

4.2 Word Problems

Objective

- Solve word problems involving the product of fractions.

Notes

Students have previously encountered some multistep word problems involving the product of a fraction and a whole number, and some simple word problems involving the product of fractions. In this chapter, they will solve multistep problems involving the product of fractions. Some of these problems involve finding a fraction of a fraction. If the whole is 1, then to find a fraction of a fraction of the whole we find the product of the two fractions. $\frac{1}{4}$ of $\frac{2}{3}$ of 1 whole is $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$ of the whole. If the whole is a set, we can either find the product of the fractions, or first find a fraction of a whole. $\frac{1}{4}$ of $\frac{2}{3}$ of 360 is $\frac{1}{4} \times \frac{2}{3}$ of 360 = $\frac{1}{6} \times 360 = 60$. Or $\frac{1}{4} \times \frac{2}{3}$ of 360 is $\frac{1}{4} \times (\frac{2}{3} \times 360) = \frac{1}{4} \times 240 = 60$.

Bar models (essentially fraction bars in these problems) can help students determine a method to solve the problems. Using the bar model emphasizes the relationship between the fractions: division to find the number in each part and multiplication to find the number in more than one part. Students can either write equations with fractions, or with units, a unit being a fractional part of the whole.

In these problems, we will either be given the value of the whole, or of a fractional part of the whole. Bar models are particularly effective with word problems where we are given the value of a fractional part of the whole. (Without a model, the concepts essentially involve division of a fraction, as will be seen in later chapters of this unit, since we are given the value of an equal part, rather than the value of a whole.)

4.2a Word problems

Objective

- Solve word problems involving product of fractions.



Common Core State Standards

- 5.NF.4a
- 5.NF.4b

Mathematical Practices

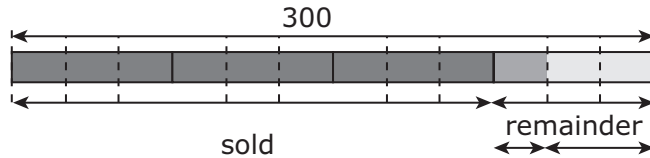
- MP1
- MP2
- MP3
- MP4

Discussion	Textbook, pp. 95–96
<ul style="list-style-type: none"> Discuss the four methods shown on these two pages. Point out that the first step used here is to draw a model using fraction bars. The whole is the total number of stamps Jim has, 360. Since he sold 1 third of them, the bar is divided into thirds. The second bar shows only the remainder and that we need to find 1 fourth of the remainder. We can either find the amount in the remainder first, or we can find how many he sold on Tuesday as a fraction of the whole rather than as a fraction of the remainder. Relate each step in the different methods to the model. The fourth method requires the least number of computations, and so can be the easiest, if a model has been used. The remainder is two units, so to find 1 fourth of the remainder, each of those units needs to be divided into two equal units to have fourths. If all the units are divided into two units, there are six in total, and from the model it is easy to see that we need to find the value of one unit. Point out that we could also use the idea of units to find the value of the remainder and then a half unit, which is a fourth of the remainder. 3 units = 360, 1 unit = 120, half of a unit = 60. Point out that it is not necessary to draw two separate bars; and we could have divided up the units on the first bar to show fourths for the remainder without first drawing a separate bar for the remainder. <p>Answers:</p> <p>Method 1: $\frac{1}{6} \times 360 = \mathbf{60}$; 60</p> <p>Method 2: $\frac{1}{4} \times 240 = \mathbf{60}$; 60</p> <p>Method 3: $\frac{1}{4} \times 240 = \mathbf{60}$; 60</p> <p>Method 4: 1 unit = 60</p>	<p>2 Word Problems</p> <p>Jim had 360 stamps. He sold $\frac{1}{3}$ of them on Monday and $\frac{1}{4}$ of the remainder on Tuesday. How many stamps did he sell on Tuesday?</p> <p>Method 1:</p> <p>$1 - \frac{1}{3} = \frac{2}{3}$</p> <p>First, find what fraction of the stamps were left on Monday.</p> <p>He had $\frac{2}{3}$ of the stamps left on Monday. The remainder is $\frac{2}{3}$.</p> <p>$\frac{1}{4} \times \frac{2}{3} = \frac{1}{2 \times 3} = \frac{1}{6}$</p> <p>Next, find $\frac{1}{4}$ of the remainder.</p> <p>He sold $\frac{1}{6}$ of the stamps on Tuesday.</p> <p>$\frac{1}{6} \times 360 = \square$</p> <p>Jim sold \square stamps on Tuesday.</p> <p>95</p> <p>Method 2:</p> <p>$1 - \frac{1}{3} = \frac{2}{3}$</p> <p>He had $\frac{2}{3}$ of the stamps left on Monday.</p> <p>$\frac{2}{3} \times 360 = 2 \times 120 = 240$</p> <p>He had 240 stamps left on Monday.</p> <p>$\frac{1}{4} \times 240 = \square$</p> <p>Jim sold \square stamps on Tuesday.</p> <p>Method 3:</p> <p>$\frac{1}{3} \times 360 = 120$</p> <p>He sold 120 stamps on Monday.</p> <p>$360 - 120 = 240$</p> <p>He had 240 stamps left on Monday.</p> <p>$\frac{1}{4} \times 240 = \square$</p> <p>Jim sold \square stamps on Tuesday.</p> <p>Method 4:</p> <p>$\frac{1}{3} \times 360 = 120$</p> <p>He sold 120 stamps on Monday.</p> <p>$360 - 120 = 240$</p> <p>He had 240 stamps left on Monday.</p> <p>$\frac{1}{4} \times 240 = \square$</p> <p>Jim sold \square stamps on Tuesday.</p> <p>I divide all the units into 2 parts, so there are now 6 parts.</p> <p>Total number of stamps = 6 units = 360 Number of stamps sold on Tuesday = 1 unit = \square</p> <p>96</p>
<p>Assessment</p> <ul style="list-style-type: none"> Have students solve these problems and discuss their solutions. Tell them they do not have to follow the method in the text only. Try to discuss at least two alternative solutions. 	<p>Textbook, p. 97</p>

- **Task 1:** The drawing in the text shows finding the amount unsold first, then 2 thirds of that. We can also divide the remainder into thirds, and then all the units, giving a total of $4 \times 3 = 12$ units, and find the value of 2 units. Your student might also notice that 2 units is 1 sixth of the total and simply divide 300 by 6 to find the answer.

Answer:

1.



$$12 \text{ units} = 300$$

$$1 \text{ unit} = 300 \div 12 = 25$$

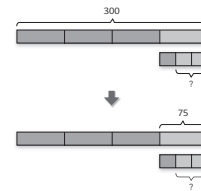
$$2 \text{ units} = 25 \times 2 = 50$$

$$\text{Or } 2 \text{ units} = \frac{300}{12} \times 2 = 50$$

Or:

$$\frac{2}{12} \times 300 = 50$$

1. Marisol made 300 tarts. She sold $\frac{2}{3}$ of them and gave $\frac{1}{3}$ of the remainder to her neighbor. How many tarts did she have left?



$$4 \text{ parts} = 300$$

$$1 \text{ part} = 300 \div 4 = 75$$

$$3 \text{ units} = 75$$

$$1 \text{ unit} = 75 \div 3 = 25$$

$$2 \text{ units} = 25 \times 2 = 50$$

Or:

$$\frac{1}{4} \times 300$$

$$= 75$$

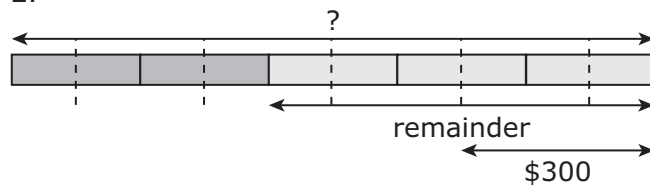
$$\frac{2}{3} \times 75$$

$$= 50$$

- **Task 2:** Rather than being given the whole, we are given the value for a fractional part of the whole.
- The drawing in the text shows first finding the value of the remainder.
- We can also combine the two bars and find a unit that works for both fifths and halves. Since one part in the bottom bar is $1\frac{1}{2}$ unit in the top bar, we can cut up each of the units in the top bar in half. One way to determine how to cut up the bar is to use the lowest common multiple of the number of units in the remainder, 3, and the denominator of the fraction of the remainder, 2. To divide the remainder into 6 units, we divide each unit into half. The entire bar will then have 10 units. (In this case, 10 turns out to be the lowest common multiple of the two denominators, but that is not always the best choice.)

Answer:

2.

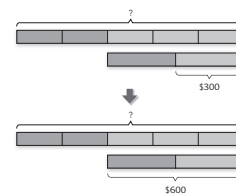


$$3 \text{ units} = \$300$$

$$1 \text{ unit} = \$300 \div 3 = \$100$$

$$10 \text{ units} = \$100 \times 10 = \$1,000$$

2. Mr. Anderson gave $\frac{2}{5}$ of his money to his wife and spent $\frac{1}{2}$ of the remainder. If he had \$300 left, how much money did he have at first?



$$1 \text{ part} = \$300$$

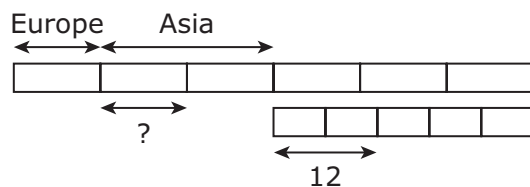
$$2 \text{ parts} = \$300 \times 2 = \$600$$

$$3 \text{ units} = \$600$$

$$1 \text{ unit} = \$600 \div 3 = \$200$$

$$5 \text{ units} = \$200 \times 5 = \$1,000$$

- Write the problem on the right on to the board and have students solve it and share their solutions.
- Students may start by creating a bar with 30 units, which does lead to a solution. The lowest common denominator of 6, 3, and 5 is 30. Then the 2 fifths of the remainder is 6 of the 30 units. However, if they count the bars first before deciding on a method of solution, they will see that the remainder is half of the total. So they might find it easier to find the value of the remainder rather than drawing 30 units. Modeling the problem should come first, then deciding on a method of solution, rather than following specific steps that worked well with another problem.



1 small unit = 6
 Half the total = $6 \times 5 = 30$
 1 part = $30 \div 3 = 10$
 There are 10 more coins from Asia than from Europe.

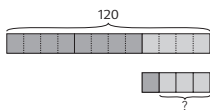
One sixth of the coins in John's collection are from Europe, $\frac{1}{3}$ are from Asia, and $\frac{2}{5}$ of the remainder are from Africa. If there are 12 coins from Africa, how many more coins are from Asia than from Europe?

Practice

Workbook Exercise 4, pp. 102–105

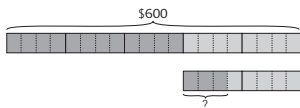
EXERCISE 4

1. Tracy bought 120 eggs. She fried $\frac{2}{3}$ of them and boiled $\frac{1}{4}$ of the remainder. How many eggs did she have left?



12 units = 120
 3 units = $120 \div 4 = 30$
 Tracy had 30 eggs left.

2. Mr. Ramirez had \$600. He saved $\frac{3}{5}$ and spent $\frac{3}{8}$ of the remainder. How much did he spend?

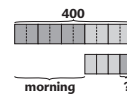


5 parts = \$600
 2 parts = $(\$600 \div 5) \times 2 = \$120 \times 2 = \$240$
 8 units = \$240
 3 units = $(\$240 \div 8) \times 3 = \$30 \times 3 = \$90$
 Mr. Ramirez spent \$90.

102

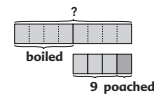
Unit 4: Multiply and Divide Fractions

3. Lindsey made 400 tarts. She sold $\frac{3}{5}$ of them in the morning and $\frac{1}{4}$ of the remainder in the afternoon. How many tarts did she sell in the afternoon?



10 units = 400
 1 unit = 40
 Lindsey sold 40 tarts in the afternoon.

4. Mrs. Gray bought some eggs. She boiled $\frac{1}{2}$ of them and poached $\frac{1}{4}$ of the remainder. She had 9 eggs left. How many eggs did she buy?

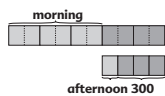


3 units = 9
 1 unit = 3
 8 units = $3 \times 8 = 24$
 Mrs. Gray bought 24 eggs.

Unit 4: Multiply and Divide Fractions

103

5. Christina made some pancakes. She sold $\frac{3}{5}$ of them in the morning and $\frac{1}{4}$ of the remainder in the afternoon. If she had 300 pancakes left, how many pancakes did she make?



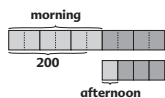
3 units were left.

3 units = 300

10 units = $(300 \div 3) \times 10 = 1,000$

Christina made 1,000 pancakes.

6. Mrs. Klein made some fruit buns. She sold $\frac{3}{5}$ of them in the morning and $\frac{1}{4}$ of the remainder in the afternoon. If she sold 200 more fruit buns in the morning than in the afternoon, how many fruit buns did she make?



5 units = 200

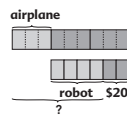
10 units = $200 \times 2 = 400$

Mrs. Klein made 400 buns.

104

Unit 4: Multiply and Divide Fractions

7. Alex spent $\frac{1}{3}$ of his pocket money on a toy airplane and $\frac{2}{3}$ of the remainder on a toy robot. He had \$20 left. How much did he spend altogether?

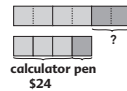


2 units = \$20

7 units = $(\$20 \div 2) \times 7 = \$10 \times 7 = \$70$

Alex spent \$70.

8. John spent $\frac{2}{5}$ of his money on a pen and a calculator. The calculator cost 3 times as much as the pen. If the calculator cost \$24, how much money did he have left?



3 units = \$24

2 units = $(\$24 \div 3) \times 2 = \$8 \times 2 = \$16$

John had \$16 left.

105

Unit 4: Multiply and Divide Fractions