[JR HIGH – YEAR 1 – STUDENT]

BOOK 1

PRINCIPLES OF MATHEMATICS

BIBLICAL WORLDVIEW CURRICULUM

[Katherine A. Loop]

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Acknowledgments

This curriculum was a major part of my life for several years, but I don't believe you would be holding it in your hands today if it were not for some very special people. I'd like to acknowledge and thank

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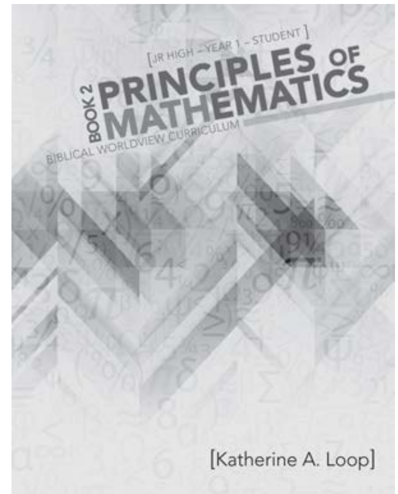
Above all, I'd like to thank the Lord, without whom all our labors are in vain.

Once you're done with Book 1 . . . you'll be ready for Book 2!



Get ready to continue exploring principles of mathematics! Now that you know the core principles of arithmetic and geometry, you're ready to move on to learning even more skills that will allow you to explore more aspects of God's creation.

In Book 2, we'll focus on the core principles of algebra, coordinate graphing, probability, statistics, functions, and other important areas of mathematics. The topics may sound intimidating, but you'll discover that they are simply useful techniques that serve a wide range of practical used. As we do in this book, we'll continue to discover that all of math boils down to a way of describing God's creation and a useful tool we can use to serve God, all while worshiping Him!

About the Author

Katherine Loop is a homeschool graduate from Northern Virginia. Understanding the biblical worldview in math made a tremendous difference in her life and started her on a journey of researching and sharing on the topic. For over a decade now, she's been researching, writing, and speaking on math, along with other topics. Her previous books on math and a biblical worldview have been used by various Christian colleges, homeschool groups, and individuals.

Website and Blog: www.ChristianPerspective.net
(Check out the free e-newsletter.)

Facebook: https://www.facebook.com/katherinealoop
(Join the conversation!)

YouTube: https://www.youtube.com/user/mathisnotneutral

About This Curriculum

What Is This Curriculum?

This is Year 1 of a two-year math course designed to give students a firm mathematical foundation, both academically and spiritually. Not only does the curriculum build mathematical thinking and problem-solving skills, it also shows students how a biblical worldview affects our approach to math's various concepts. Students learn to see math, not as an academic exercise, but as a way of exploring and describing consistencies God created and sustains. The worldview is not just an addition to the curriculum, but the starting point. Science, history, and real life are integrated throughout.

How Does a Biblical Worldview Apply to Math… and Why Does It Matter?

Please see lessons 1.1–1.3 and 2.7 for a brief introduction to how a biblical worldview applies to math and why it matters.

Who Is This Curriculum For?

This curriculum is aimed at **grades 6-8**, fitting into most math approaches the **year or two years prior to starting high school algebra**. If following traditional grade levels, Year 1 should be completed in grade 6 or 7, and Year 2 in grade 7 or 8.

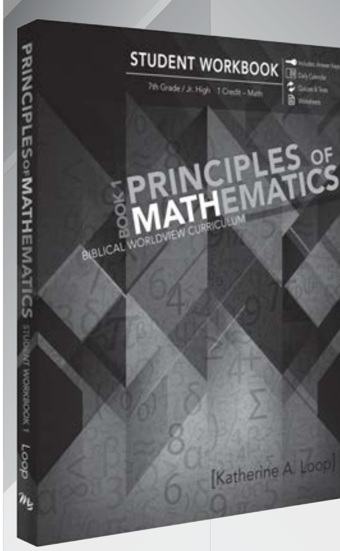
The curriculum also works well for **high school students** looking to firm up math's foundational concepts and grasp how a biblical worldview applies to math. High school students may want to follow the alternate accelerated schedule in the *Student Workbook* and complete each year of the program in a semester, or use the material alongside a high school course.

Where Do I Go Upon Completion?

Upon completion of Year 1, students will be ready to move on to Year 2 (coming Spring 2016). Upon completion of both years, **students should be prepared to begin or return to any high school algebra course.**

Are There Any Prerequisites?

Year 1: Students should have a **basic knowledge of arithmetic** (basic arithmetic will be reviewed, but at a fast pace and while teaching problem-solving skills and a biblical worldview of math) and **sufficient mental development** to think through the concepts and examples given. Typically, anyone in 6th grade or higher should be prepared to begin.



Student Workbook

Year 2: It is strongly recommended that students complete Year 1 before beginning Year 2 (coming Spring 2016), as math builds on itself.

What Are the Curriculum's Components?

The curriculum consists of the ***Student Text*** and the ***Student Workbook***. The *Student Text* contains the lessons, and the *Student Workbook* contains all the worksheets, quizzes, and tests, along with an Answer Key and suggested schedule.

How Do I Use This Curriculum?

General Structure — This curriculum is designed to be self-taught, so students should be able to read the material and complete assignments on their own, with a parent or teacher available for questions. This student book is divided into chapters and then into lessons. The number system used to label the lessons expresses this order. The first lesson is labeled as 1.1 because it is Chapter 1, Lesson 1.

Worksheets, Quizzes, and Tests — The accompanying *Student Workbook* includes worksheets, quizzes, and tests to go along with the material in this book, along with a suggested schedule and answer key.

Answer Key — A complete answer key is located in the *Student Workbook*.

Schedule — A suggested schedule for completing the material in 1 year, along with an accelerated 1-semester schedule, is located in the *Student Workbook.*

General Notes to Students

Review — If at any point you hit a concept that does not make sense, *back up and review the preceding concepts.*

Showing Your Work — Except for mental arithmetic problems, you should show your work on all word problems — this means you should write down enough steps of what you did that someone can see how you solved the problem (what you added, subtracted, etc.). Unless otherwise specified, it does not matter how you show your work (it doesn't have to be as in-depth as the answer key) — the important thing is that you can see how you obtained your answer. While showing your work may seem like busy work on simple problems, forming the habit of organizing your steps on paper from the beginning will greatly help you when you come to in-depth problems involving numerous steps.

Units of Measurement — If a unit is given in the problem (miles, feet, etc.), you should always include a unit in your answer.

Fractions — From 5.3 on, fractional answers should be denoted in simplest terms, unless otherwise specified. This includes rewriting mixed numbers as improper fractions. If a question is asked using only fractions, your answer should be listed as a fraction.

Decimals — From 7.4 on, decimal answers should be rounded to the hundredth digit unless otherwise specified.

CONTENTS

Introduction and Place Value

1.1 Math Misconceptions

Math — what does the word bring to mind? Numbers in a textbook? Lists of multiplication and division facts? Problems to solve?

That about sums up the typical view of math, doesn't it? Yet while math does have numbers, facts, and problems, there's much more to math than typically presented.

But before we look at what math is, let's start by examining what it is not. Specifically, let's take a look at three common — but dangerous — misconceptions about math.

Misconception 1: Math Is Neutral

Most math books approach math as a neutral subject. And at first glance, math certainly appears neutral. Neutral means "not engaged on either side; not aligned with a political or ideological grouping."[1] Christians and atheists all can agree that "1 + 1 = 2." This makes math neutral, right?

To answer this, consider a tree. A tree seems pretty neutral too, doesn't it? People of all religions can see a tree, touch a tree, smell a tree, and study a tree, agreeing on a tree's basic features. But this does not mean a tree is neutral. A tree's very existence begs for an explanation. Where did trees originate? Why does a tree have intricate parts that all work and grow together?

Our underlying perspective regarding a tree is determined by what we would call a **worldview**. In *Understanding the Times*, David Noebel (founder of Summit Ministries) defines a worldview this way: "A worldview is like a pair of glasses — it is something through which you view everything. And the fact is, everyone has a worldview, a way he or she looks at the world."[2] In other words, a worldview is a set of truths (or falsehoods we believe to be true) through which we interpret life.

Those with a biblical worldview — those looking at life in light of what the Bible teaches — would see a tree as part of God's originally perfect but now fallen creation, while those with a naturalistic worldview might say a tree evolved from a cosmic bang. When we look at the essential questions of a tree — where it came from, how we should use it, etc. — we see a tree is not really neutral.

In a similar way, math facts may seem neutral. People of all religions can use math and agree that "1 + 1 = 2." But this does not mean math is neutral. Where did math originate? Why does math work the way it does? Why are we able to use math?

Just as it does in the case of a tree, the Bible gives us a framework from which we can answer these questions and build our understanding of math. As we'll discover, only the biblical explanation for math's very existence makes sense out of math and transforms math from a dry list of facts to an exciting exploration.

The point here is simply that math cannot be neutral. The Bible teaches Jesus is Lord of all — the Creator and Sustainer of *all* things (Colossians 1:16–17). He doesn't exempt math from that. Math cannot be separated into a "neutral" box.

Misconception 2: A Biblical Math Curriculum Is the Same as Any Other, with a Bible Verse or Problem Thrown In Now and Then

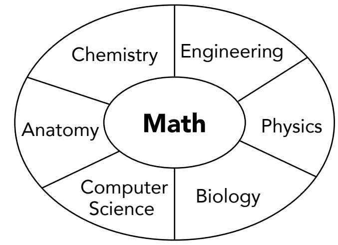
If you're wondering if we're just going to add a Bible verse to the top of the page, mention God dividing the Red Sea when we discuss division, and have you solve Bible-based word problems, let me assure that this is *not* what this curriculum aims to do. Although thinking about how God divided the Red Sea might be helpful in turning our eyes to the Lord, it does nothing for helping us understand how to view *division itself* in light of biblical principles. In this course, we're aiming to let the Bible's principles transform our view of *math itself.*

Misconception 3: Math Is a Textbook Exercise

Quite often, math comes across as a textbook exercise. We memorize this and solve that. There are so many seemingly arbitrary rules to follow that it's easy to scratch your head and wonder who invented this complex system in the first place.

If your view of math is confined to rules and problems — or even if you know there's more to math but are not sure why it feels so dry — there is good news! Math is *not* a mere textbook exercise. Math helps us observe the design throughout creation, design instruments, draw, build boats, operate a business, work with computers, cook, sew, and more. In this course, we'll incorporate history, science, and real-life applications as we go, exploring math both inside *and* outside of a textbook.

1.2 What Is Math?



If you were to try to work in nearly any field of science — be it chemistry, engineering, or anatomy — you would need to study and use math. Why? *Because math is the tool scientists use to explore creation.*

Not only do scientists use math, but artists, pilots, musicians, business managers, clerks, sailors, and homemakers do too. All occupations use math to one extent or another!

Math also shows up in everyday life. Every time you go shopping, you use math — math helps you know how much an item costs (price tags use numbers!), find the unit price of an item, estimate your total, etc. You use math in the kitchen when you measure ingredients. You use math to count the number of silverware to put on the table, to balance a checkbook and track your finances, to understand loans and car payments, to figure out how many bags of bark you need to landscape a flowerbed or how many square feet of carpet to cover a room — the list of math's everyday uses goes on and on.

Math is clearly more than intellectual rules and techniques found in a textbook. Which brings us to the question: what *is* math?

When we **start with the Bible** — God's revealed Word to man — as our source of truth, it makes sense out of every area of life, including math. It gives us a framework for answering not only *what* math is, but also *where* it came from and *why* it works. Take a look at just a few truths with me.

- **All things were created and are sustained by the eternal, triune God of the Bible.**

> *In the beginning God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters (Genesis 1:1–2).*
>
> *In the beginning was the Word, and the Word was with God, and the Word was God. The same was in the beginning with God. All things were made by him; and without him was not any thing made that was made. . . . And the Word was made flesh, and dwelt among us (and we beheld his glory, the glory as of the only begotten of the Father,) full of grace and truth (John 1:1–3, 14).*
>
> *Jesus answered them. . . . "I and my Father are one" (John 10:25, 30).*
>
> *For by him [Jesus] were all things created, that are in heaven, and that are in earth, visible and invisible, whether they be thrones, or dominions, or principalities, or powers: all things were created by him, and for him: And he is before all things, and by him all things consist (Colossians 1:16–17).*

[referring to Jesus] . . . upholding all things by the word of his power . . . (Hebrews 1:3).

- **God is a consistent God who never changes — and who has appointed the ordinances of heaven and earth.**

  For I am the LORD, I change not; therefore ye sons of Jacob are not consumed (Malachi 3:6).

  Thus saith the LORD; If my covenant be not with day and night, and if I have not appointed the ordinances of heaven and earth; Then will I cast away the seed of Jacob and David my servant, so that I will not take any of his seed to be rulers over the seed of Abraham, Isaac, and Jacob: for I will cause their captivity to return, and have mercy on them (Jeremiah 33:25–26).

- **God created man in His own image and gave him the task of subduing the earth.**

  So God created man in his own image, in the image of God created he him; male and female created he them. And God blessed them, and God said unto them, Be fruitful, and multiply, and replenish the earth, and subdue it: and have dominion over the fish of the sea, and over the fowl of the air, and over every living thing that moveth upon the earth (Genesis 1:27–28).

Let's look at how these truths apply to math. A never-changing God is holding *all* things together and has appointed the ordinances — or the decrees — by which heaven and earth operate. God created and sustains a consistent universe. God also created man in His image, capable of subduing and ruling over the earth.

We already established that math is the tool scientists use to describe creation. In other words, math is a way of describing the consistencies God created and sustains! Man is able to use math to, in a very limited way, think "God's thoughts after Him" (Johannes Kepler) because God made us in His image and gave us the ability to subdue the earth.

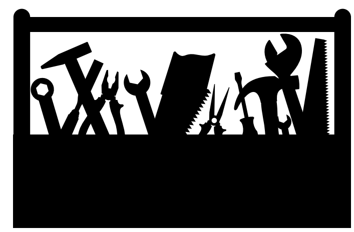
The Bible teaches that God created all things — and math is no exception. The symbols and techniques we think of as math describe on paper the ordinances by which God governs all things. Men might develop different symbols (people have used many different numerals and techniques throughout history, as we'll see throughout this course), but men have no control over the principles. No matter what symbols we use to describe it, one plus one consistently equals two because God both decided it would and, day in and day out, keeps this ordinance in place!

Math, in essence, is **a way of describing the consistent way this universe operates.** It is the language, so to speak, we use to express the quantities and consistencies around us — quantities and consistencies God created and sustains.

Math works outside a textbook *because* God is faithful to uphold all things. Math facts never change *because* God never changes. We can rely on math *because* we can rely on God. Math is complex and complicated *because* God created a complex universe and it takes a lot of different rules and methods to even begin to describe it! Math applies universally *because* God's rule is universal — He's present everywhere. Math helps us see the incredible wisdom and care displayed throughout creation — an order, wisdom, and care that is there *because* we have a wise and caring Creator! At the same time, math reveals the effects of sin that mar God's original design — effects that are there *because* of man's sin, but which remind us of the mercy found in Jesus.

> *Mathematics transfigures the fortuitous concourse of atoms into the tracery of the finger of God. — Herbert Westren Turnbull*[3]

Do you catch how this understanding could revolutionize our view of math? Math doesn't have to be a dry subject of mere numbers and techniques. Numbers and techniques are tools to describe God's creation and help us with the real-life tasks God's given us to do. As Walter W. Sawyer points out, mathematics is like a chest of tools.



> *Mathematics is like a chest of tools. — Walter W. Sawyer*[4]

I love that imagery. Picture a chest of tools for a moment. Some tools — such as a screwdriver — are easy to use and apply in thousands of situations. Other tools — such as a router — take more time and dedication to master and serve a more limited, although just as necessary, purpose.

In a similar fashion, some math concepts — such as addition — are fairly easy to grasp and frequent in their applications. Others — such as some aspects of algebra or calculus — require more dedication to grasp. These higher-level concepts, while they might not come in handy as often as addition, have *very* powerful applications.

From basic to advanced, *all* of math is a tool that points us to God and can be used to complete the tasks He gives us to do!

Ready to Begin?

Some of you reading this course probably dislike math or find it an incredibly challenging task. Others of you may love it and be gifted in it.

Whatever your current view of math, I invite you to take a journey with me. While we'll be seeking to approach concepts from a biblical worldview throughout the course, these first two chapters will be extra-intensive in that department, as we want to lay a firm foundation upon which to build the rest. So please bear with the extra amount of reading.

My prayer is that you'll acquire a deeper appreciation for God's greatness and faithfulness and be encouraged in your walk with Him as you delve into the world of mathematics.

1.3 The Spiritual Battle in Math

The Bible gives us a solid foundation for why math works. Math is a tremendous testimony to God's faithfulness and power. Yet math has been sadly twisted.

Let's take a deeper look at the spiritual battle within math, at how men try to explain math apart from God, and at how ultimately only a biblical worldview makes sense out of math.

Naturalism in Math

Consider the following quote:

> *One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.*
> *— Heinrich Hertz (German physicist)*[5]

Notice that Mr. Hertz is claiming that mathematical formulae themselves are wise and have an independent existence. Rather than acknowledging God, he is giving *math itself* the credit for math's amazing ability to work. This is a very naturalistic view of math — an attempt to explain math from only natural causes, apart from God.

Let's think about this claim for a moment. Can math itself explain its own existence? Remember, math goes hand in hand with creation. Things don't just "happen." We live in a universe consistent enough that we can describe gravity using math and call it a law. If the universe were run by random processes, why would we see such order, design, and consistency?

Besides this, viewing math as a self-existent truth still doesn't answer the fundamental question of how we *know* it's true in the first place. Is the foundation for truth our experience? Do we know that one plus one equals two because we experience that it does, and therefore assume that it always will?

Our experience in itself is not a solid foundation for truth. For one, we can never experience everything, so therefore could never truly know anything for sure! Math is so useful because it helps us solve problems we have not experienced. It allows us to calculate the force needed to get a brand-new rocket into the air — and to predict how a bridge will hold weight before we build it. Much of math deals with things that we can never actually experience, but which help us solve a variety of real-life problems. In order to use math, we *have* to assume it works consistently in areas we have never — and never can — experience.

Humanism in Math

Now consider this quote:

> *The German mathematician Leopold Kronecker (1823–91) once said, "God made the integers, all else is the work of man." First causes, this comment suggests, are divine, while the complexities, minutiae, and refinements of mathematics are a human creation. For Kronecker's contemporary Dedekind, however, the integers too were the "free creations of the human mind." . . . For him, as for many modern mathematicians and theorists, mathematics stood as an independent and secular discipline.* — Denis Guedj[6]

Who did Kronecker and Dedekind give credit for math? *Man*. Both these men viewed math as the product of the human mind. Rather than giving God the credit for math's ability to work, they gave it to man. This is a humanistic view of math — a view that focuses on *man* and *his* achievements, ignoring his Creator.

While it is true that man developed math symbols and techniques, it makes no sense why those symbols and techniques mean anything in real life if they truly are the "free creations of the human mind" as Dedekind stated.

Let's think about the problem with basing truth on human reasoning. Time and again, math concepts men think up using mathematical reasoning end up applying in creation. Why is this? Why do men's thoughts line up with reality? Why do we find such an orderly, mathematical world all around us?

Albert Einstein expressed the problem this way — and admits there's something miraculous in the world that can't be explained by reasoning alone.

> *Even if the axioms of the theory are posited by man, the success of such a procedure supposes in the objective world a high degree of order which we are in no way entitled to expect a priori [based on man's reasoning]. Therein lies the "miracle" which becomes more and more evident as our knowledge develops. . . . And here is the weak point of positivists and of professional atheists, who feel happy because they think that they have not only pre-empted the world of the divine, but also of the miraculous.*[7]

Also, why are there universal laws of logic we rely on to be true? Why can't one person decide that 1 plus 1 will equal 2 and another that it will equal 3 and they both be right? This sort of thinking, if applied consistently, would completely make math, as well as logic itself, meaningless and useless!

For more information different worldviews on math and how the biblical worldview makes sense of math, see James D. Nickel, *Mathematics: Is God Silent?* rev. ed. (Vallecito, CA: Ross House Books, 2001). For more information on how logic itself can't be explained apart from God, see Dr. Jason Lisle, *The Ultimate Proof of Creation: Resolving the Origins Debate* (Green Forest, AR: Master books, 2009).

The Battle Defined

The spiritual battle over math is the same as the battle we find in other areas of life. Will we recognize our *dependency* on God, or claim *independence* from Him?

Our view of any area of life — including math — is either going to stem from a dependent perspective on life (one that recognizes our dependency on God and His Word) or an independent one. When we get down to the fundamental level, there is no such thing as neutrality. Even a tree is not neutral — as we saw in the first lesson, the tree was either created by God or got here some other way.

Likewise, math is either dependent on God or it is not. If God does not receive the glory for math's ability to work, that glory goes somewhere else. As R.J. Rushdoony points out:

> *. . . mathematics is not the means of denying the idea of God's pre-established world in order to play god and create our own cosmos, but rather is a means whereby we can think God's thoughts after Him. It is a means towards furthering our knowledge of God's creation and towards establishing our dominion over it under God. The issue in mathematics today is root and branch a religious one.*[8]

The Bible is clear: we are to trust and worship God; He gives us our every breath, He controls each aspect of life, and He determines truth — apart from Him we are nothing. If man ignores this truth, he does so to his own demise.

> *For the wrath of God is revealed from heaven against all ungodliness and unrighteousness of men, who hold the truth in unrighteousness; Because that which may be known of God is manifest in them; for God hath shewed it unto them. For the invisible things of him from the creation of the world are clearly seen, being understood by the things that are made, even his eternal power and Godhead; so that they are without excuse: Because that, when they knew God, they glorified him not as God, neither were thankful; but became vain in their imaginations, and their foolish heart was darkened. Professing themselves to be wise, they became fools, And changed the glory of the uncorruptible God into an image made like to corruptible man, and to birds, and fourfooted beasts, and creeping things (Romans 1:18–23).*

The Depth of the Battle

The battle over math is much more than a theological squabble over numbers. It ultimately affects our entire approach to truth.

If we look at math as something spiritually neutral — a self-existent or man-made fact — then math becomes an independent source of truth. We find ourselves viewing math as the ultimate standard rather than God's Word.

Millions of people have embraced the lie of evolution because they believe it has been scientifically proven to be true. At the root of their belief is the false notion that deductive reasoning or mathematical principles are the ultimate standard ruling the universe.

Yet, apart from God, it does not even make sense why we can reason or why the universe is consistent! Science and math would be impossible in a universe without God. The very tool skeptics try to use to disprove God cannot be explained apart from God. Even honest unbelievers acknowledge their inability to explain math in their worldview. Most simply ignore the *why*.

In this article I shall not attempt any deep philosophical discussion of the reasons why mathematics supplies so much power to physics. . . . The vast majority of working scientists, myself included, find comfort in the words of the French mathematician Henri Lebesgue: "In my opinion a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy — an opinion, moreover, which has been expressed by many philosophers."
— Freeman J. Dyson[9]

But when we look at math from a biblical perspective, we understand that math is not a source of truth; it is a description of the consistencies of God. God is the source of truth. We can only rely on math to work because we can trust God. Thus, as we study math in this course, we will not approach it as a means to determine truth or as the source of truth, but rather as a tool to help us understand the trustworthy principles our trustworthy God created and sustains.

Math and the Gospel

Although we might try our hardest, we cannot change math. We can change the symbols or names we use in math, but we cannot change what the names and symbols represent — 1 of something plus 1 of something else will consistently equal 2. Math is not relative. Why?

Because God is God and we are not! He, not us, decides how things will be. He set and keeps certain principles in place, and if we want a math that will actually work, we *have* to conform to those principles.

Math reminds us that God decides truth, not us. We need to be careful in every area that we take head to the truth He has revealed to us in His Word, the Bible, and that we don't try to change those truths to fit what we think or want. For example, the Bible tells us that salvation is by faith in Jesus, and not by works or any other way (Ephesians 2:8; James 14:6), and that hell is real (Revelation 21:8).

It's tempting to try to change this truth, thinking there must be some good in ourselves or that God would not really send people to hell (especially those whom we love and think are nice), but God's truth is not open to negotiation. He's God, not us. If we want salvation, we have to conform to what *God* says will save us.

Over and over again, the Bible, God's Word, urges us to trust in God's way of salvation — Jesus. Only He could pay the penalty for sin. Only by believing upon Him — admitting our own helplessness — can we be saved. Just as God is faithful to hold this universe together consistently, He will be faithful to everything else He says in His Word. You can rely on what God says.

If you've not responded to God's gift of salvation, today is the day to do so! He will keep His Word — both to save and to punish.

If you're not sure if you have trusted God's way of salvation, don't delay in making sure. If you are sure, then take tremendous comfort in the knowledge that God is faithful and will complete what He began in you.

For more information on God's plan of salvation, please see www.biblicalperspective.net.

Keeping Perspective

The battle we face in math is ultimately a battle to remember our complete dependency on God. Even our ability to count comes from Him! Each math concept works only *because* of His faithfulness. Apart from Him, we truly can do *nothing*.

Ever since the Garden of Eden, Satan has been trying to distort the truth and get men to trust themselves instead of God. He has done this very thing in math — turning what should be a testimony to God into a testimony to man and math.

We can all see that math works. Someone or something has to be responsible for math's ability to work. If we're not giving the glory for math to God, then we're ending up giving it to man or to math. If math is not encouraging us that we can depend on our faithful, all-powerful God, then it is subtly telling us we can live independently from Him and determine our own truth.

Yes, indeed, there is a spiritual battle in math — and it's the same battle we face in every area of life.

1.4 Numbers, Place Value, and Comparisons

Now that we've seen the overall foundation the Bible gives us and explored a little about the spiritual battle in math, let's begin applying what we've discussed to specific aspects of math. In order to build our understanding of math from the foundation up, we'll be exploring simple review concepts for these first few chapters. As we do, though, we'll be learning important principles that apply to more advanced concepts.

An Overview of Mathematical Symbols and Terms

Math is filled with symbols and terms. Just as it is helpful if we use the same words to refer to objects (a book, sink, couch, etc.), it's helpful to use standardized symbols and terms in math.

Before we jump into looking at specific symbols and terms, though, let's take a minute to look at the big picture. Much of math is a naming process — a way of describing quantities and consistencies God created. So let's take a look at the first "naming" process the Bible describes: Adam naming the animals.

> *And out of the ground the LORD God formed every beast of the field, and every fowl of the air; and brought them unto Adam to see what he would call them: and whatsoever Adam called every living creature, that was the name thereof (Genesis 2:19).*

In naming the animals, Adam

1. observed God's creation (the animals) and
2. assigned names to describe the different animals.

In describing quantities, we

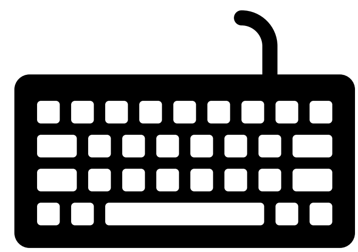
1. observe God's creation (the quantities around us) and
2. assign names (or symbols) to different quantities.

So what can we learn from Adam naming the animals? Well, notice that God brought the animals to Adam for naming — Adam was in God's presence while observing and naming. While sin separated man from his Creator, through Jesus we can again **know God and worship Him while using math to describe His creation.** This holds true not just for basic math, but for *every* area of math we'll explore. In fact, the Bible urges us to do "whatsoever" we do "as to the Lord"!

> *And whatsoever ye do, do it heartily, as to the Lord, and not unto men; (Colossians 3:23).*

Number Systems: Beyond Quantities

Number systems prove useful in other ways besides recording quantities too. For example, house numbers and telephone numbers don't record quantities — instead, they give us a way of "naming" homes and telephone lines. As another example, numbers and math are used in cryptography ("the art of writing or solving codes")[11] to help code messages. And before you picture coding as only wartime messages across enemy lines, did you realize that computers use a code to translate the letters or symbols you type on a keyboard? There's a number assigned to every letter or symbol that can be typed on a keyboard!

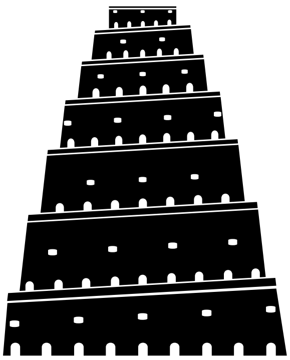


Reviewing Numerals and Place Value

Undoubtedly, you already know how to count (use words — like "one" — to describe quantities) and write numerals (use symbols — like "1" — to describe quantities). Below is just a quick review.

"Zero" is the name we mainly use in English to describe having nothing (you may also sometimes hear other names, such as "nought," "oh," or "nil," used to mean nothing). "One" is the name for a single unit — a single pen, dollar, CD, etc. "Two" is the name for a group of 2 units of anything.

Rather than words, we commonly use symbols. It's a lot easier to write "1" than to spell out "one" all the time! At the same time, though, it would be impossible to have a different symbol for *every* number. Instead, we use a concept known as place value.



Different Words for Quantities — The Tower of Babel

Different cultures use different words to describe quantities. For example, a single quantity is called *one* in English, *uno* in Spanish, *eine* in German, and *один* in Russian. Once again, the Bible tells us why.

Genesis 11:1–9 tell us about an event that changed the world — the Tower of Babel. Prior to the Tower of Babel, "the whole earth was of one language, and of one speech" (Genesis 11:1). Thus, men would have used the same words to describe quantities.

At the Tower of Babel, men misused the ability God had given them to communicate and sought to unite against God and make a name for themselves. The project stopped abruptly when God came down and confused their languages. The Tower of Babel accounts for the many different language systems we find, including the different words used to describe numbers.

In place value, the place, or location, of a number determines its value. So we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent up to nine. Once we reach ten, we move to the next "place" over, using the same digits, but knowing that each one represents a set, or group, of ten. So "10" represents 1 set of ten, and 0 sets of one, "20," represents 2 sets of ten, and 0 sets of one, and "21" represents 2 sets of ten, and 1 set of one.

This place-value concept can be extended as far as necessary to represent numbers. Once we have ten tens, we move on to hundreds, then thousands, then ten thousands, then hundred thousands, and so forth.

Hundred trillion	Ten trillion	One trillion	Hundred billion	Ten billion	One billion	Hundred million	Ten million	One million	Hundred thousand	Ten thousand	One thousand	Hundred	Ten	One

Using our place-value system, we would represent the approximate world population in 2011 as 6,946,043,989 (or six billion, nine hundred forty-six million, forty-three thousand, nine hundred eighty-nine).[10]

Hundred trillion	Ten trillion	One trillion	Hundred billion	Ten billion	One billion	Hundred million	Ten million	One million	Hundred thousand	Ten thousand	One thousand	Hundred	Ten	One
					6	9	4	6	0	4	3	9	8	9

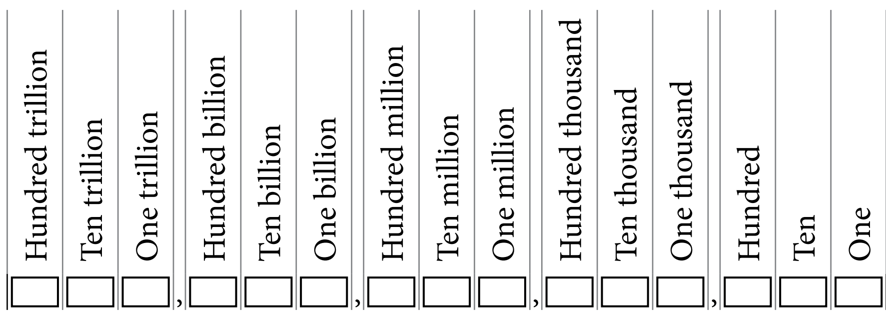
Notice how the commas every three places help our eyes count the places and determine the value.

4444444 4,444,444

In other countries, decimal points (4.444.444) or other separators are used instead of commas. (An important thing to keep in mind if ordering something online from another country!) Spaces (4 444 444) are also a recognized way of separating the places.

Reading Numbers

Notice how when reading numbers, we recycle terms. We start with ones (our basic units), tens, and hundreds. Then we have thousands (our new unit), followed by *ten* thousands and *hundred* thousands. We repeat this for millions, billions, etc.



Notice also that in writing, we use commas every three digits, thereby separating the "thousands," "millions," etc.

To read a number, we read the number from left to right. If a digit has a zero, we don't read that place, as there's nothing to "report" there (as in the 0 in the hundred's place in 123,456,567,087).

123,456,567,087 would be read "one hundred twenty-three billion, four hundred fifty-six million, five hundred sixty-seven thousand, eighty-seven."

Now, I am sure you already know how to read numbers in English, but did you realize that there are variations in how to read numbers? The British often add an "and" (example: "one hundred *and* twenty-one" instead of "one hundred twenty-one"). 1,325 could also be read as "thirteen twenty-five" instead of as "one thousand three hundred twenty-five." This might make sense for dates ("the year thirteen twenty-five") or even house numbers ("I live at thirteen twenty-five Pleasant Lane"). When reading a street address over the phone, you might even just read each digit by itself, as in "one, three, two, five Pleasant Lane" to avoid confusion. These variations remind us that **names are a tool to help us communicate**, so clearly communicating is the most important thing.

When asked to write the word you would use to read a number in this course, use the traditional American method ("one thousand, three hundred twenty-five" for 1,325).

Reviewing Basic Comparison Terms and Symbols

If a number is larger, or has more, than another number, we say it is **greater than** the other number. If it is smaller/has less, we say it is **less than** the other number. If two quantities are the same, we say they are **equal**. If they are not the same and we do not want to make a specific comparison as to which one is greater, we say they are **not equal**. (Any number that is greater than or less than another number is also not equal to it — it's just a matter of what point we want to make.)

Notice that the "less than" and "greater than" signs are the same, but pointing the opposite directions. You can remember which direction to put the symbol by remembering that the **larger side goes with the larger number.**

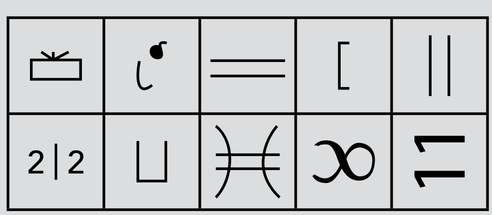
The symbols <, >, =, and ≠ are merely "shortcuts" for describing how numbers compare. They save our fingers from having to write the word out every time. It's a lot easier to write < than "less than." It also makes equations easier to read.

● ● ● ● ● 5	5 is less than or does not equal 6. < or ≠	● ● ● ● ● ● 6	● ● ● ● ● 5	5 equals 5 =	● ● ● ● ● 5
● ● ● ● ● ● 6	6 is greater than or does not equal 5 > or ≠	● ● ● ● ● 5	● ● ● ● ● ● 6	6 equals 6 =	● ● ● ● ● ● 6

Would it surprise you to learn that >, <, =, and ≠ are algebraic symbols? Any time we use a non-numerical symbol in math, we are actually using algebra. So >, <, =, and ≠ are actually part of algebra! Algebra is nothing to fear — it's just a way of using symbols to describe God's creation. In the case of >, <, =, and ≠ , we're using symbols to describe how numbers compare.

Different Ways to Compare Numbers

Much as symbols for writing numbers have varied, so have symbols for comparing them. While we're used to using the "=" sign to mean "equal to," other symbols have been used throughout history — the box shows just a few. Instead of symbols, many cultures also used words or contractions to describe equality (*pha, equantur, aequales, gleich*, etc.).[12] Once again, history helps us see that the symbols we study in math are just an agreed-upon language system we use today to describe the quantities and consistencies God created and sustains.



Keeping Perspective

We looked today at a few names (one, two, three, etc.) and symbols (1, 2, 3, =, >, <, etc.) used in math. As we continue our study of math, we're going to learn various names and symbols men have adopted to describe different consistencies or operations. Keep in mind that **terms and symbols are like a language** — agreed-upon ways of communicating about the quantities and consistencies around us.

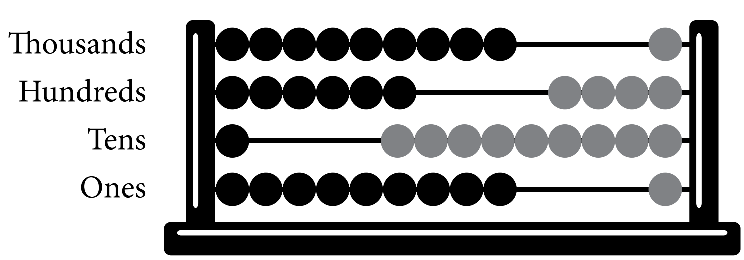
1.5 Different Number Systems

It's all too easy to start viewing the terms, symbols, and methods we learn in math as math itself, thereby subtly thinking of math as a man-made system. A look at history, however, reveals many other approaches to representing quantities. Let's take a look at a few of them and at how they compare with our place-value system.

Place-Value Systems

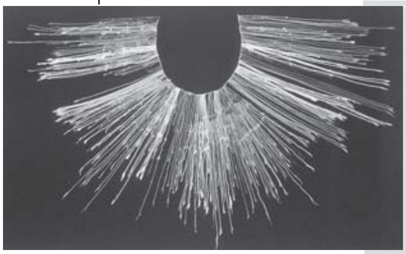
In the last lesson, we reviewed how the number system we're mainly familiar with uses the place, or location, of a digit to determine its value. This is known as a **place-value system.**

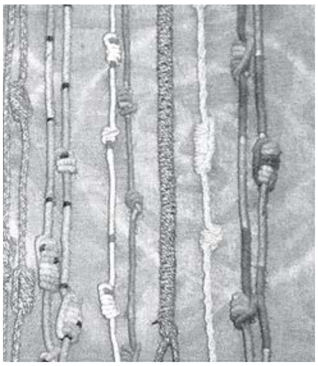
Perhaps place value is easiest to picture using a device used extensively throughout the Middle Ages: an abacus. Each bead on the bottom wire of an abacus represents one; on the next, ten; on the third, one hundred; and on the fourth, one thousand. To represent a quantity on an abacus, we move the appropriate number of beads from each wire to the right. In the abacus shown, the 1 bead to the right on the thousands wire represents 1 thousand, the 4 beads to the right on the hundreds wire represent 4 hundred, the 9 beads to the right on the tens wire represent 9 tens, and the 1 bead on the ones wire represent 1. Altogether, that makes 1,491.



1,491

Just as the place, or line, of a bead changes its value, the place, or location, of a symbol in a place-value system changes its value. The number system commonly used today is called the **Hindu-Arabic decimal system** (or just the "**decimal system**" for short). This system came from the Hindu system, which the Arabs adopted and brought to Europe.





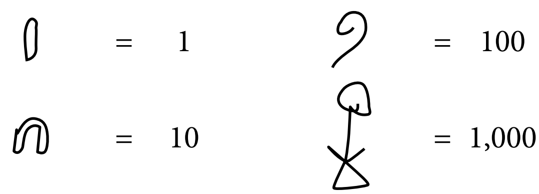
The Quipu — An Intriguing Approach

The Incas — an extensive empire in South America spanning more than 15,000 miles — had a fun approach to recording quantities. They tied knots on a device called a quipu (kē pōō).[13] The quipu system was extremely complicated, and only special quipu makers, called quipucamayocs, were able to interpret them. Although we do not know a lot about quipus, we do know they used place value. The location of the knot, along with some other factors, determined its value.

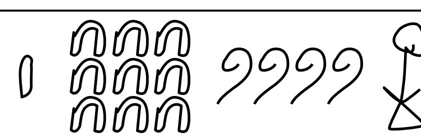
Apparently, the Incas were very successful with this innovative approach. Not only did they operate a huge empire, but the Incas baffled the Spanish conquerors by their ability to record the tiniest details as well as the largest ones on their quipus.[14]

Fixed-Value Systems

A different approach to recording quantities is to *repeat* symbols to represent other numbers. For example, here are some symbols in Egyptian numerals (hieroglyphic style).[15]



The next figure shows two different quantities represented using Egyptian numerals and our decimal place-value system. Notice how when writing twenty-two, the Egyptians repeated their symbol for one and their symbol for ten twice. They put the smaller values on the left and the larger values on the right. Thus the symbol for ten (𓎆) is to the right of the symbol for one (𓏺).

Decimal System a place-value system	**Egyptian System** a fixed-value system
22	
1,491	

We'll refer to number systems that use repeated symbols like this as **fixed-value systems**.

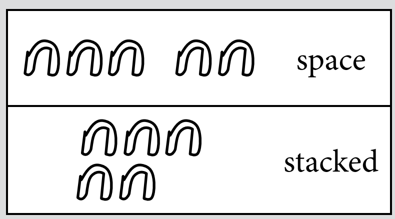
A Deeper Look at the Egyptian System

Notice that in the Egyptian version of 1,491, the symbols representing "ninety" are stacked on top of each other.

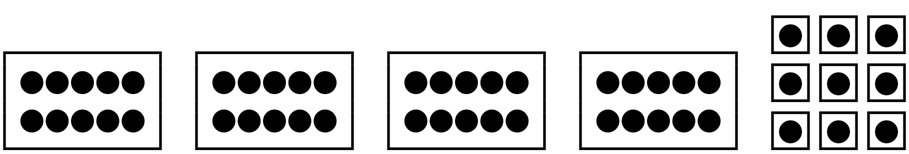
This: Rather than this:

While there were many variations within the Egyptian system over time, in general, when writing more than four of each symbol, the Egyptians **spaced, stacked, or grouped the symbols in sets (groups) of four or less**, with the larger set on top or first.[16] This practice made it easier to count the symbols (and thus to read the number!) at a glance.

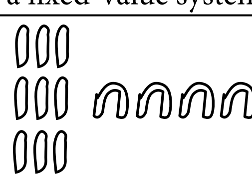
50



Let's compare our decimal place-value system with the Egyptian system. To record forty-nine objects in the Egyptian system, we would repeat the symbol for "one" nine times to show we had nine ones, and then repeat our symbol for "ten" four times to show we had four sets of ten. In the decimal system, we would instead use our symbols for four and nine, putting the 4 in the tens column so it represents four sets of ten and 9 in the ones column, representing nine sets of one.

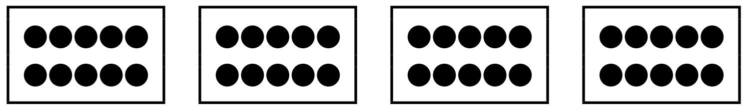


"Forty-nine" = four sets of ten and nine ones

Decimal System a place-value system	**Egyptian System** a fixed-value system
49	

When we compare forty-nine in both systems, we see it takes significantly fewer symbols to represent the number in the decimal system. Place value saves a lot of extra writing!

To represent a number like forty in the decimal system, we would again use a 4, adding a zero (0) to represent that we have no (0) sets of one. Notice the importance of a zero (0) in a place-value system; without it, we would have no way of showing that the 4 represents 4 sets of ten instead of 4 sets of one.



set of 10 set of 10 set of 10 set of 10

"Forty" = four sets of ten and no ones

Decimal System a place-value system	**Egyptian System** a fixed-value system
40	𓎆𓎆𓎆𓎆

Ordered Fixed-Value Systems

Another approach to recording quantities is to again use a limited number of symbols and repeat those symbols, but to add rules regarding their order that change the symbols' meaning. Roman numerals are an example of an ordered fixed-value system.

Take a look at these symbols used for quantities in Roman numerals:

I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

As with the Egyptians, quantities in Roman numerals are represented by repeating symbols, although this time with the larger quantities on the left.

22 is written XXII in Roman numerals.

But unlike in the Egyptian system, the same symbol is generally not repeated more than three times. Instead, it is assumed that whenever a symbol representing a smaller quantity is to the *left* of a symbol representing a larger quantity, one should *subtract* the value of the smaller quantity from the value of larger quantity to get the value the two symbols represent.

Notice that the smaller quantity is to the *left* of the larger—this means to subtract I from V, giving us 5 – 1, or 4.

Notice that the smaller quantity is to the *right* of the larger—this means to add I to V, giving us 5 + 1, or 6.

I	1	XI	11
II	2	XII	12
III	3	XIII	13
IV	4	XIV	14
V	5	XV	15
VI	6	XVI	16
VII	7	XVII	17
VIII	8	XVIII	18
IX	9	XIX	19
X	10	XX	20

There was a time when "four" was written IIII instead of IV. But IV is easier to read, as there are fewer symbols involved.

Now let's take a look at the same number we looked at with the Egyptians: 1,491.

Decimal System a place-value system	**Roman Numeral System** an ordered fixed-value system
1,491	**MCDXCI** M = 1,000 CD = 500 – 100 = 400 XC = 100 – 10 = 90 I = 1 1,000 + 400 + 90 + 1 = 1,491

Notice that Roman numerals would not lend themselves well to quickly adding or subtracting on paper! There is a reason we use the decimal place-value system for most purposes.

Keeping Perspective

While you may use only our current decimal place-value system on a regular basis, being aware of other systems will help you learn to better see our place-value system as just one system to help us describe quantities.

1.6 Binary and Hexadecimal Place-value Systems

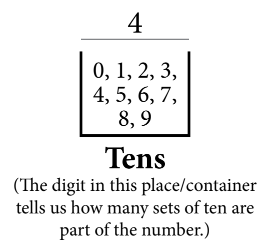
Before we move on, we're going to take one more look at the concept of place value, as it's a pretty important concept. While I'm sure you're quite familiar with our current place-value system, did you realize computers use place-value systems based on a value besides ten?

Well, they do! They use what's known as a binary place-value system. Exploring this system, along with the hexadecimal place-value system, is not only cool, but it can also help provide an even firmer grasp of the decimal place-value system. Let's take a look.

Unwrapping Place-Value Systems

The value we choose for each place in the system is called our **base**. You can picture a base like a container — the size of the container determines how much it can hold. In the decimal system, each place, or container, can hold *ten* digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9); once we reach *ten* of a unit, we move to the next place over, using the same digits, but knowing that each one represents ten of the previous place's value.

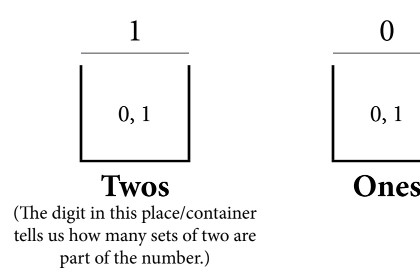
We write forty-two as "42" to represent 4 sets of ten (or 40) plus 2.



Binary System

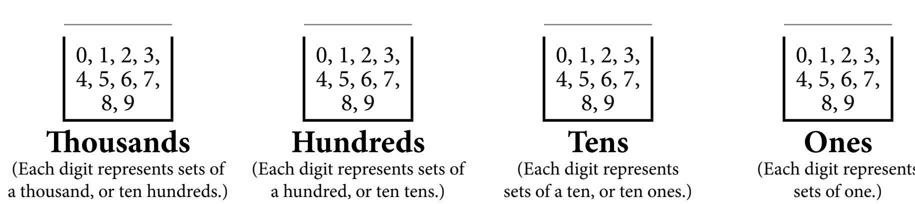
Computers actually operate off a base-two place-value system called the **binary system** (*bi* means *two*). In a binary system, instead of allowing *ten* values (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in each place, we only allow *two* (0, 1). It's as if each place, or container, can only hold the *two* digits: 0 and 1. Once we reach two, we move to the next place over. While in the decimal system, each place is worth ten times the previous place, each place in the binary system is worth two times the previous place.

In binary, the number "10" represents 1 set of *two* and 0 sets of *one*, or two!



To make things clearer, take a look at the first four places, or containers, for both systems.

Decimal (base 10)



Binary (base 2)

0, 1	0, 1	0, 1	0, 1
Eights (Each digit represents sets of eight, or two fours.)	**Fours** (Each digit represents sets of fours, or two twos.)	**Twos** (Each digit represents sets of two, or two ones.)	**Ones** (Each digit represents sets of one.)

Let's take a look at how this plays out with a few numbers.

Example: Find the decimal value of the binary number 1010.

1	0	1	0
0, 1	0, 1	0, 1	0, 1
Eights (Each digit represents sets of eight, or two fours.)	**Fours** (Each digit represents sets of fours, or two twos.)	**Twos** (Each digit represents sets of two, or two ones.)	**Ones** (Each digit represents sets of one.)

1 set of 8 = 1 x 8 = *8*
0 sets of 4 = 0 x 4 = *0*
1 set of 2 = 1 x 2 = *2*
0 sets of 1 = 0 x 1 = *0*
8 + *0* + *2* + *0* = 10

1010 in binary is the same as the decimal number 10.

Example: Find the decimal value of the binary number 1111.

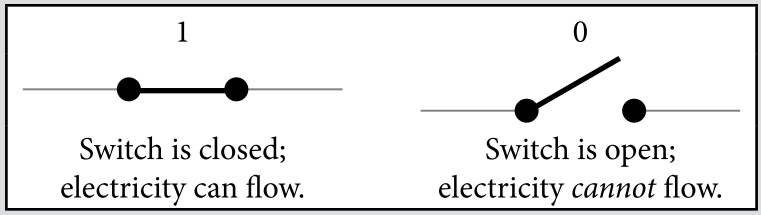
1	1	1	1
0, 1	0, 1	0, 1	0, 1
Eights (Each digit represents sets of eight, or two fours.)	**Fours** (Each digit represents sets of fours, or two twos.)	**Twos** (Each digit represents sets of two, or two ones.)	**Ones** (Each digit represents sets of one.)

1 set of 8 = 1 x 8 = *8*
1 set of 4 = 1 x 4 = *4*
1 set of 2 = 1 x 2 = *2*
1 set of 1 = 1 x 1 = *1*
8 + *4* + *2* + *1* = 15

1111 in binary is the same as the decimal number 15.

Computer Circuits

Because computer circuits run on electricity, the 0 and 1 used in binary numbers can easily describe the "off" and "on" flows of electricity controlled by an open or closed switch. Whenever there's electricity, the computer interprets it as a 1. When there's no electricity, it interprets it as a 0.



Making Computer Talk More Concise: Hexadecimal Numbers

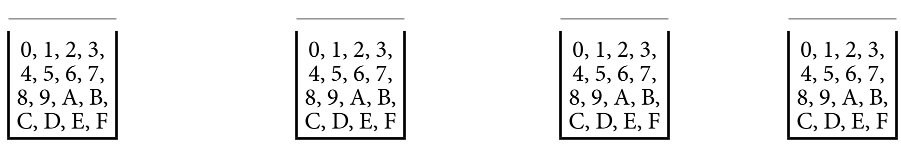
Although binary numbers translate well to electrical pulses, they tend to get long quickly (eight is written 1000), making them difficult for us to read. To help make numbers more readable, computer programs often use hexadecimal numbers (a place-value system based on 16) to represent binary numbers. Because it has a larger base (i.e., a container that can hold more digits), the hexadecimal system can represent very large numbers with fewer digits.

Decimal: 1,200
Binary: 10010110000
Hexadecimal: 4B0

Base 16-Hexadecimal System

16 Symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

A represents the decimal value of 10.
B represents the decimal value of 11.
C represents the decimal value of 12.
D represents the decimal value of 13.
E represents the decimal value of 14.
F represents the decimal value of 15.



Four thousand ninety-sixes (Each digit represents sets of four thousand ninety-six, or sixteen two hundred fifty-sixes.)

Two hundred fifty-sixes (Each digit represents sets of two hundred fifty-six, or sixteen sixteens.)

Sixteens (Each digit represents sets of sixteen, or sixteen ones.)

Ones (Each digit represents sets of one.)

Example: Find the decimal value of the hexadecimal number 4B0.

4	**B**	**0**
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
Two hundred fifty-sixes (Each digit represents sets of two hundred fifty-six, or sixteen sixteens.)	**Sixteens** (Each digit represents sets of sixteen, or sixteen ones.)	**Ones** (Each digit represents sets of one.)

4 sets of 256 = 4 x 256 = *1,024*
11 sets of 16 = 11 x 16 = *176*
0 sets of 1 = 0 x 1 = *0*
1,024 + *176* = 1,200

4B0 in hexadecimal is the same as the decimal number 1,200.

Keeping Perspective

Place-value systems can be based off *any* quantity — and other systems besides the decimal one are in common use today! Each system is a tool to help us describe quantities . . . and each works best in different situations.

While it's not necessary for you to learn the binary or hexadecimal systems (unless you plan to go into computer programming), take some time to explore them a little. Thinking outside the box this way will help you develop your mathematical skills and grow in your ability to use math as a tool.

Decimals

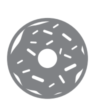
7.1 Introducing Decimals

Throughout this course, we've been using the decimal system (i.e., our base-10 place-value system) to represent whole numbers. It's time now to look at extending this system to also represent partial quantities. (The word *decimal* actually comes from the Latin root *decimus*, which means "tenth."[1])

Most of us were exposed to representing partial amounts in the decimal system since we were little, mainly because we use them to write parts of a dollar. But let's take a closer look at this notation and at how to use it as an effective "tool" in our mathematical toolbox.

Decimals — Extending the Notation

To better understand how to use our decimal system to describe partial quantities, picture a grocery store without decimals. How would you describe the price of items less than $1? You could use fractions.

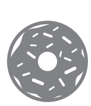


$\$\frac{4}{10}$

$\$\frac{2}{5}$

$\$\frac{1}{2}$

Most likely, you would write all your fractions with the same denominator to make them easier to compare. Since there are 100 cents in $1, it would make sense to use 100 as the denominator.



$\$\frac{40}{100}$

$\$\frac{40}{100}$

$\$\frac{50}{100}$

Now it is easier to compare the cost of each item. We could further simplify expressing these costs by writing these costs in our base-10 decimal system, letting the place, or location, of the number show its denominator.

\$ 0.40 \$ 0.40 \$ 0.50

Let's take a look at what we just did. We basically added what we call a **decimal point** to the right of the ones digits and extended place value to represent partial quantities.

Each place to the left in our decimal system is worth 10 times the previous place, and each place to the right is worth $\frac{1}{10}$ of the previous place. The decimal point separates the whole numbers from the partial ones.

Thousands	,	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
$1{,}000$ or $(10 \bullet 100)$		100 or $(10 \bullet 10)$	10 or $(10 \bullet 1)$	1 or $(10 \bullet \frac{1}{10})$		$\frac{1}{10}$ or $(10 \bullet \frac{1}{100})$	$\frac{1}{100}$ or $(10 \bullet \frac{1}{1{,}000})$	$\frac{1}{1{,}000}$ or $(10 \bullet \frac{1}{10{,}000})$

We're going to begin using a dot (•) rather than the times sign (x) to represent multiplication in our presentations. Since an "x" has a different meaning in algebra, it's important to become familiar with other ways to show multiplication. Just remember, 4 x 2, 4 • 2, and 4(2) all mean four *times* two!

Whether representing a partial quantity or a whole one in the decimal system, the place, or location, of each digit determines its value. Notice how the place of the digit "6" gives it radically different meanings!

$$0.006 = 6 \text{ thousandths or } \frac{6}{1{,}000}$$

$$0.06 = 6 \text{ hundredths or } \frac{6}{100}$$

$$0.6 = 6 \text{ tenths or } \frac{6}{10}$$

$$6 = 6 \text{ ones or six}$$

$$60 = 6 \text{ tens or sixty}$$

$$600 = 6 \text{ hundreds or six hundred}$$

From now on, we'll refer to partial quantities written in the decimal system as **decimals.**

Switching Between Fractions and Decimals

Example: Express 0.6 as a fraction.

What fraction does 0.6 represent? Well, the 6 is in the tenths place ($\frac{1}{10}$), so the 6 represents 6 tenths, or $\frac{6}{10}$.

Example: Express 0.61 as a fraction.

Again, the 6 is in the tenths place, so we have 6 tenths ($\frac{6}{10}$). However, we also have a 1 in the hundredths place, giving us $\frac{1}{100}$. So we have $\frac{6}{10} + \frac{1}{100}$. Rewriting with the same denominator and adding gives us $\frac{60}{100} + \frac{1}{100}$, or $\frac{61}{100}$.

Now, in the last example, we didn't really need to do the addition. Because of how place value works, we could have looked at 0.61 as $\frac{61}{100}$ to begin with. All we really needed to do was remove the decimal point and add the denominator from the right-most place value represented.

> You can always look at the entire decimal part of a number as a fraction of the right-most decimal place represented.

Example: Express 0.612 as a fraction.

We can view this as a fraction of the right-most decimal place represented. So we can think of it as $\frac{612}{1{,}000}$.

We could also have found that via addition:

$$\frac{6}{10} + \frac{1}{100} + \frac{2}{1{,}000} = \frac{600}{1{,}000} + \frac{10}{1{,}000} + \frac{2}{1{,}000} = \frac{612}{1{,}000}$$

You may already know how to convert $\frac{4}{5}$ to a decimal by dividing. We'll get to that later in this chapter, but for now, practice the way shown in the example.

Note: We could then simplify the fraction if we need a simple answer.

$$\frac{612}{1{,}000} \div \frac{4}{4} = \frac{153}{250}$$

Example: Express $\frac{4}{5}$ as a decimal.

Remember, our decimal place-value system expresses partial quantities in terms of tenths or multiples of tenths. So to write this quantity as a decimal, we first need to make its denominator 10 or a multiple of 10.

$$\frac{4}{5} \bullet \frac{2}{2} = \frac{8}{10}$$

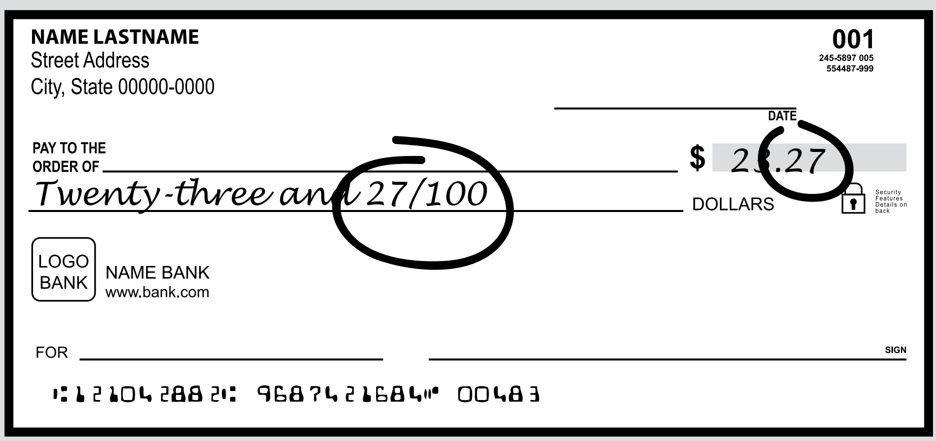
Now we can rewrite it in the decimal system.

0.8

Remember, the first place to the right of the decimal point represents fractions of 10, so 0.8 is another way of saying 8 tenths, or $\frac{8}{10}$.

Applying It: Writing Checks

When writing a check, we write the amount using our decimal system, and then rewrite it in words. Instead of using words, though, the cents are typically written as a fraction (notice that the fraction takes up less space than the words "twenty-seven hundredths" would).



Adding Zeros

You can always add zeros to the right of a decimal point.

$$0.6 = 0.60 = 0.600 = 0.6000 = \frac{6}{10} = \frac{60}{100} = \frac{600}{1{,}000} = \frac{6{,}000}{10{,}000}$$

Adding (or removing) a zero to the right of the last number of a decimal does not change its value; adding (or removing) a 0 actually multiplies (or divides) the fraction by $\frac{10}{10}$, which is worth 1.

$$0.6 = \frac{6}{10}$$

$$\frac{6}{10} \bullet \frac{10}{10} = \frac{60}{100} = 0.60$$

For example, when working with money in America, we represent amounts in terms of dollars and cents. Since there are **100 cents in a dollar**, we always represent the partial amount of a dollar in terms of hundredths.

For example, rather than writing \$0.1, we would add a zero, making this \$0.10 (which is essentially just rewriting $\frac{1}{10}$ as $\frac{10}{100}$). Now it's easy to see that this represents 10 cents.

Reading Partial Quantities in the Decimal System

0.1 could be read "one tenth" just as you would read $\frac{1}{10}$; 3.24 could be read "three and twenty-four hundredths" (notice the "and" used to break up the whole and partial portion of the number).

However, you'll sometimes hear 0.1 read as "point one" and 3.24 as "three point two four" or "three point twenty-four" instead.

Or, if we were dealing with money, we'd read 0.1 as "10 cents" and 3.24 "three dollars and twenty-four cents."

A Look at History

While we're quite used to seeing quantities written with a decimal point today, that hasn't always been the case. Notice some of the different ways "2.16" has been expressed — one mathematician even used our current equal sign as a way of separating whole numbers from partial amounts![2]

2⓪1①6②	216②	$2.\overset{\text{i}}{1}.\overset{\text{ii}}{6}.$	$2,\overset{/}{1}\overset{//}{6}$	2:16
$2\lfloor\underline{16}$	2=16	2▲16	2^{16}	2‘16

Over the years, mathematicians have tried to standardize notation. After all, it is a lot easier not to have to learn a whole new set of symbols for every math book you read! In America, we use a decimal point. In some other countries, however, a comma is used instead of a decimal point. Our current notation is only *one* way to describe quantities on paper. God gave man the creativity to develop different ways to express quantities.

Keeping Perspective

Decimals (i.e., partial quantities written in our base-10 decimal system) are another way of describing and working with quantities. We are only able to explore creation this way *because God gave us this ability*. Mice do not do math, but man can because God created man in His image. Isn't it wonderful that God designed us differently than animals and gave us the ability to fellowship with Him?

The account of King Nebuchadnezzar (Daniel 4) really brings this point home. King Nebuchadnezzar boasted to himself about Babylon, taking credit for building such a great empire. God humbled him and made him like the beasts of the earth. God was showing him *he could not even think or function apart from God's enabling.* We are utterly and completely dependent on God for *everything*, including the ability to think and name quantities!

7.2 Adding and Subtracting Decimals

One of the biggest advantages to expressing partial quantities using the decimal system is that, we can then use the same basic methods to work with them as we do with whole numbers! Again, although you probably already have used these methods with decimal numbers, we're going to take a deeper look at *why* we're able to apply these methods to decimals and *what* we're really doing when we do.

Addition and Subtraction

Notice that ***each* digit — including those to the right of the decimal point — in the decimal system is 10 times the previous one.**

Each place to the left is 10 times the previous place.

$\longleftarrow$

Thousands		Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
1	,	**1**	**1**	**1**	.	**1**	**1**	**1**
1,000 or (10 • 100)		100 or (10 • 10)	10 or (10 • 1)	1 or $(10 \bullet \frac{1}{10})$		$\frac{1}{10}$ or $(10 \bullet \frac{1}{100})$	$\frac{1}{100}$ or $(10 \bullet \frac{1}{1{,}000})$	$\frac{1}{1{,}000}$ or $(10 \bullet \frac{1}{10{,}000})$

Keep this in mind as we look at some simple additions.

Say we need to add 0.9 + 0.1. Notice that if we rewrite this as fractions, we see that it equals 1.

$$0.9 + 0.1$$

$$\frac{9}{10} + \frac{1}{10} = \frac{10}{10} = 1$$

Now say we want to add 0.09 + 0.01. Again, let's do it with fractions.

$$0.09 + 0.01$$

$$\frac{9}{100} + \frac{1}{100} = \frac{10}{100}$$

We could simplify the answer down to $\frac{1}{10}$, as $\frac{10}{100} \div \frac{10}{10} = \frac{1}{10}$.

Notice how we could have found both of these answers using the standard addition algorithm we use for whole numbers. **We can rename partial quantities just as we can whole quantities** because *each* place is still 10 times the previous one. Thus, we can add and subtract decimals (i.e., partial quantities written in the decimal system) using the *same basic processes* we do for whole numbers.

Rename $\frac{10}{10}$ as 1:

$$\begin{array}{r} \scriptstyle 1 \\ 0.9 \\ +\ 0.1 \\ \hline 1.0 \end{array}$$

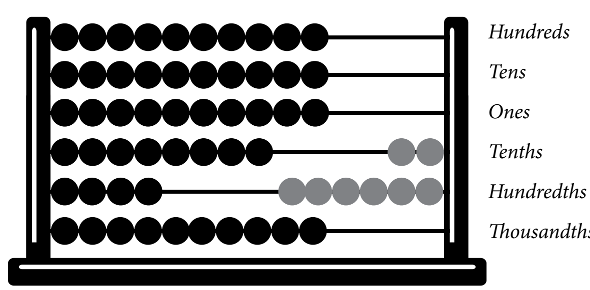
Rename $\frac{10}{100}$ as $\frac{1}{10}$:

$$\begin{array}{r} \scriptstyle 1 \\ 0.09 \\ +\ 0.01 \\ \hline 0.10 \end{array}$$

Now let's picture adding 0.26 + 0.17 on an abacus for a moment. Notice how we labeled the bottom three rows of our abacus as partial quantities, and that we exchanged 10 from one row for 1 from the row above. This is *exactly* what we do on paper when we add partial amounts using our traditional addition algorithm.

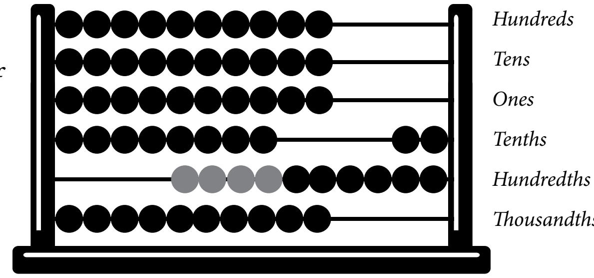
Example: Find 0.26 + 0.07 on an abacus.

Step 1: *Form the starting quantity of 0.26.*

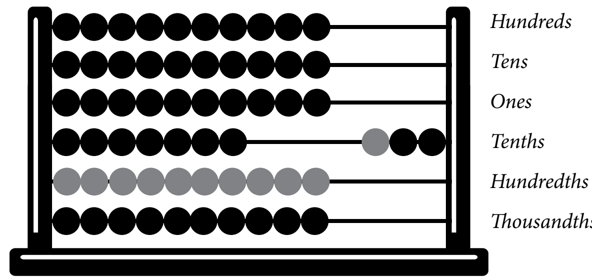


Step 2: *Add 0.07.*

Notice how we run out of beads after adding 4 out of the 7 hundredths.

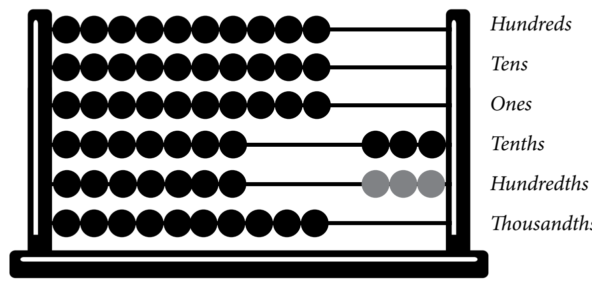


We have to rename 10 hundredths as 1 tenth.



And then we can add the remaining 3 hundredths.

Answer: *0.33*



Example: Find 0.26 + 0.07 on paper.

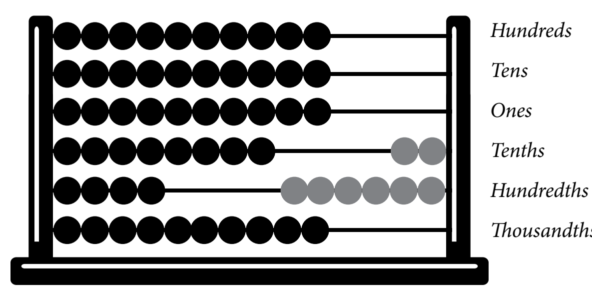
Rename $\frac{10}{100}$ as $\frac{1}{10}$:

$$\begin{array}{r} \overset{1}{0.2}6 \\ +\,0.07 \\ \hline 0.33 \end{array}$$

Likewise, the same process works for subtracting partial amounts written as decimals that we use for whole numbers.

Example: Find 0.26 – 0.07 on an abacus.

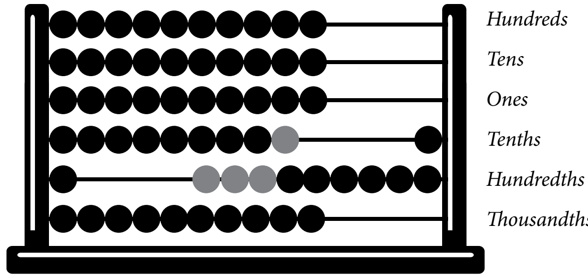
Step 1: *Form the starting quantity of 0.26.*



Step 2: *Subtract 0.07.*

We only have 6 hundredths and we need to subtract 7. So we have to rename 1 tenth as 10 hundredths and mentally subtract 7 from that, leaving 3 additional hundredths.

Another way of thinking about it is that renaming the 1 tenth to 10 hundredths gives us a total of 16 hundredths, which minus 7 equals 9.



Answer: 0.19

Example: Find 0.26 – 0.07 on paper:

Rename $\frac{1}{10}$ as $\frac{10}{100}$:

$$\begin{array}{r} {}^{1\ 1} \\ 0.\cancel{2}6 \\ -\,0.07 \\ \hline 0.19 \end{array}$$

Expanding It Further

It doesn't matter if the numbers we're adding and subtracting contain whole numbers, partial quantities, or a combination — if they're written using the decimal system, we can use our standard algorithm to keep track of place value.

$$\begin{array}{r} {}^{1\ \ 1\ 1} \\ 4.157 \\ +\ \ 5.878 \\ \hline 10.035 \end{array} \qquad \begin{array}{r} {}^{0\ 11\ \ 12\ 13\ 1} \\ \cancel{1}\cancel{2}.\cancel{3}\cancel{4}2 \\ -\ \ 8.978 \\ \hline 3.364 \end{array}$$

Adding 0s

Note that it's important to keep our digits lined up so we subtract tenths from tenths, hundredths from hundredths, etc. Otherwise, we could end up with a totally incorrect answer! Adding zeros to the right of a number so it has the same number of total digits as the other numbers we're working with can help guard against accidentally lining up digits incorrectly.

$$\begin{array}{r} 48.23 \\ -\ \ \ 4.1 \\ \hline \end{array} \qquad \begin{array}{r} 48.23 \\ -\ \ 4.10 \\ \hline \end{array}$$

Incorrect Correct

Keeping Perspective

Place value allows us to describe partial quantities in a way that makes it possible for us to use the *same algorithms*, or methods, to easily add or subtract them on paper. As a result, we'll find decimals an invaluable tool in describing the quantities God has placed all around us and in serving Him wherever He calls us.

7.3 Multiplying Decimals

What about multiplying numbers with decimals? While you probably already know this rule, let's try to arrive at it step-by-step as if you didn't.

Remember, in math we try to build on what we know to find simple methods for working with quantities. So we want to adapt our traditional multiplication method to work with decimals. If we could find a simple way to remove the decimal point temporarily and then add it back again when we were finished, we would be able to apply the method we already use to numbers with decimals.

Let's take a look at a problem:

$$7.5 \bullet 5$$

Since our place-value system is based on 10, multiplying by 10, 100, 1,000, etc., is just a matter of moving the decimal point to the right the appropriate number of times, which increases the value of each digit by 10, 100, 1,000, etc. So if we multiply 7.5 by 10, this would remove the decimal point, leaving us 75.

$$7.5 \bullet 10 = 75. \rightarrow \text{Decimal point moved to the right.}$$

We can now find the answer to 75 • 5 using the method we have already learned. After we have finished, we can divide by 10 again to put the decimal point back in the correct place. Again, since our decimal system is based on 10, to divide by 10, we just move the decimal over one place to the left.

Since we are both multiplying and dividing by the same number (in this case, 10), the multiplication and division will cancel each other out and not affect the final result.

	Multiply to remove the decimal.	Solve.	Divide to add decimal back.
$\begin{array}{r} 7.5 \\ \text{x}\quad 5 \\ \hline \end{array}$	$\begin{array}{r} 7.5 \\ \text{x}\quad 5 \\ \hline \end{array}$	$\begin{array}{r} \overset{2}{7}5 \\ \text{x}\quad 5 \\ \hline 375 \end{array}$	$\begin{array}{r} \overset{2}{7}5 \\ \text{x}\quad 5 \\ \hline 37.5 \end{array}$
	(7.5 x 10 = 75)		(375 ÷ 10 = 37.5)

What if we have more than one digit to the right of the decimal? We would follow the same guideline, just multiplying by 100, 1,000, etc., as needed. Multiplying by 100 would move the decimal two places to the right, by 1,000 three places, etc.

	Multiply to remove the decimal.	Solve.	Divide to add decimal back.
$\begin{array}{r} 7.50 \\ \underline{\times \quad 5} \end{array}$	$\begin{array}{r} 7.\underbrace{50} \\ \underline{\times \ 100} \\ 750 \end{array}$	$\begin{array}{r} \overset{2}{7}50 \\ \underline{\times \quad 5} \\ 3750 \end{array}$	$\begin{array}{r} 3750 \\ \underline{\div \ 100} \\ 37.\underbrace{50} \end{array}$
$\begin{array}{r} 7.500 \\ \underline{\times \quad 5} \end{array}$	$\begin{array}{r} 7.\underbrace{500} \\ \underline{\times \ 1000} \\ 7500 \end{array}$	$\begin{array}{r} \overset{2}{7}500 \\ \underline{\times \quad 5} \\ 37500 \end{array}$	$\begin{array}{r} 37500 \\ \underline{\div \ 1000} \\ 37.\underbrace{500} \end{array}$
$\begin{array}{r} 7.5000 \\ \underline{\times \quad 5} \end{array}$	$\begin{array}{r} 7.\underbrace{5000} \\ \underline{\times \ 10000} \\ 75000 \end{array}$	$\begin{array}{r} \overset{2}{7}5000 \\ \underline{\times \quad 5} \\ 375000 \end{array}$	$\begin{array}{r} 375000 \\ \underline{\div \ 10000} \\ 37.\underbrace{5000} \end{array}$

Now that we've found a way to multiply decimals, we want to find a way to simplify the process. Rather than actually writing out the multiplication and the division, we can just ignore the decimal point to start with and multiply as normal. After multiplying, we could then count the number of digits to the right of the decimal points in the numbers being multiplied, and add a decimal point in the answer so as to keep the same number of total digits to the right of the decimal point. This would reduce multiplying and dividing by 10, 100, 1,000, etc., to a mechanical process we do not even have to think about.

Example: Solve 7.51 • 5

$$\begin{array}{rll} 7.51 & \longleftarrow & \text{Total of two digits to the} \\ \underline{\times \quad 5} & & \text{right of the decimal.} \\ 37.55 & \longleftarrow & \text{Total of two digits to the} \\ & & \text{right of the decimal.} \end{array}$$

Example: Solve 7.51 • 6.45

$$\begin{array}{rll} 7.51 & \longleftarrow & \text{Total of four digits to} \\ \underline{\times 6.45} & \longleftarrow & \text{the right of the decimal.} \\ 3755 & & \\ 30040 & & \\ \underline{450600} & & \text{Total of four digits to} \\ 48.4395 & \longleftarrow & \text{the right of the decimal.} \end{array}$$

Keeping Perspective

Once again, we're building on what we know about place value to find an easy way of working with quantities. Don't forget, though, that we can only do this because of the amazingly consistent way God governs all things. If 10 times a number didn't always equal the same thing, we couldn't multiply decimals like this! Multiplication ultimately rests on God's faithfulness.

7.4 Dividing and Rounding with Decimals

It's time now to explore division yet again. This time, we're going to combine what we know about decimals and rounding to find yet another way to express remainders.

Division — From Remainder to Decimal

As we've seen before, when we divide two numbers, we don't always end up with a whole number — sometimes we have a remainder.

Let's say you spent \$46 for a package containing 5 DVDs. If you wanted to find out how much each DVD cost, you would need to divide \$46 by 5.

$$\begin{array}{r} 9 \\ 5\overline{)46} \\ -\ \underline{45} \\ 1 \end{array}$$

As you can see, we have a remainder of 1. In the past, we would have written this as r1 or $\frac{1}{5}$. However, sometimes it's more helpful to represent these remainders using decimals. Because our place-value system is based on 10, we can use the *same rules for dividing the remainder as we do for whole numbers*. We simply add a decimal point to show we're now dealing with tenths, add a zero to the dividend, and keep dividing!

$$\begin{array}{r} 9.2 \\ 5\overline{)46.0} \\ -\ \underline{45} \\ 1\,0 \\ -\underline{1\,0} \\ 0 \end{array}$$

If we buy a package of 5 for \$46, then each DVD costs \$9.20.

Note that we added a 0 after the 9.2 to make it 9.20; remember, adding zeros to the right of the decimal point does not change the meaning, as $\frac{2}{10}$ is equivalent to $\frac{20}{100}$. It's common to add a zero when working with dollars if we have 0 hundredths so that it's easier to quickly assess the cost. We know \$9.20 means 9 dollars and 20 cents.

Rounding

Let's say we spent \$1,189 to make 42 quilt racks and want to find the price per quilt rack. How much did we spend per rack?

```
         28.309
42)1,189.000
   - 84
     349
   - 336
      130
    - 126
        400
      - 378
         22
```

Okay, we've divided a *lot* of digits, and we still have a remainder. Often when dividing numbers, the answer keeps going on, not expressing evenly as a fraction of 10, 100, 1,000, etc., for a long time, if at all.

The good news is that we don't typically need that exact of an answer. For most purposes, we can round our answers after a certain number of decimals. In this case, we'll round to the nearest cent. \$28.309 rounds to \$28.31. So we spent \$28.31 per rack.

Remember, when rounding, you **look at the digit to the right of the digit you're rounding.** If it is 5 or higher, you round up; if it is less than five, you round down.

1.25 rounds to **1** (if rounding to ones) or to **1.3** (if rounding to tenths).

1.23 rounds to **1** (if rounding to ones) or to **1.2** (if rounding to tenths).

1.527 rounds to **2** (if rounding to ones) or to **1.5** (if rounding to tenths) or to **1.53** (if rounding to hundredths).

> Unless otherwise specified in this course, you can **round all answers to two decimal places (hundredths)**. Since we only go to hundredths in money (100 pennies equals \$1), rounding to the hundredths makes sense whenever dealing with money.
>
> Note that in order to round to the hundredths place, you will need to keep dividing through the thousandths place. That way you will be able to look at the thousandths-place digit to determine if you should round up or round down.

We have to round to simplify problems. Our need to round is another reminder that we can't keep track of everything. Unlike God, we are limited in what we can handle.

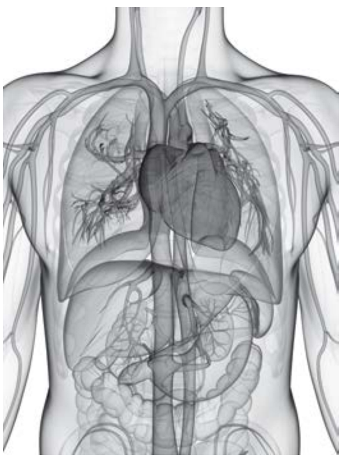
Finding an Approximate Answer

In real life, if all we're looking for is an approximate answer, we'll frequently round to the nearest whole number.

For example, if you were buying items at the store, you might want to know about how much you were committing to spend, but you might not need to know the exact amount. Approximating your answer will typically be enough to let you know if you have enough cash for your purchase.

$$\$5.99 + \$7.99 + \$1.98 \approx \$6 + \$8 + \$2 = \$16$$

However, we need to use our judgment with rounding. For instance, if you're asked to find how many shirts a certain yardage of fabric can make and the answer comes back 4.75 shirts, you cannot round and assume the fabric will yield 5 shirts. Even though 4.75 rounds to 5, if you don't have enough fabric to finish the fifth shirt, you'll only be able to make 4 shirts. **Always make sure your answer makes sense.**



Keeping Perspective — 60,000 Blood Vessels

In this lesson, we used the decimal notation to help us record the answers to division . . . including divisions that have remainders. As we learn these skills, keep in mind that the skills we learn can help us in real life . . . including in exploring and better appreciating God's creation. In the corresponding worksheet in your *Student Workbook*, you are going to use division and decimals to explore the blood vessels in your body. Have fun using math to get a fresh glimpse of how we truly are fearfully and wonderfully made (Psalm 139:14)!

7.5 Conversion and More with Decimals

It's time to dig just a little deeper into decimals. While you may already know some of these techniques, take advantage of the opportunity to better understand why the techniques work.

Conversion

We've already seen how to convert between fractions with a denominator of 10, 100, 1,000, etc., and decimals — we just put the numerator after the decimal point.

$$\frac{4}{10} = 0.4$$

$$\frac{40}{100} = 0.40$$

$$\frac{400}{1{,}000} = 0.400$$

But often, our denominator cannot be renamed into a denominator of 10, 100, 1,000, etc. Take $\frac{1}{7}$ — how can we convert it?

While you likely already know how to convert this fraction (by dividing the numerator by the denominator), let's think about why this is the case. Fractions represent division, so it would make sense to convert them to decimals simply by completing the division!

$$\frac{1}{7} = \begin{array}{r} 0.142 \\ 7\overline{)1.000} \\ -\underline{7} \\ 30 \\ -\underline{28} \\ 20 \\ -\underline{14} \\ 6 \end{array}$$

If we keep dividing, we'll end up with 0.14285714285. For our purposes, though, let's just round the answer to the nearest hundredth: 0.14. The decimal equivalent of $\frac{1}{7}$ is approximately 0.14.

Dividing a Decimal Number by a Decimal Number

The "rule" for dividing a decimal number by a decimal number is to **count the number of digits to the right of the decimal point in the divisor and move the decimal point that number of digits to the right in the dividend.** Then divide as usual.

$$8.23\overline{)3.776} \text{ changes to } 8\underbrace{23}\overline{)3\underbrace{77}.6}$$

The decimal point in both the divisor and the dividend moved two spaces to the right.

$$2.686\overline{)4.67} \text{ changes to } 2\underbrace{686}\overline{)4\underbrace{670}.}$$

The decimal point in both the divisor and the dividend moved three spaces to the right; notice that to move the dividend three spaces we had to add a 0.

$$3.41\overline{)5.893} \text{ changes to } 3\underbrace{41}\overline{)5\underbrace{89}.3}$$

The decimal point in the dividend and the divisor moved two spaces. It is okay to have a decimal in the dividend, just not in the divisor.

Any guesses why this rule works? Look at a problem written as a fraction, remembering that the fraction line is a way of representing division.

$$0.86\overline{)4.6} = \frac{4.6}{0.86}$$

Notice how if we were to multiply this fraction by $\frac{100}{100}$, we would end up removing the decimal from the divisor (0.86).

$$\frac{4.6}{0.86} \bullet \frac{100}{100} = \frac{460}{86}$$

This fraction could now be written as $86\overline{)460}$.

When we move the decimal point in the dividend and divisor in a division problem, we're really multiplying both the dividend and the divisor by a fraction worth 1, which doesn't change the value (yet another application of the identity property of multiplication!).

Keeping Perspective

Remember that you're learning all these mechanics so that you'll be equipped to use decimals in everyday life. Decimals help us when shopping, designing greeting cards, reading temperatures, comparing distances — the list could go on and on. Since decimals give us a way to represent partial quantities as part of our base-10 place-value system, they are an incredibly useful tool.

7.6 Chapter Synopsis

Decimals, and the rules for working with decimals, serve as useful tools we can use while depending on God and joyfully doing the work He has given us. Writing partial quantities in the decimal system lets us work with them with the same ease as we can whole quantities.

- Our **decimal system** extends to include partial quantities. Partial quantities written in the decimal system (i.e., **decimals**) have assumed denominators of 10, 100, 1,000, etc., with each digit to the right being $\frac{1}{10}$ of the previous one.
- Because decimals are part of the same place-value system we use for whole numbers, we can **add and subtract using the same method** we do for whole numbers, **being careful to correctly line up the digits**. We can also use the **same methods for multiplying and dividing**, using simple rules to deal with the decimal points. Always remember that rules in math are typically a shortcut for working with some consistency God created and sustains.
- When dividing with decimals, some numbers go on and on. We often round, as we rarely need that precise of an answer. **Unless otherwise specified, in this course we are rounding to the nearest hundredth.**
- Because fractions represent division, we can **convert fractions to decimals by simply dividing the numerator by the denominator.**

Measuring Distance

14.1 Units for Measuring Distance

Imagine for a moment life without measurements. How would we know how much flour to put in a recipe? How would we survey land or record and read maps? How would architects design buildings and articulate that information to builders? How would quantities of food be described on packages? Measurements are important!

God created us with the ability to measure and develop different systems for measuring. To start with, we'll focus on measuring *distance* or *length*.

To measure something, we need a standard we can compare it to—that is, a unit. A **unit** is what we call "a special quantity in terms of which other quantities are expressed."[1]

If we each used our own unit to measure distance, we'd have to go through a lengthy process to communicate to others a specific distance. A builder, for instance, would have to learn a brand-new set of units for every architectural drawing.

To avoid this sort of confusion, there are standardized systems for measurement. Let's take a look at measuring distance using two common measuring systems: the **U.S. Customary System** and the **Metric System**.

The Basic Unit — a Meter

Both the U.S. Customary System and the Metric System base their distance units on a unit called the **meter**.[2] And just how big is a meter?

Its approximate length is easiest to see on a measuring tape. Pull one out and measure 100 centimeters or 39.3701 inches. That's approximately a meter.

1 meter = 100 centimeters = 39.3701 inches

While the meter is not an actual unit in the U.S. Customary System, the yard is defined in terms of a meter.

I say approximately, because while we typically use devices such as rulers and measuring tapes to measure objects, these devices are not perfect representations (although they're definitely close enough for most practical purposes!). These devices are called **standards** — a "standard" is "a physical realization or representation of a unit."[3]

So what *is* the exact definition of a meter? The meter has actually had various definitions. You might think it would be the length of a specific bar or stick, of which duplicate sticks could be made — and it was at one point in time. The meter is currently defined, though, in terms of "the speed of light in a vacuum" — specifically as "the length of the path traveled by light in a vacuum during an interval of $\frac{1}{299{,}792{,}458}$ of a second."[4] This is a repeatable distance (because God causes light to travel consistently!) measurable by devices worldwide rather than a distance that must be measured against one specific bar.

Expanding from the Meter

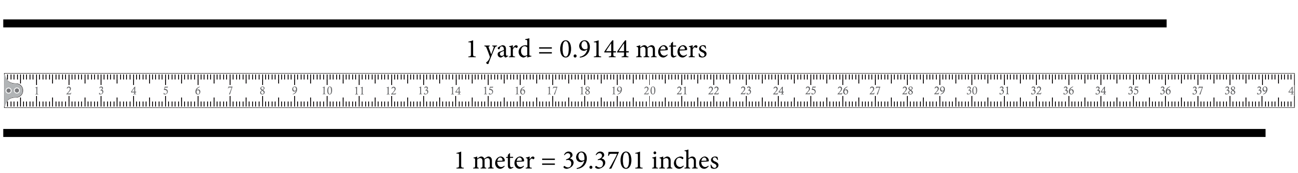
Now, it would be *quite* difficult to measure some things with a meter. Imagine a stick a meter long with no smaller markings. How would you describe the length of a paper clip? And how many meters would you need to describe a long distance, like the distance between two towns?

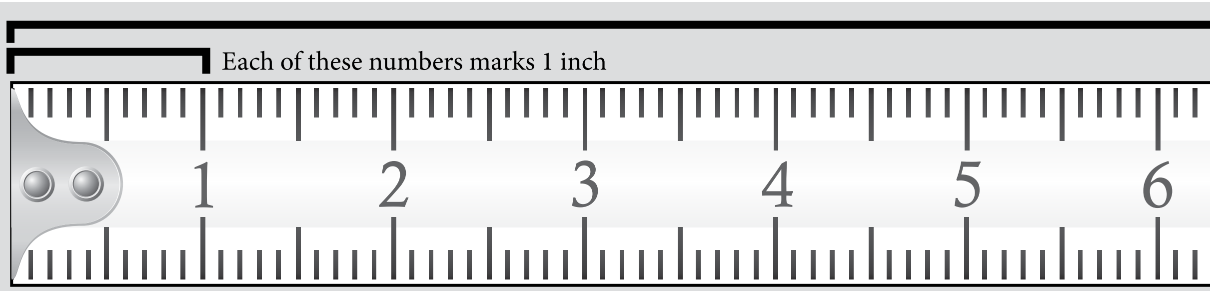
While you could describe short distances as portions of a meter and long distances as multiple meters, it would be easier to use longer or shorter units for these distances. The U.S. Customary and the Metric System both have a variety of units to cover both short and long distances.

The U.S. Customary System

In America, the U.S. Customary System of measurement is the system used for most daily purposes.

In terms of a meter, the **yard** is defined as 0.9144 meters. In other words, the yard is just shy of one meter. Pull out that measuring tape again and look for 3 feet or 36 inches — that's one yard. Notice that it's just shy of a meter.





How about some smaller units to help us measure shorter distances? We've got a **foot**, which is $\frac{1}{3}$ of a yard, and an **inch**, which is technically defined in terms of a portion of a meter, but is more commonly known as $\frac{1}{36^{\text{th}}}$ of a yard or $\frac{1}{12^{\text{th}}}$ of a foot. Notice the foot and inch marks on your measuring tape.

And for longer distances, we have a **mile**, which equals 1,760 yards or 5,280 feet. (Which, as you can imagine, is much too long to show on a measuring tape!)

Below are the units with their common abbreviations, showing how they compare to one another.

12 inches (in) = 1 foot (ft)

3 feet (ft) or 36 inches (in) = 1 yard (yd)

1 mile (mi) = 1,760 yards (yd) or **5,280 feet (ft)**

You will need to memorize the bolded relationships, as you'll need them often in everyday life.

Metric System/SI

The Metric System, or the International System of Units (SI), is a measuring system used around the world, including the United States, especially in scientific fields. We already looked at the definition of a meter. But, like the U.S. Customary System, the Metric System has other units to make it easier to measure shorter and longer distances.

Below are the four most common metric units typically used to express lengths in the Metric System:

1,000 millimeters (mm) = 1 meter (m)

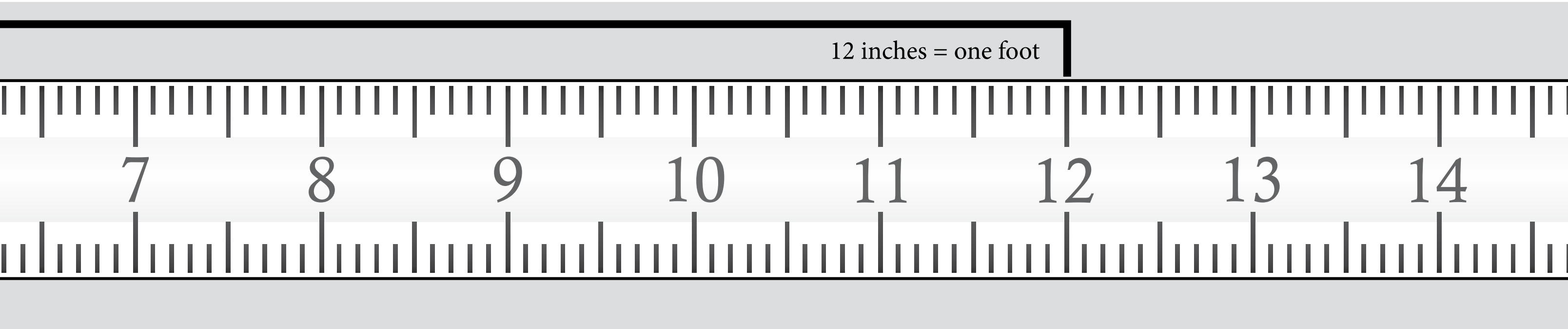
10 millimeters (mm) = 1 centimeter (cm)

100 centimeters (cm) = 1 meter (m)

1 kilometer (km) = 1,000 meters (m)

You will need to memorize these relationships, as you'll need them in everyday life.

As you might guess, millimeters and centimeters, like inches, are helpful for expressing shorter distances (like household objects), while kilometers, like miles, help measure longer distances.



Keeping the Metric Names Straight

The prefix "milli" comes from the Latin word *mīlle*, which means "thousand," and a **milli**meter is $\frac{1}{1{,}000^{th}}$ of a meter (that is, 1,000 equal a meter). Think of a **milli**on (a million is 1,000 thousands), **mill**ennial ("a span of one thousand years"),[5] etc.

The prefix "centi" comes from the Latin word *centum*, which means "hundred," and a **centi**meter is $\frac{1}{100^{th}}$ of a meter (that is, 100 of them equal a meter). Think of a **centi**pede (they have a lot of legs), a **cent** ($\frac{1}{100^{th}}$ of $1), a **cent**ennial (a 100-year celebration), etc.

The prefix "kilo" is from the Greek *khilioi*, which means "thousand," and a **kilo**meter means 1,000 meters. Any guesses what a **kilo**watt means? Yup — 1,000 watts.[6]

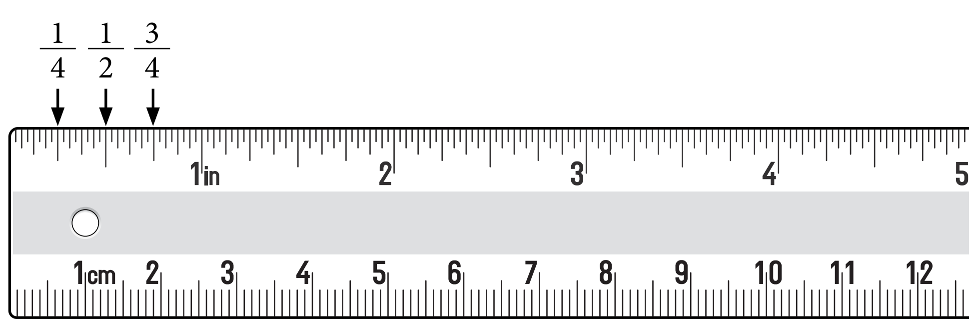
Why both Latin *and* Greek prefixes? While I couldn't find this specifically explained anywhere, it's interesting to note that the derivatives *milli*, *centi*, and *kilo* are all French, and the Metric System traces much of its roots to France.[7]

The names aren't arbitrary — remembering the meaning of the prefixes can help you keep them straight.

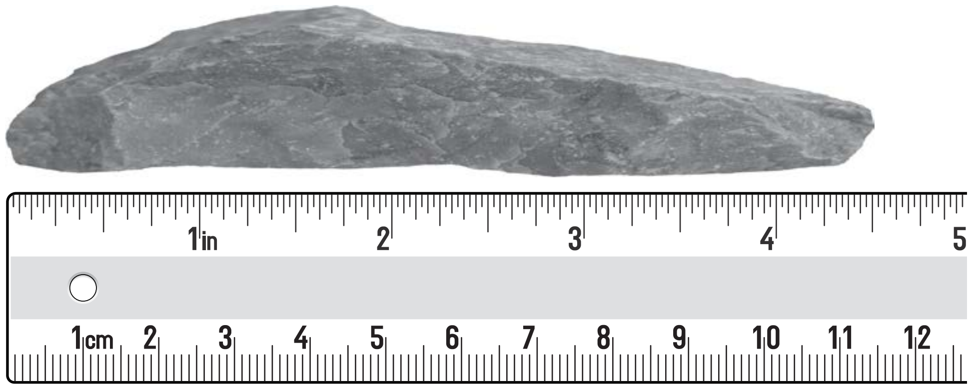
Measuring

Pull out a ruler and a measuring tape. Many of these devices list the U.S. Customary measurements on one side, and the metric measurements on the other, making it easy to measure in both systems.

For now, let's look at the U.S. Customary side — the numbers mark off the inches, and the tiny tick marks in between inches mark fractions of an inch. Each inch is broken into 16 fractional markings. Longer markings mark off $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of an inch.



To measure a distance, simply hold your ruler or measuring tape to the distance you're trying to measure. For example, the rock shown is $4\frac{1}{2}$ inches long if we measure all the way to the farthest edge.



Whose Standard?

When we measure a length, we measure it against a standard — a yard, ruler, etc. — and see how it compares. If we use a faulty standard, we'll get very misleading information.

The same is true spiritually. Many people think of themselves as "good enough" to get to heaven because they see themselves as better than other people. When we die, though, we won't be judged based on how well we did compared to others — we'll be held against *God's standard*. So we'd be wise to look at that standard now and see how we measure up.

Consider just a few of God's commandments from Exodus 20. Be honest with yourself. Are you really a good person when compared with this standard of holiness?

"*Thou shalt have no other gods before me.*" Have you ever loved or desired anything before God?

"*Honour thy father and thy mother: that thy days may be long upon the land which the LORD thy God giveth thee.*" Have you always honored your parents perfectly?

"Thou shalt not kill." Jesus clarified this: *"Ye have heard that it was said of them of old time, Thou shalt not kill; and whosoever shall kill shall be in danger of the judgment: But I say unto you, That whosoever is angry with his brother without a cause shall be in danger of the judgment"* (Matthew 5:21–22). Whoa! Who hasn't been angry without cause?

"*Thou shalt not commit adultery.*" Again, Jesus clarifies: "*Ye have heard that it was said by them of old time, Thou shalt not commit adultery: But I say unto you, That whosoever looketh on a woman to lust after her hath committed adultery with her already in his heart*" (Matthew 5:27–28). The impure thoughts in our hearts and minds make us guilty in God's eyes!

"*Thou shalt not bear false witness against thy neighbour.*" "Bearing false witness" is telling a lie. Have you ever told a lie . . . even a small one? What about when you were little?

"*Thou shalt not covet. . . .*" Have you ever wanted something that wasn't yours?

When held against God's standard, we are *all* guilty — we're all idolaters, murders, adulterers, coveters, and liars. And that's just a few of God's commandments. Plus, in God's eyes, if we're guilty of breaking just *one* of His commandments, we're guilty of breaking them all (James 2:10). The Bible compares even our righteous deeds to filthy rags (Isaiah 64:6). And He warns that **all** sin will be punished — and the punishment for sin is eternal death (Romans 6:23) in a torturous lake of fire (Revelation 20:15; 21:8).

But there's good news! God knew before the world began that man would rebel against Him, and He had a plan. You see, mankind was not always so hopelessly lost in sin — originally, God created us perfect and without any sin at all. But man chose to rebel, bringing sin and death into the world.

Yet God, knowing all the evil you and I would do (and think), chose to come down as a man and claim all that evil as His own, dying on a cross to bear the penalty we deserved. He now offers His righteousness and eternal life in Heaven to all who place their trust in Jesus.

Question: *Have you placed your trust in Jesus, or are you still trying to attain God's standard of perfection on your own? Have you unconsciously raised your own standard to get to Heaven or are you looking at God's standard and His solution? If you haven't trusted Jesus, today is the day! None of us know what tomorrow will hold. If you have trusted Jesus, rejoice in His undeserved righteousness and go tell someone else about what He has done.*

See www.LivingWaters.com for more information, both on the gospel and on how to use the 10 Commandments in sharing it with others.

Solving Problems with Measurements

Let's take a look now at solving problems involving measurements.

Example: If I'm $5\frac{1}{2}$ feet tall and I stand on a 6-foot ladder, at what height will my head be?

We can easily add our measurements together to find the answer.

$5\frac{1}{2}$ feet + 6 feet = $11\frac{1}{2}$ feet

Example: If I'm 66 *inches* tall and I stand on a 6-*foot* ladder, how tall will my head be?

The answer to this question is not as obvious, is it? We cannot simply add 66 inches and 6 feet, or we will get an entirely bogus answer! Since the units of measure are not the same, we need to first reexpress the inches as feet or the feet as inches.

We'll go into more details on how to express distance in different units in the next lesson; for now I just wanted you to be aware that when dealing with measurements, **you need to make sure you're dealing with the same units**!

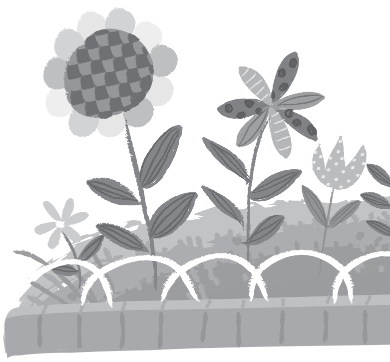
Keeping Perspective — A Glance at History

While we focused on the U.S. Customary System and the Metric System units of length in this lesson, throughout history, men have used different units of length. In reading your Bible, you've probably read about the cubit. The cubit was defined as the length from a man's elbow to his middle fingertip. Since this length varies based on the height of a man, this measurement could lead to different standards. (For example, the measurement from Goliath's elbow to fingertip was much different than David's!) The ancient Egyptians used the "Royal Cubit," which was somewhere around 20.6 inches; many other cubits were closer to 18 inches. Can you see why standardization is important?

According to the National Institute of Standards and Technology (NIST), the branch of the Department of Commerce in charge of providing "measurement standards for science and industry" and a "national scientific laboratory in the physical sciences,"[8] our inch, foot, and yard trace their origin back to the cubit, with the Romans influencing some of the smaller units, along with introducing the idea of a mile.[9]

No matter what units we use, the principle is the same: we're using a standard to describe distances. We can do this because God created us with this ability.

14.2 Conversions via Proportions



Say you measured an edge of your garden, and it was 72 *inches* long. You want to buy edging to go along that edge, which is sold by the *foot*. How many *feet* of edging do you need?

In order to figure out how many feet you need, you need to **convert** 72 inches to feet — that is, you need to figure out how to describe 72 *inches* using *feet* as your unit of measurement instead. Any ideas how to do that? There are actually many ways — let's take a look at one of them.

Conversions via Proportions

Think back to what you know about ratios and proportions. As we've seen, a ratio is "the relative size of two quantities expressed as the quotient of one divided by the other,"[10] or basically a fancy name for using division to compare quantities. A proportion is two equal ratios.

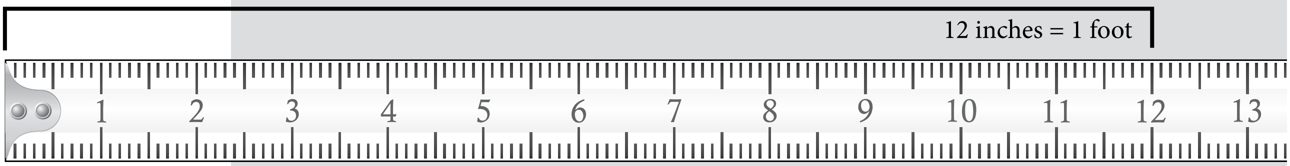
Let's use a ratio to compare how inches and feet relate. There are 12 inches in 1 foot, so we could write this ratio as a fraction like this:

$$\frac{12 \text{ in}}{1 \text{ ft}} \quad or \quad \frac{1 \text{ ft}}{12 \text{ in}}.$$

It's important to note that this ratio is really worth 1. After all, 12 inches and 1 foot represent the *same distance*, just in different units. This means that we really could substitute either one for the other in the ratio. When we do, we see that the ratio is really worth 1, as any number divided by itself equals 1.

substitute 12 in for 1 ft $\frac{12 \text{ in}}{1 \text{ ft}} = \frac{12 \text{ in}}{12 \text{ in}} = 1$

substitute 1 ft for 12 in $\frac{1 \text{ ft}}{12 \text{ in}} = \frac{1 \text{ ft}}{1 \text{ ft}} = 1$



In this course, we'll refer to a ratio between two units that is worth 1 as a **conversion ratio**. It shows us how the two units compare.

Conversion ratio between inches and feet: $\frac{12 \text{ in}}{1 \text{ ft}}$ or $\frac{1 \text{ ft}}{12 \text{ in}}$

Now that we've expressed the relationship between inches and feet as a ratio, it's easy to convert 72 inches to feet. All we need to do is form an equivalent ratio—that is, another ratio that expresses the same distance in both feet and inches, but using 72 inches instead of 12 inches. We can figure out the number of feet to use in the ratio using a proportion!

$$\frac{12 \text{ in}}{1 \text{ ft}} = \frac{72 \text{ in}}{? \text{ ft}}$$

Now we can think through what number would **finish creating an equivalent ratio** (see 5.3). Since 72 ÷ 12 = 6, if we multiply by $\frac{6}{6}$, we'll form an equivalent ratio with 72 as our numerator.

$$\frac{12 \text{ in}}{1 \text{ ft}} \bullet \frac{6}{6} = \frac{72 \text{ in}}{6 \text{ ft}}$$

$$\frac{12 \text{ in}}{1 \text{ ft}} = \frac{72 \text{ in}}{6 \text{ ft}}$$

Important! Notice that in both ratios we've put inches in the numerator and feet in the denominator. We could have reversed this ($\frac{1 \text{ ft}}{12 \text{ in}} = \frac{? \text{ ft}}{72 \text{ in}}$), but we have to be consistent. We could *not* put feet in the numerator in one ratio and in the denominator in the other, as we'd no longer be comparing like units to like units.

$$\cancel{\frac{1\text{ ft}}{12\text{ in}} = \frac{72\text{ in}}{?\text{ ft}}} \qquad \cancel{\frac{12\text{ in}}{1\text{ ft}} = \frac{?\text{ ft}}{72\text{ in}}}$$

$$\frac{12\text{ in}}{1\text{ ft}} = \frac{72\text{ in}}{?\text{ ft}} \qquad \frac{1\text{ ft}}{12\text{ in}} = \frac{?\text{ ft}}{72\text{ in}}$$

72 in equals 6 ft.

Feet must be compared with feet and inches with inches.

Watch Your Units

When doing problems, pay attention to the units used, and make sure to **include the units in your answer**. An answer without units when units were involved will be considered partially incorrect, as you've not included what that number represents. Watching your units carefully will serve you well, both in real life and in upper-level sciences.

Keeping Perspective

In math, there's often more than one way to solve a problem. In fact, in the next lesson you're going to learn yet another way to convert units. Each one will come in handy at different times.

In coming up with these methods, all we're really doing is looking at the notations and skills we've already learned (ratios, proportions, etc.) and seeing if there's a way to apply them to help us in expressing distances. We're then walking away with a "rule" or "method" about how to convert units that simplifies what we discovered. You'll see this process repeated over and over again in math.

14.3 Different Conversion Methods

We saw in the last lesson that we can express the conversion ratio as a fraction: $\frac{12\text{ in}}{1\text{ ft}}$ *or* $\frac{1\text{ ft}}{12\text{ in}}$. We also saw that since both 12 inches and 1 foot represent the *same* length, both these fractions really represent 1.

We then used that knowledge to find the answer via a proportion:

$$\frac{12\text{ in}}{1\text{ ft}} = \frac{72\text{ in}}{?\text{ ft}}$$

$$\frac{12\text{ in}}{1\text{ ft}} \bullet \frac{6}{6} = \frac{72\text{ in}}{6\text{ ft}}$$

Answer: 6 ft

It's time now to look at two additional methods to convert units. Both these methods prove quite useful, as we'll see.

Conversion via the Ratio Shortcut

Since multiplying by a fraction worth 1 doesn't change the value and since the conversion ratio is worth 1, rather than setting up a proportion, we could *multiply* 72 inches by our conversion ratio instead.

$$72 \text{ in } \bullet \frac{1 \text{ ft}}{12 \text{ in}} = \frac{72 \text{ in} \bullet 1 \text{ ft}}{12 \text{ in}}$$

Remember, 72 in can be thought of as $\frac{72 \text{ in}}{1}$*. Since dividing by 1 doesn't change the value of the number, we ignored the 1, but* ***we treated 72 as a numerator****, as if it were written* $\frac{72 \text{ in}}{1}$.

Dealing with the Units

Now, what does $\frac{72 \text{ in} \bullet 1 \text{ ft}}{12 \text{ in}}$ equal? How do we handle the units? Just as we can divide by $\frac{12}{12}$ to simplify the numbers, we can divide by $\frac{\text{in}}{\text{in}}$ to simplify the units.

$$\frac{72 \text{ in} \bullet 1 \text{ ft}}{12 \text{ in}} \div \frac{\text{in}}{\text{in}} = \frac{72 \bullet 1 \text{ ft}}{12} = \frac{72 \text{ ft}}{12} \div \frac{12}{12} = 6 \text{ ft}$$

Notice that we could have simplified both the units and the numbers as we went. Remember, a fraction line means to divide. Since both the numerator and the denominator have "in", the division will cancel out the multiplication, just as it does with numbers.

$$\overset{6}{\cancel{72}} \cancel{\text{in}} \bullet \frac{1 \text{ ft}}{\underset{1}{\cancel{12}} \cancel{\text{in}}} = 6 \text{ ft}$$

The "Rule"

We've finally arrived at another "rule," or method, for unit conversions. Once again, this method applies the principles we've learned to a new situation, reducing the amount of thinking we have to do each time.

To convert a unit into another unit, just multiply it by the conversion ratio! Be sure to write the conversion ratio so the unit you're trying to convert will cancel out, leaving you with the unit of measure you need.

We'll refer to this method of converting units as **conversion via the ratio shortcut**, and the proportion method we looked at in the last lesson as **conversion via a proportion**.

Example: Convert 8 miles into feet.

$$8 \cancel{\text{mi}} \bullet \frac{5{,}280 \text{ ft}}{1 \cancel{\text{mi}}} = 42{,}240 \text{ ft}$$

Notice that we put the miles as the denominator of our conversion ratio so the miles would cancel out and the answer would be in feet.

Example: Convert 42,240 feet into miles.

$$42{,}240 \cancel{\text{ft}} \bullet \frac{1 \text{ mi}}{5{,}280 \cancel{\text{ft}}} = \frac{42{,}240 \text{ mi}}{5{,}280} = 8 \text{ mi}$$

Again, notice that we put the feet in the denominator of our conversion ratio so it would cancel out. ***Always write the conversion ratio so the unit you're trying to convert will cancel out.*** *Otherwise, you won't succeed in converting to a new unit.*

The conversion via the ratio shortcut method may seem more involved at first, but it will save you lots of time once you become familiar with it, especially when dealing with multistep conversions.

Conversion via Mental Math

On simple conversion problems, we could convert mentally. Notice that when we converted 72 inches to feet we ended up dividing 72 by 12.

$$72 \cancel{\text{in}} \bullet \frac{1 \text{ ft}}{12 \cancel{\text{in}}} = \frac{72 \bullet 1 \text{ ft}}{12} = \frac{72 \text{ ft}}{12} = 6 \text{ ft}$$

If we had needed to find the answer mentally, we could have simply divided 72 inches by 12 inches. When you think about it, we know that 1 foot equals 12 inches. It follows then that if we were to divide the 72 inches by 12, we'd get the equivalent measurement in feet.

If, on the other hand, we'd started with 6 feet and needed to find inches, we could find it using multiplication. If 12 inches equals 1 foot, than 6 feet is going to equal 6 • 12 inches, or 72 inches.

72 inches and 6 feet represent the *same distance*, but in different units.

While it is pretty easy to convert between inches and feet mentally since we're familiar with the units, it's also easy to make a mistake and divide when we need to multiply or vice versa. So be cautious when doing unit conversions mentally.

Also, sometimes the numbers involved make it hard to solve problems completely mentally. For example, converting 83.54 inches to feet would require this division:

$$83.54 \div 12 = ?$$

While that's a little complicated to solve mentally, we could still figure out what we need to multiply or divide mentally and then use paper or a calculator to find the answer.

$$83.54 \div 12 = 6.96$$

On example problems, we'll still show how to set up problems like this one mentally; know, though, that you might need to use paper or a calculator to perform the math.

Which Method Is Best?

We've now talked about three different methods for unit conversion:

- Conversion via a proportion
- Conversion via the ratio shortcut
- Conversion via mental math

Which one is best? It depends! In general, it's a good habit to convert via the ratio shortcut, as it makes multistep conversions much easier, as we'll see soon.

Conversion via a Proportion	Conversion via the Ratio Shortcut	Conversion via Mental Math
$\frac{12 \text{ in}}{1 \text{ ft}} = \frac{72 \text{ in}}{? \text{ ft}}$	$\overset{6}{\cancel{72}}\ \cancel{\text{in}} \bullet \frac{1 \text{ ft}}{\underset{1}{\cancel{12}}\ \cancel{\text{in}}} = 6 \text{ ft}$	We know 1 foot equals 12 inches, so 72 inches divided by 12 will give us our feet $72 \div 12 = 6$
Answer: 6 ft	Answer: 6 ft	Answer: 6 ft

Keeping Perspective

Once again, we're continuing to build on what we know to find shortcuts to deal with additional situations. The process of unit conversion we looked at in this lesson is one you'll continue to build on later. As you familiarize yourself with it, remember that each conversion method is ultimately a way of helping us work with real-life distances. It's math in action — a tool to help us measure and describe God's creation.

14.4 Currency Conversions

We often need to convert between a lot more than distance units. The good news is that you have all the skills you need! Let's practice applying these skills to currency.

In America, we use the dollar as our currency, but other countries use other currencies. When traveling, it's often necessary to convert between currencies, exchanging dollars for pounds, euros, etc. The exchange rate is the conversion ratio (also sometimes called the conversion rate; remember, a rate is just a specific type of ratio) between two currencies. If the exchange rate is $\frac{1}{3}$, that means that for every 1 unit of one currency you could receive 3 of the other.

Searching the Internet for the exchange rate between two countries should yield the current rate (although that rate may vary throughout the day, and there may be an additional fee from the vendor who converts the money). Once you know the exchange rate, you can use any of the methods you've learned to convert between the two currencies.

The exchange rate can vary based on a variety of factors, including the economy of the different nations.

Example: If 2 British pounds = 1 U.S. dollar, how much is 8 pounds in U.S. dollars?

We've been given this ratio, or rate: $\frac{1 \text{ dollar}}{2 \text{ pounds}}$.

Conversion via a Proportion	Conversion via the Ratio Shortcut	Conversion via Mental Math
$\frac{? \text{ dollars}}{8 \text{ pounds}} = \frac{1 \text{ dollar}}{2 \text{ pounds}}$	$\cancel{8}^{4}\ \cancel{\text{pounds}} \bullet \frac{1 \text{ dollar}}{\cancel{2}_{1}\ \cancel{\text{pounds}}} = \frac{4}{1}$ dollars	$8 \div 2 = 4$
Answer: $4	Answer: $4	Answer: $4

Example: While in Britain, you find an item marked 45 pounds. How much will it cost you in U.S. dollars, assuming a conversion rate of 1.602 British pounds to 1 U.S. dollar?

Rate: $\frac{1 \text{ dollar}}{1.602 \text{ pounds}}$

Conversion via a Proportion	Conversion via the Ratio Shortcut	Conversion via Mental Math
$\frac{45 \text{ pounds}}{? \text{ dollars}} = \frac{1.602 \text{ pounds}}{1 \text{ dollar}}$	$45\ \cancel{\text{pounds}} \bullet \frac{1 \text{ dollar}}{1.602\ \cancel{\text{pounds}}} = \frac{45}{1.602}$ dollars	$45 \div 1.602 = 28.09$
Answer: $28.09	Answer: $28.09	Answer: $28.09

You might need a paper or a calculator to complete the conversion via Mental Math on this one.

Conversion and Missionaries

During tough economic times, missionaries have it extra, extra tough — not only can the dollars they receive decrease because people have less to give, but when the value of the U.S. dollar decreases, as is often the case during an economic depression, the missionaries often get fewer foreign currency per U.S. dollar, meaning their support doesn't go as far. So keep missionaries you support in mind (and prayer) during tough economic times.

Keeping Perspective

The conversion methods you're learning apply in all sorts of situations — including exchanging money in a foreign country. Math can even help you while you're traveling.

14.5 Metric Conversions

It's time to dig deeper into the metric measurement system, applying the same conversion methods to metric units.

Understanding the Metric System

While the units we looked at in 14.1 (millimeters, centimeters, meters, and kilometers) are the most commonly known, the Metric System actually contains other units too.

10 **millimeters** (mm) = 1 **centimeter** (cm)
10 **centimeters** = 1 decimeter (dm)
10 decimeters = 1 **meter** (m)
10 **meters** = 1 decameter (dam)
10 decameters = 1 hectometer (hm)
10 hectometers = 1 **kilometer** (km)

Notice how, starting with centimeters, **each unit is worth 10 of the previous unit**. We'll see shortly that this makes the Metric System incredible easy to work with, since our decimal system is also based on 10.

Conversions within the Metric System

Because each unit in the Metric System is worth 10 of the previous unit, to convert from one unit to the next largest unit, we need only to divide by 10.
1 mm = 0.1 cm (decimal moved over one place to the left, as we divided by 10).
Likewise, to convert from one unit to the next smallest, we need only to multiply by 10.
0.1 cm = 1 mm (decimal moved over one place to the right as we multiplied by 10).

The same methods for conversions that we've looked at so far apply to the Metric System — but because the Metric System is based on 10 and because multiplying or dividing by 10 is simply a matter of moving our decimal point (see 7.3), the math involved is much simpler.

Example: Convert 1 millimeter to centimeters.

Conversion via a Proportion	**Conversion via the Ratio Shortcut**	**Conversion via Mental Math**
$\frac{1 \text{ mm}}{? \text{ cm}} = \frac{10 \text{ mm}}{1 \text{ cm}}$	$1 \cancel{\text{mm}} \bullet \frac{1 \text{ cm}}{10 \cancel{\text{mm}}} = \frac{1\text{cm}}{10} = 0.1 \text{ cm}$	$1 \div 10 = 0.1$
Answer: 0.1 cm	Answer: 0.1 cm	Answer: 0.1 cm

Example: Convert 3 cm to millimeters.

Conversion via a Proportion	Conversion via the Ratio Shortcut	Conversion via Mental Math
$\frac{3 \text{ cm}}{? \text{ mm}} = \frac{1 \text{ cm}}{10 \text{ mm}}$	$3 \cancel{\text{cm}} \bullet \frac{10 \text{ mm}}{1 \cancel{\text{cm}}} = 30 \text{ mm}$	$3 \bullet 10 = 30$
Answer: 30 mm	Answer: 30 mm	Answer: 30 mm

Example: Convert 2 meters to centimeters.

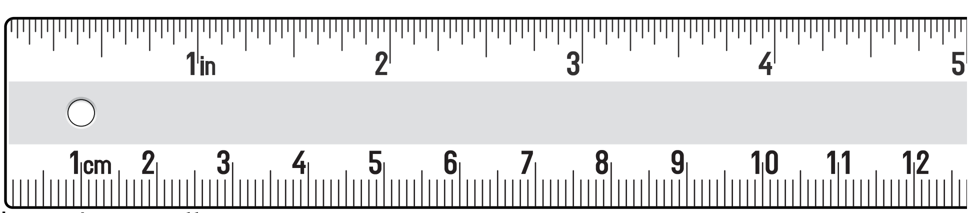
Here we're switching from meters to centimeters — those are more than 1 unit apart. But the math is still quite simple! We know there are 100 centimeters in 1 meter.

Since every conversion between metric units is a multiple of 10, conversions within the Metric System can easily be done mentally.

Conversion via a Proportion	Conversion via the Ratio Shortcut	Conversion via Mental Math
$\frac{2 \text{ m}}{? \text{ cm}} = \frac{1 \text{ m}}{100 \text{ cm}}$	$2 \cancel{\text{m}} \bullet \frac{100 \text{ cm}}{1 \cancel{\text{m}}} = 200 \text{ cm}$	$2 \bullet 100 = 200$
Answer: 200 cm	Answer: 200 cm	Answer: 200 cm

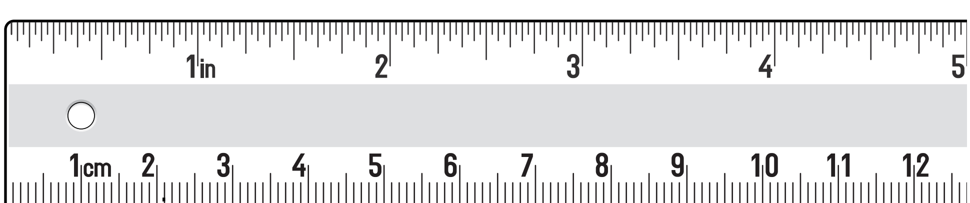
Measuring with Metric

Pull out a ruler and take a look at the metric side of it for a moment. The numbers mark off the centimeters, and the tiny tick marks mark off millimeters. There are 10 millimeters per 1 centimeter.



10 millimeters = 1 centimeter

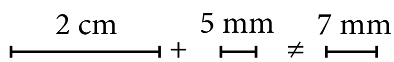
The line shown below is 2 centimeters and 1 millimeter long. We could represent it in centimeters alone as 2.1 cm, since 1 millimeter equals 0.1 cm. (Remember, dividing by 10 just means moving the decimal over to the left.) Notice how easy it is to express the portion representing a part of a centimeter as a decimal!



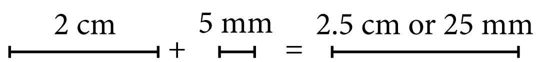
2 centimeters and 1 millimeter long or 2.1 cm

Remember to Convert First!

It's easy to add, subtract, multiply, or divide the wrong numbers and reach an entirely wrong answer if you add unlike units. For instance, if you're asked to add 2 centimeters and 5 millimeters, you can't just add 2 and 5, because they represent different units of measure.



You have to first convert the centimeters to millimeters or the millimeters to centimeters, and *then* add them.



$$2 \text{ cm} + 5 \text{ mm} = 2.5 \text{ cm}$$

$$2 \text{ cm} + 5 \text{ mm} = 25 \text{ mm}$$

Is Morality Like Measurements?

Most people acknowledge that murder is wrong (God has written His laws upon our hearts — Romans 2:15), but few can explain *why* it's wrong. Without acknowledging a Creator, we have no basis for an absolute standard for right and wrong. Some people will argue that society as a whole determines right and wrong, just as they determine measurement units. But in that case, who is to say that Hitler was wrong for murdering the Jews? His German society didn't view it as wrong!

You see, only the biblical worldview gives us a basis for morality. Morality is not an arbitrary rule like a unit of measure that man can change — it's given to us by God, based on the character of God.

When people tell you they don't believe in God, consider asking them how they explain right and wrong. Point out that by condemning the "Hitlers" of this world, they are acting contrary to their worldview.

The Bible gives us a firm foundation that makes sense out of every area of life — let's share it with others in love!

See Dr. Jason Lisle's *The Ultimate Proof of Creation: Resolving the Origins Debate* (Green Forest, AR: Master Books, 2009) for more details on how morality only makes sense in a biblical worldview, and on how to use that to challenge other worldviews.

Keeping Perspective

The Metric and the U.S. Customary System are both different ways to measure distances. While you will likely use the U.S. Customary System more in daily life, becoming familiar with them both is important, as other countries (and many technical fields) use the Metric System.

14.6 Multistep Conversions

Sometimes it takes more than one step to convert between two units. Let's say we need to convert 4 miles into yards, but we can't remember how many yards are in a mile, only that there are 5,280 feet in a mile.

We could look up the ratio between yards and miles, or we could find the answer by breaking this problem down into further steps, converting our miles to feet and *then* to yards.

Conversion via Proportions:

Converting to feet:

$$\frac{4\text{ mi}}{?\text{ ft}} = \frac{1\text{ mi}}{5{,}280\text{ ft}}$$

Answer: 21,120 ft

Now that we have found the feet, we can convert to yards.

$$\frac{21{,}120\text{ ft}}{?\text{ yd}} = \frac{3\text{ ft}}{1\text{ yd}}$$

Answer: 7,040 yd

Conversion via the Ratio Shortcut:

Converting to feet:

$$4\ \cancel{\text{mi}} \bullet \frac{5{,}280\text{ ft}}{1\ \cancel{\text{mi}}} = 21{,}120\text{ ft}$$

Converting to yards:

$$21{,}120\ \cancel{\text{ft}} \bullet \frac{1\text{ yd}}{3\ \cancel{\text{ft}}} = 7{,}040\text{ yd}$$

Answer: 7,040 yd

Conversion via Mental Math:

Converting to feet:

$4 \bullet 5{,}280\text{ ft} = 21{,}120\text{ ft}$

Converting to yards:

$21{,}120 \div 3 = 7{,}040$

Answer: 7,040 yd

Notice it's a little challenging to do multistep conversions mentally.

Doing It in One Step

Using the conversion via the ratio shortcut method can greatly simplify multistep conversion problems, as we can do the conversion in a single step by multiplying by more than one conversion ratio until we have the answer in the desired unit.

$$4\,\cancel{\text{mi}} \bullet \frac{\overset{1,760}{\cancel{5,280}}\,\cancel{\text{ft}}}{1\,\cancel{\text{mi}}} \bullet \frac{1\text{ yd}}{\underset{1}{\cancel{3}}\,\cancel{\text{ft}}} = \frac{4 \bullet 1,760 \bullet 1\text{ yd}}{1} = 7,040\text{ yd}$$

Because the conversion via the ratio shortcut makes multistep conversions so much simpler, we'll be emphasizing it throughout this course.

Notice how each unit cancelled out *as we went*!

Also notice that we used $\frac{5,280\text{ ft}}{1\text{ mi}}$ instead of $\frac{1\text{ mi}}{5,280\text{ ft}}$ because we needed to have miles in the denominator in order to cancel it out. And we used $\frac{1\text{ yd}}{3\text{ ft}}$ instead of 3 ft over 1 yd for the same reason — we needed the feet to cancel.

If you're ever unsure which unit you should put on the numerator and the denominator in a conversion ratio, just **think about what you need to use in order to cancel out the units you don't want and leave only the unit you do.**

Keeping Perspective

Hopefully, you now see why multiplying by a conversion ratio is such a valuable method to know. Sometimes methods that initially seem silly end up saving time in more complicated situations. The more you learn math, the more you'll realize how different tools combine. Yet all these tools only work because of the inherent consistency God created and sustains. Don't lose sight of the fact that He is the One ". . .upholding all things by the word of his power. . ." (Hebrews 1:3).

14.7 Conversions Between U.S. Customary and Metric

Guess what? The same methods you've been using to convert within a system apply for conversions *between* systems. **All we need to know is the ratio between two units (i.e., the conversion ratio).**

Below are the ratios between some common U.S. Customary and Metric units.

1 in = 2.54 cm

1 ft = 30.48 cm

1 yd = 0.9144 m

1 mi = 1.60934 km

Now we can convert away! The examples show the conversions using the conversion via the ratio shortcut method, but any of the methods we've looked at would work. We're just using this method as it saves time on multistep conversions, as we saw in the last lesson.

Example: Convert 80 yards to meters.

Ratio between meters and yards: $\frac{0.9144 \text{ m}}{1 \text{ yd}}$

Multiply:

$$80 \cancel{\text{yd}} \bullet \frac{0.9144 \text{ m}}{1 \cancel{\text{yd}}} = 73.152 \text{ m}$$

Example: You drive into Canada and see a sign saying the town you're going to is 100 km away. How many miles is that?

Ratio between kilometers and miles: $\frac{1 \text{ mi}}{1.60934 \text{ km}}$

Multiply:

$$100 \cancel{\text{km}} \bullet \frac{1 \text{ mi}}{1.60934 \cancel{\text{km}}} = \frac{100}{1.60934} \text{ mi} = 62.14 \text{ mi}$$

Remember to Think It Through

One common confusion in unit conversion regards which way to express the conversion ratio — should it be $\frac{1 \text{ in}}{2.54 \text{ cm}}$ or $\frac{2.54 \text{ cm}}{1 \text{ in}}$?

It depends on what unit of measure we want in the end.

If we want to convert from 50 inches to centimeters, we would place the inches on the bottom of the ratio so they would cancel out.

$$50 \cancel{\text{in}} \bullet \frac{2.54 \text{ cm}}{1 \cancel{\text{in}}} = 127 \text{ cm}$$

If, on the other hand, we wanted to convert 127 centimeters into inches, we would place the centimeters on the bottom of the ratio to cancel them out.

$$127 \cancel{\text{cm}} \bullet \frac{1 \text{ in}}{2.54 \cancel{\text{cm}}} = \frac{127 \text{ in}}{2.54} = 50 \text{ in}$$

Remember, we can write the ratio either way because 1 in and 2.54 cm represent the *same quantity*. Thus, either way, the resulting fraction is worth 1.

$$1 \text{ in} = 2.54 \text{ cm}$$

Substitute 2.54 cm for 1 in:

$$\frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{2.54 \text{ cm}}{2.54 \text{ cm}} = 1$$

Substitute 1 in for 2.54 cm:

$$\frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{1 \text{ in}}{1 \text{ in}} = 1$$

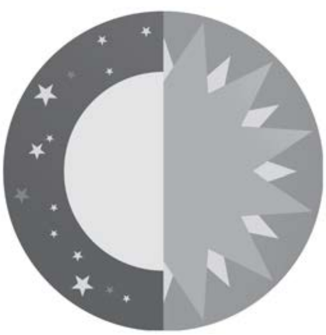
Keeping Perspective

Do you see a pattern? We keep applying the same methods to different situations. A lot of math class is expanding on what you know or applying it to new settings. If you ever encounter a problem you don't know how to solve (whether in a textbook or in real life), don't be afraid to try on your own to think through how to apply what you know to it. Chances are you have all the tools you need!

14.8 Time Conversions

It's time to apply unit conversion to a different area altogether: time. Let's start by taking a look at some units we use for measuring time, and then at how to convert between them.

Units of Time



God created time and gave us ways of keeping track of time (day and night, stars, etc.) when He created the universe. Unlike us, God had no beginning — He is outside of time.

> *In the beginning God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters. And God said, Let there be light: and there was light. And God saw the light, that it was good: and God divided the light from the darkness. And God called the light Day, and the darkness he called Night. And the evening and the morning were the first day (Genesis 1:1–5).*
>
> *And God said, Let there be lights in the firmament of the heaven to divide the day from the night; and let them be for signs, and for seasons, and for days, and years: (Genesis 1:14).*
>
> *Hast thou not known? hast thou not heard, that the everlasting God, the LORD, the Creator of the ends of the earth, fainteth not, neither is weary? there is no searching of his understanding (Isaiah 40:28).*

Every day, God causes the earth to rotate in a consistent fashion. Whichever side of the earth is tilted away from the sun experiences darkness, while the other side experiences light. As the earth rotates, the stars and moon appear to change their positions overhead. One complete rotation forms a **day**.

Sometimes we need to know specifically how much of a day has passed. If we go outside during the morning, the sun will be in a different spot than if we go outside in the evening. So the sun's position helps us keep track of approximately what time of the day it is. But it's not always possible to see the sun, especially on a cloudy day. Nor is it easy to tell someone exactly when we want to meet using the sun alone. Thus we use **hours** and **minutes** to refer to portions of a day.

God gave us the 7-day **week** when He created the world in six days and rested on the seventh (Genesis 2:2-3; Exodus 20:8-11). A **year** is based on how long it takes the earth to travel around the sun (approximately 365 days).

Every day and year that passes testifies that God is continuing to hold this world together in a consistent fashion, just as He promised.

> *While the earth remaineth, seedtime and harvest, and cold and heat, and summer and winter, and day and night shall not cease. (Genesis 8:22).*

60 seconds = 1 minute (min)
60 minutes = 1 hour (hr)
24 hours = 1 day (d)
7 days = 1 week (wk)
365 days = 1 year (yr *or* y)

Conversions between Units

Because we all deal with time every single day, working with and converting between various units of time is essential . . . and you already have all the skills you need! You can convert between units of time the same way you have been between units of distance. Again, we'll focus on the conversion via the ratio shortcut method, as it will save time on multistep problems and is an important method to learn.

Example: How many minutes are in 1 day?

$$1\ \cancel{\text{d}} \bullet \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{d}}} \bullet \frac{60\text{ min}}{1\ \cancel{\text{hr}}} = 1{,}440\text{ min}$$

Notice how all we did is set up the ratios so that everything but the unit we wanted — minutes — crossed out. Again, this works because we're multiplying by a value worth 1. $\frac{24\text{ hr}}{1\text{ d}}$ is equivalent to 1, as both the numerator and the denominator represent the same quantity. The same holds true for $\frac{60\text{ min}}{1\text{ hr}}$.

Applications to Time Problems

Because time is such an intricate part of life, we face time problems all the time (pun intended). Some require multiple "tools" from our mathematical toolbox to solve.

For example, let's say that you know you run 8 miles per hour. You want to figure out how far you can run in 2 hours at that pace.

You are probably used to solving this type of problem like this:

8 • 2 = 16. You can run 16 miles in 2 hours.

Now that you know about units, it's time to begin including them in the problems. Here is another way to write the problem that shows what's really happening with the units:

$$\frac{8 \text{ mi}}{1 \cancel{\text{hr}}} \bullet 2 \cancel{\text{hr}} = 16 \text{ mi}$$

Notice that we put 8 mi over 1hr: $\frac{8 \text{ mi}}{1 \text{ hr}}$. This is a ratio showing 8 miles per hour. Remember, *per* is a good clue that you're dealing with a ratio.

Always include units in problems when units are given. This practice will keep you from a lot of accidental errors, as well as help you solve more intricate problems. If you do the math correctly, you'll end up in the correct unit. If you don't end up in the correct unit, you'll know you did something wrong.

Example: Jenny can run 8 miles per hour. How far can she run in 90 minutes?

Notice that our speed is given in miles per *hour*, but we're asked how far we can go in 90 *minutes*. In order to get an accurate answer, we have to make sure that we use the same units.

While we could convert the 90 minutes to hours, it's easy enough to **substitute 60 min for the 1 hr** (after all, 60 min = 1 hr), making our speed $\frac{8 \text{ mi}}{60 \text{ min}}$.

Now we can multiply to find the answer.

$$\frac{8 \text{ mi}}{\underset{2}{\cancel{60}} \cancel{\text{min}}} \bullet \overset{3}{\cancel{90}} \cancel{\text{min}} = \frac{8 \text{ mi} \bullet 3}{2} = 12 \text{ mi}$$

Example: Jenny runs 8 miles an hour. How far can she run in 20 minutes?

Once again, we'll rewrite our speed as $\frac{8 \text{ mi}}{60 \text{ min}}$ since our time was given in minutes instead of hours.

$$\frac{8 \text{ mi}}{\underset{3}{\cancel{60}} \cancel{\text{min}}} \bullet \overset{1}{\cancel{20}} \cancel{\text{min}} = \frac{8 \text{ mi}}{3} = 2.67 \text{ mi}$$

Everyday Time Conversions

While it's important to know how to convert time units on paper, we often also need to convert them mentally. For example, if you know it takes you 25 minutes to drive to a store, and that you need about 35 minutes in the store plus another 10 minutes to get to your 3 p.m. appointment, at what time do you need to leave your house?

To find the answer, start by figuring out how many hours and minutes you need altogether. Mentally add 25 + 35 + 10, which is 70 minutes. Now, you could convert 70 minutes to hours by multiplying by the conversion ratio

$$\overset{7}{\cancel{70}}\ \cancel{\text{min}} \bullet \frac{1 \text{ hr}}{\underset{6}{\cancel{60}}\ \cancel{\text{min}}} = \frac{7 \text{ hr}}{6} = 1\frac{1}{6} \text{ hr}$$

Now you could convert the fractional amount back to minutes.

$$\frac{1}{\underset{1}{\cancel{6}}}\ \cancel{\text{hr}} \bullet \frac{\overset{10}{\cancel{60}} \text{ min}}{1\ \cancel{\text{hr}}} = \frac{10 \text{ min}}{1} = 10 \text{ min}$$

Total Time Needed: 1 hr, 10 min

But, that's a long way to go about it for this problem. You can really solve it *all* mentally. You know 60 minutes makes 1 hour, so 70 minutes would be 1 hour and 10 minutes.

Now that you know how long you need, you can figure out when to leave. An hour earlier than 3 p.m. is 2 p.m., and 10 minutes earlier than that is 1:50 p.m.

Keeping Perspective — Don't Get Stuck!

When learning math, it's easy to get stuck on a concept and assume that every problem has to be solved the same way. But in real life, we encounter a variety of concepts all the time. We can't rely on just one mathematical tool — we need to be able to think through what tool to use for each situation.

Since one of this course's goals is to help equip you to use math wherever you may need to (it is, after all, a tool that can be used for God's glory), we'll sometimes throw a problem into a worksheet that requires different tools to solve than those covered in that lesson. Be sure to think through every problem to make sure your answer — and method of solving — makes sense.

14.9 Chapter Synopsis

I hope you had some fun this chapter with measurements! Here's a quick review of what we covered.

- **Finding standards: units** — To measure, we need a standard, or unit, we can use. We explored the distance units in two measurement systems — the U.S. Customary System and the Metric/SI System.

U.S. Customary System

12 inches (in) = 1 foot (ft)
3 feet or 36 inches = 1 yard (yd)
1 mile (mi) = 1,760 yards (yd) or 5,280 feet (ft)

Metric System/SI

10 millimeters (mm) = 1 centimeter (cm)
10 centimeters = 1 decimeter (dm)
10 decimeters = 1 meter (m)
10 meters = 1 decameters (dam)
10 decameters = 1 hectometer (hm)
10 hectometers = 1 kilometer (km)

- **Conversions** — We explored various methods for unit conversion (via a proportion, the ratio shortcut, and mental math) and practiced converting distance, currency, and time units. We also learned some common conversion ratios that help convert between the Metric and Customary Systems.

Common Conversions between Systems

1 in = 2.54 cm
1 ft = 30.48 cm
1 yd = 0.9144 m
1 mi = 1.60934 km

Common Conversions of Time

60 seconds = 1 minute (min)
60 minutes = 1 hour (hr)
24 hours = 1 day (d)
7 days = 1 week (wk)
365 days = 1 year (yr *or* y)

As you move on from this chapter, remember that units of measuring distance are simply predefined distances against which we can compare and describe the distances of objects. And methods for unit conversion are just shortcuts that use the consistencies and conventions we know about multiplication, fractions, etc., to easily convert units. Once again, we're using the abilities God gave us to help us more easily name and describe God's creation.

As you continue to learn about and use measurements, ponder the fact that God knows not just the measure of the things easy to measure, but the measure of things we can't possibly measure, such as the dust of the earth, the waters in the oceans, and the stars in the sky. While we can measure some things, our inability to measure so many aspects of creation reminds us again of how much greater God is than we are.

> *Who hath measured the waters in the hollow of his hand, and meted out heaven with the span, and comprehended the dust of the earth in a measure, and weighed the mountains in scales, and the hills in a balance? (Isaiah 40:12).*