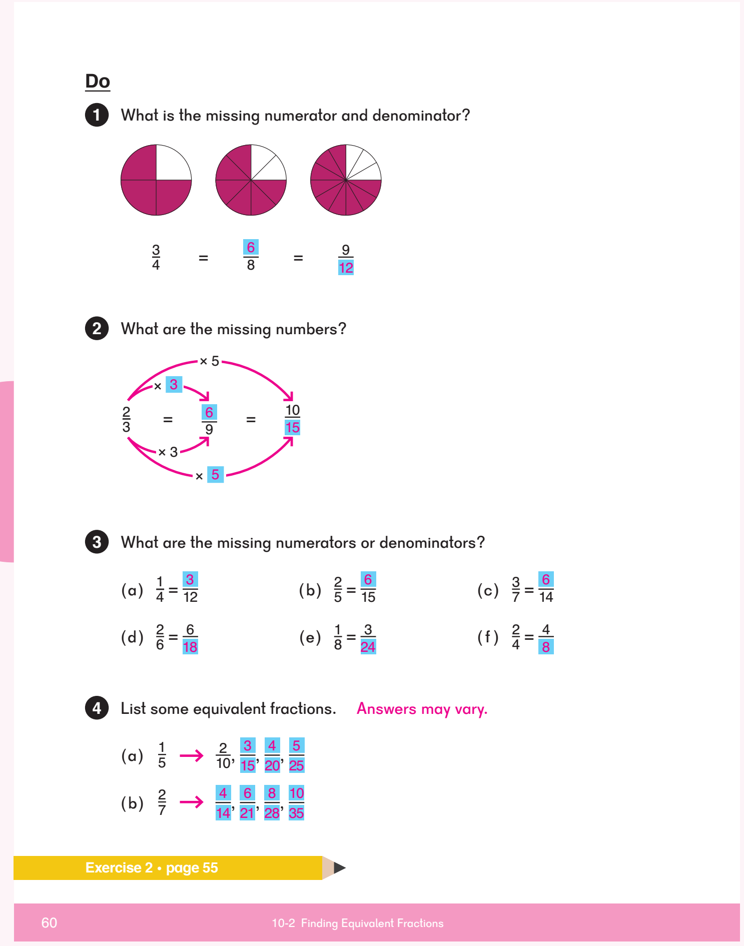
Do

❶ What is the missing numerator and denominator?

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

❷ What are the missing numbers?

$\frac{2}{3} = \frac{6}{9} = \frac{10}{15}$ (× 3, × 3; × 5, × 5)

❸ What are the missing numerators or denominators?

(a) $\frac{1}{4} = \frac{3}{12}$ (b) $\frac{2}{5} = \frac{6}{15}$ (c) $\frac{3}{7} = \frac{6}{14}$

(d) $\frac{2}{6} = \frac{6}{18}$ (e) $\frac{1}{8} = \frac{3}{24}$ (f) $\frac{2}{4} = \frac{4}{8}$

❹ List some equivalent fractions. Answers may vary.

(a) $\frac{1}{5} \rightarrow \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}$

(b) $\frac{2}{7} \rightarrow \frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \frac{10}{35}$

Exercise 2 • page 55

60 10-2 Finding Equivalent Fractions

★ Extend the activity by highlighting non-consecutive rows:

$$\frac{2}{5} \underset{\times 4}{\overset{\times 4}{=}} \frac{8}{20}$$

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	8	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50

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Review 3

Objective

- Review topics from Chapter 1 through Chapter 11.

Use this cumulative review as needed to practice and reinforce content and skills from the first 11 chapters.

Brain Works

★ Problems Without Numbers

Often when students encounter a word problem, they focus on computing some quantity rather than what is being asked. These numberless problems can be a good way to get students to think about the relevant underlying mathematical ideas.

(a) I know the length and width of a room and the number of square meters that a can of paint will cover. How can I find how many cans of paint I will need to buy to paint the walls of that room?

(b) How can I find how many times a bicycle wheel will turn going 2 kilometers?

(c) How can I find the distance around a square dog park if I know how long two-thirds of one side is?

(d) I know how many centimeters long and wide a tile is, and how many meters long and wide a room is. How do I find how many tiles will cover the floor?

(e) I know how many milliliters of juice it takes to fill a small pitcher. How can I find how many liters it takes to fill 5 such pitchers?

(f) I know how much my dog weighs when standing on four feet. How do I find his weight when standing on three feet?

(g) I know the weight of a box of gumballs in bags and the weight of the empty box. What else do I need to know to find the weight of each bag of gumballs in the box?

Review 3 R 3

1 Estimate, then find the value. Estimations may vary. Actual values are given.

(a) 2,306 + 3,895
6,201

(b) 7,329 + 494
7,823

(c) 6,475 + 1,785
8,260

(d) 4,926 – 1,469
3,457

(e) 8,152 – 267
7,885

(f) 5,003 – 1,308
3,695

2 Solve.

(a) 87 × 3
261

(b) 45 × 7
315

(c) 95 ÷ 6
15 R 5

(d) 57 ÷ 4
14 R 1

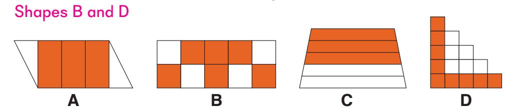
(e) 329 ÷ 5
65 R 4

(f) 268 × 9
2,412

(g) 843 × 8
6,744

(h) 915 ÷ 8
114 R 3

3 Which of the following shapes have $\frac{3}{5}$ colored?
Shapes B and D



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Objective

- Practice finding area and perimeter.

After students complete the **Practice** in the textbook, have them continue to practice by playing games from this chapter.

Activity

▲ Area Game

Materials: Centimeter Graph Paper (BLM) in a dry erase sleeve, a pair of 6-sided dice, 2 colors of dry erase markers

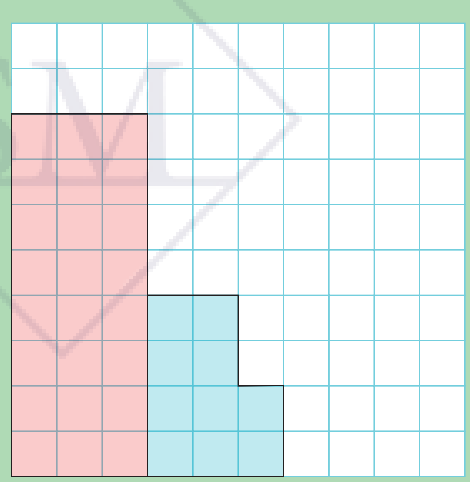
This is an extension of **Fences** from Lesson 3.

Players take turns rolling the dice. On each turn, they roll the dice, multiply the numbers, and shade an equivalent area on the Centimeter Graph Paper (BLM).

In the example shown below:

- Player One (blue) rolled a 5 and 2, and fenced an area of 10 square units.
- Player Two (red) rolled 6 and 4, and fenced in an 8 × 3 array, or 24 square units.

Play ends when a player cannot place an appropriate sized shape on the board. Each player adds up the total amount of area they have fenced in. The player with the most area is the winner.



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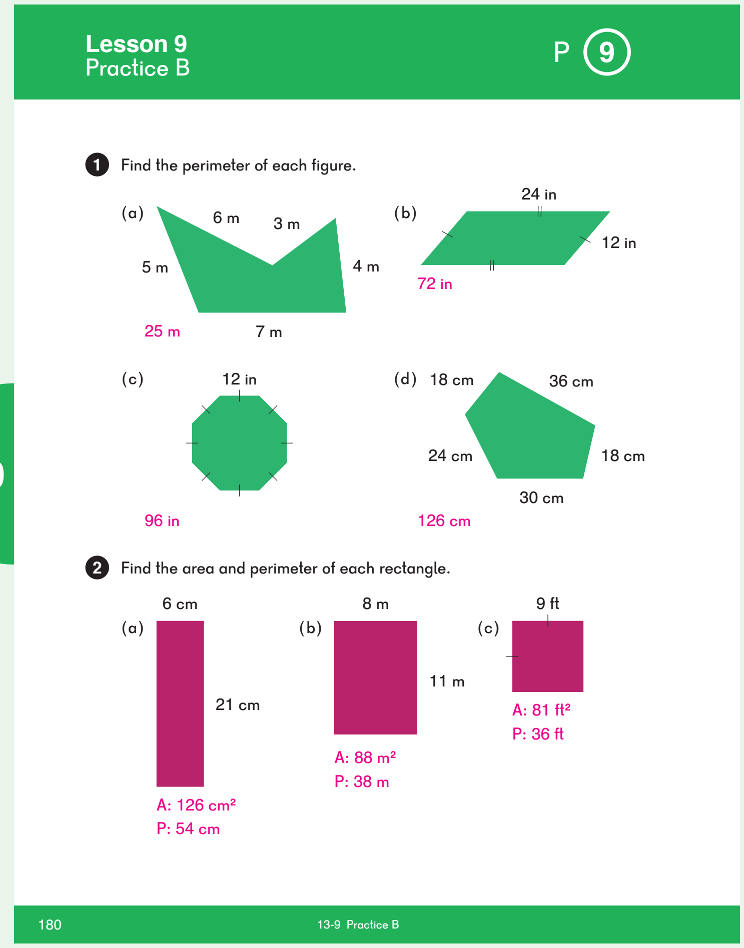

Lesson 9
Practice B
P 9

1 Find the perimeter of each figure.

(a) 6 m, 3 m, 5 m, 4 m, 7 m
25 m

(b) 24 in, 12 in
72 in

(c) 12 in
96 in

(d) 18 cm, 36 cm, 24 cm, 18 cm, 30 cm
126 cm

2 Find the area and perimeter of each rectangle.

(a) 6 cm, 21 cm
A: 126 cm²
P: 54 cm

(b) 8 m, 11 m
A: 88 m²
P: 38 m

(c) 9 ft
A: 81 ft²
P: 36 ft

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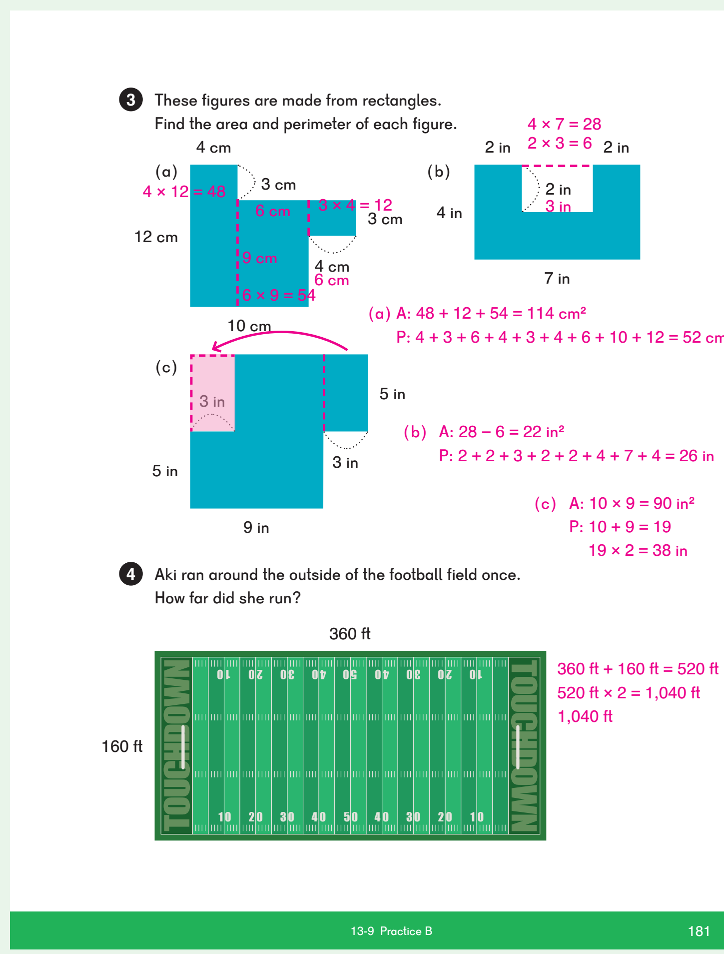

3 These figures are made from rectangles.
Find the area and perimeter of each figure.

(a) 4 cm, 3 cm, 12 cm, 3 cm, 4 cm, 10 cm
4 × 12 = 48
6 cm
3 × 4 = 12
9 cm
6 cm
6 × 9 = 54

(b) 2 in, 2 in, 2 in, 4 in, 7 in
4 × 7 = 28
2 × 3 = 6
3 in

(a) A: 48 + 12 + 54 = 114 cm²
P: 4 + 3 + 6 + 4 + 3 + 4 + 6 + 10 + 12 = 52 cm

(c) 3 in, 5 in, 5 in, 3 in, 9 in

(b) A: 28 − 6 = 22 in²
P: 2 + 2 + 3 + 2 + 2 + 4 + 7 + 4 = 26 in

(c) A: 10 × 9 = 90 in²
P: 10 + 9 = 19
19 × 2 = 38 in

4 Aki ran around the outside of the football field once.
How far did she run?

360 ft
160 ft
TOUCHDOWN TOUCHDOWN
10 20 30 40 50 40 30 20 10

360 ft + 160 ft = 520 ft
520 ft × 2 = 1,040 ft
1,040 ft

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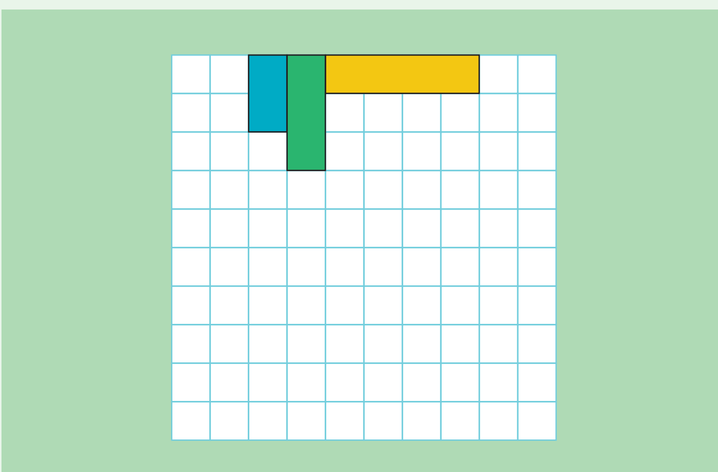
Brain Works

★ Tiling

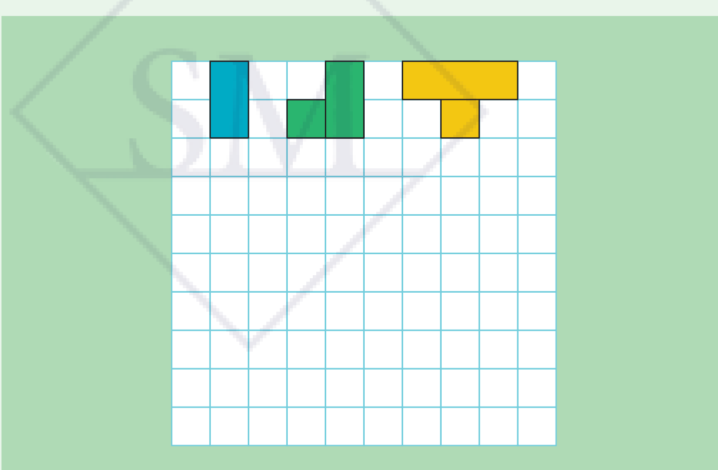
Materials: Inch Graph Paper (BLM) or graph paper

For this independent activity, provide students with Inch Graph Paper (BLM) or graph paper and the following situation:

"On a 10-square unit grid, you can lay rectangles that are 2 square units, 3 square units, and 4 square units. How many different ways can you cover the grid?"



Challenge students by asking, "On a 10-square unit grid, you can lay shapes that are 2 square units, 3 square units, and 4 square units. How many different ways can you cover the grid?"



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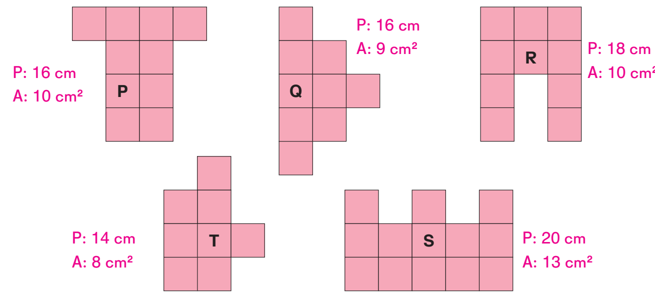
5. These figures are made up of one-centimeter squares.



(a) Which figure has the shortest perimeter?
Figure T

(b) Which figure has the longest perimeter?
Figure S

(c) Which figure has the largest area?
Figure S

(d) Which figure has the smallest area?
Figure T

(e) Which figures have the same perimeter?
Figures P and Q

(f) Which figures have the same area?
Figures P and R

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