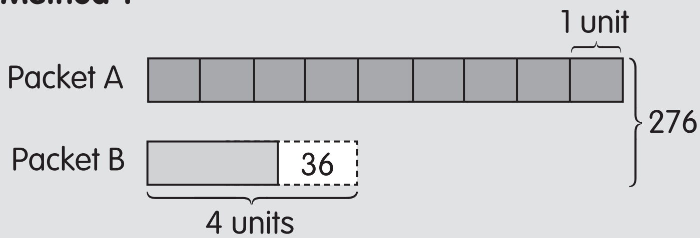
Challenging Problems

Worked Example 1

There is a total of 276 beads in Packet A and Packet B. There are 36 fewer beads in Packet B than $\frac{4}{9}$ of the number of beads in Packet A. How many beads are there in Packet A?

Method 1



From the model,

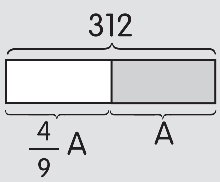
13 units $\longrightarrow$ $276 + 36 = 312$

1 unit $\longrightarrow$ $312 \div 13 = 24$

9 units $\longrightarrow$ $9 \times 24 = 216$

There are **216** beads in Packet A.

Method 2



$A + B = 276$

$\frac{4}{9}A = B + 36$

$\frac{4}{9}A = (276 - A) + 36$

$\frac{4}{9}A = 312 - A$

$A + \frac{4}{9}A = 312$

$\frac{9}{9}A + \frac{4}{9}A = 312$

$\frac{13}{9}A = 312$

$A = 312 \times \frac{9}{13}$

$= 216$

Packet A has **216** beads.

Answer all questions. Show your work and write your statements clearly.

1. Without converting the fractions to decimals, state which of these fractions are smaller than $\frac{1}{5}$.

   A. $\frac{5}{21}$ B. $\frac{7}{36}$ C. $\frac{15}{72}$ D. $\frac{26}{101}$

2. Study the pattern below.

$$\frac{1}{1 \times 2} = \frac{1}{2}$$

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} = \frac{2}{3}$$

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} = \frac{3}{4}$$

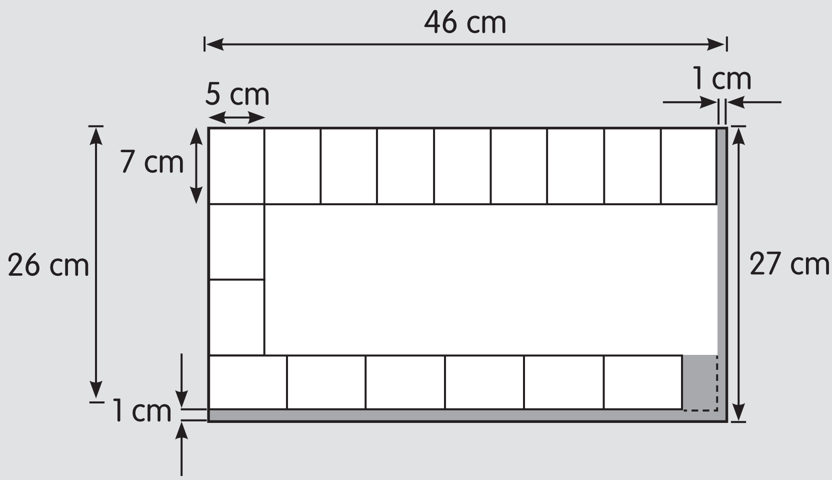
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \frac{4}{5}$$

Given that $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{2013 \times 2014} = \frac{x - 2}{x - 1}$, where x is a whole number, find the value of x.

Worked Example 2

A rectangular cardboard is 46 cm long and 27 cm wide. What is the maximum number of rectangles, each 7 cm long and 5 cm wide, that can be cut from it?

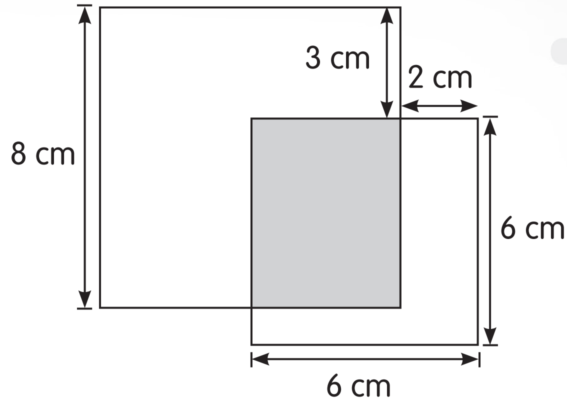
The diagram below shows that the maximum number of rectangles that can be cut out. There are 3 rows of 9 rectangles and 1 row of 6 rectangles, that is, a total of $(9 \times 3) + 6 = 33$ rectangles.
It will leave a strip of width 1 cm on the right and at the bottom, and a 5 cm long and 3 cm wide rectangle.



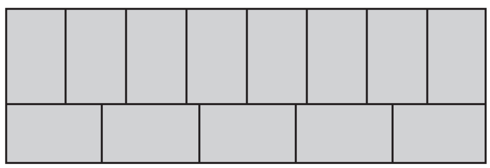
The maximum number of rectangles that can be cut from the rectangular cardboard is **33**.

Note: Without drawing a sketch or diagram, it is hard to determine the maximum number of rectangles that can be cut, such that a minimum amount of unused space is left.

9. The figure below shows two overlapping squares. What is the area of the unshaded region?



10. The figure below is made up of 13 identical rectangles. If its area is 520 cm^2, what is its perimeter?



Hint: Express the width of a rectangle in terms of its length, or the length in terms of its width.

4. *Method 1*

$2\frac{1}{3}$ cm + $2\frac{1}{3}$ cm + $2\frac{1}{3}$ cm = 7 cm

A 7-cm strip of paper yields 3 smaller pieces.

7 cm ⟶ 3 pieces

21 cm ⟶ 3 × 3 = 9 pieces

Number of pieces Tommy will have = **9**

Method 2

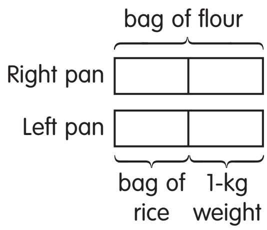
$2\frac{1}{3} = \frac{7}{3}$

Number of pieces Tommy will have $= 21 \div \frac{7}{3}$

$= 21 \times \frac{3}{7}$

$= \mathbf{9}$

5.



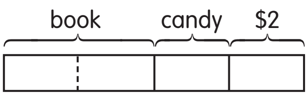
1 unit ⟶ 1 kg

2 units ⟶ 2 × 1 kg = 2 kg

Mass of 2 similar bags of flour = 2 × 2 kg

= **4 kg**

6.

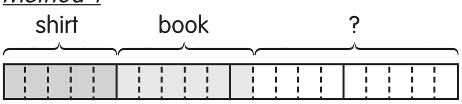


1 unit ⟶ \$2

2 units ⟶ 2 × \$2 = \$4

He paid **\$4** for the book.

7. *Method 1*



(a) Fraction of his allowance left $= \mathbf{\frac{9}{20}}$

(b) 6 units ⟶ \$18

1 unit ⟶ \$18 ÷ 6 = \$3

20 units ⟶ 20 × \$3 = \$60

Amount of allowance he had at first

= **\$60**

Method 2

$\frac{3}{4}$ of remainder ⟶ $\frac{2}{5}$ on a book

$\frac{3}{4}$ of remainder ⟶ $\frac{3}{5}$ left

(a) Fraction of his allowance left after buying a shirt $= 1 - \frac{1}{4} = \frac{3}{4}$

Fraction of his allowance left after buying a shirt and a book $= \frac{3}{5} \times \frac{3}{4} = \mathbf{\frac{9}{20}}$

(b) Fraction of his allowance spent on a book $= \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$

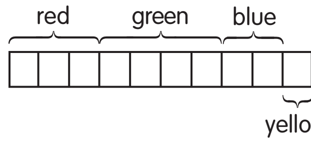
$\frac{6}{20}$ of his allowance ⟶ \$18

$\frac{1}{20}$ of his allowance ⟶ \$18 ÷ 6 = \$3

$\frac{20}{20}$ of his allowance ⟶ 20 × \$3 = \$60

Amount of allowance he had at first = **\$60**

8. $\frac{2}{5} = \frac{4}{10}$



Difference between the number of red marbles and the number of blue marbles

= 1 unit

1 unit ⟶ 17

10 units ⟶ 10 × 17 = 170

Total number of marbles = **170**

Challenging Problems (pp. 18–22)

1. Other than comparing each fraction with $\frac{1}{5}$, we can also multiply each fraction by 5 and compare with 1.

$\frac{5}{21} \times 5 = \frac{25}{21}$, which is greater than 1.

$\frac{7}{36} \times 5 = \frac{35}{36}$, which is less than 1.

$\frac{15}{72} \times 5 = \frac{75}{72}$, which is greater than 1.

$\frac{26}{101} \times 5 = \frac{130}{101}$, which is greater than 1.

Since $\frac{35}{36}$ is less than 1, $\mathbf{\frac{7}{36}}$ is less than $\frac{1}{5}$.

2. From observing the pattern, we know that

$\frac{x-2}{x-1} = \frac{2{,}013}{2{,}014} = \frac{2{,}015-2}{2{,}015-1}$

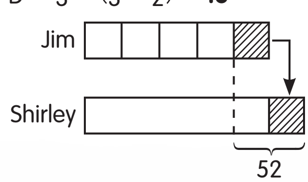
Hence, the value of x is **2,015**.

3. $\frac{N}{D} = \frac{2}{3}$; N × D = 216

216 = 2 × 2 × 2 × 3 × 3 × 3

$\frac{N}{D} = \frac{2}{3} \times \left(\frac{2}{3} \times \frac{3}{2}\right) = \mathbf{\frac{12}{18}}$

4.



8 units ⟶ 260 − 52 = 208

1 unit ⟶ 208 ÷ 8 = 26