Horizons

Algebra I
Student Book

$$|7x - 40| - 42 > -19$$

$$|15 + 18| =$$

$$\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = 6$$

$$5 \div \sqrt{5} =$$

$$2(y - 2) = 2\left(-\tfrac{3}{2}\right)(x + 1)$$

$$2y - 4 = -3(x + 1)$$

$$2y - 4 = -3x - 3$$

Introduction to...
Exploring Math through...

Often students ask:

Who uses this stuff anyway?

Will I ever have to use algebra in the real world?

I will NEVER be a math major. Why do I have to learn all this?

Math is a school subject that is used daily by people in their work, homes, and play. Many people use math in their jobs, even if those jobs do not require a college degree in mathematics. There is a good chance you will use math on an algebra level when you get a job. Math is also an integral part of recreation. Almost every sport or hobby uses math in some way.

While you may find some of the topics in algebra challenging, they will help you learn more about math and God's carefully designed world. You do not know what plans God has for your life. You may be surprised in the directions God leads you and find that you use math in ways you never expected.

Throughout this book, you will read about several sports and hobbies that require the use of math. Whether or not God's plan for your life includes college, math will play a role in your future.

> *"For I know the plans I have for you," declares the LORD, "plans to prosper you and not to harm you, plans to give you hope and a future."*
>
> —Jeremiah 29:11 NIV

Natural numbers are counting numbers. (1, 2, 3, . . .)

Whole numbers are the natural numbers and zero. (0, 1, 2, . . .)

Integers are the positive and negative whole numbers. (-1, 0, 1, . . .)

Rational numbers are numbers that can be written as a fraction. $\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{1}, 10.5\right)$

Irrational numbers are numbers that CANNOT be written as a fraction. $\left(\sqrt{2}, \pi\right)$

Real numbers are numbers in any of the above categories.

Signed Number Rules:
When adding two numbers with the same sign, add the numbers like normal, and keep the same sign in the answer.
(+2) + (+5) = (+7) and (-2) + (-5) = (-7)

When adding two numbers with opposite signs, ignore the signs (use the absolute values) and subtract the smaller number from the larger number. Keep the sign of the larger number as the sign in the answer.

(+5) + (-2) = (5 − 2) = 3. 5 is larger than 2 and 5 is positive in the problem, so the answer is positive.
(+5) + (-2) = (+3).

(-5) + (+2) = -(5 − 2) = 3. 5 is larger than 2 and 5 is negative in the problem, so the answer is negative.
(-5) + (+2) = (-3)

When subtracting signed numbers, change the sign of the second number and add.
$\left(+5\right) - \left(-2\right) = \left(+5\right) + \left(+2\right) = 5 + 2 = 7$

When multiplying two numbers with the same sign, the answer is ALWAYS positive.
$\left(+5\right) \times \left(+4\right) = 20 \quad \left(-5\right) \times \left(-4\right) = 20$

When multiplying two numbers with different signs, the answer is ALWAYS negative.
$\left(+5\right) \times \left(-4\right) = -20 \quad \left(-5\right) \times \left(+4\right) = -20$

When multiplying more than two numbers, count the number of negatives. If there is an even number of negative terms, the answer is positive. If there is an odd number of negative terms, the answer is negative.

When dividing signed numbers, follow the rules of multiplying signed numbers.

① CLASSWORK

Identify each number as *natural*, *whole*, *integer*, *rational*, *irrational*, or *real*. Some numbers may have more than one answer.

	7	−4	$\sqrt{2}$	0	$1\frac{1}{4}$	$\frac{1}{6}$	π	5.3
Natural								
Whole								
Integer								
Rational								
Irrational								
Real								

Solve, using the rules for signed numbers.

(+42) + (+61) =

(+42) + (-61) =

(+42) − (-61) =

(-42) − (-61) =

(-3)(-4) =

(-3)(4) =

(-3)(4)(2) =

(-3)(-4)(2) =

$\left(+12\right) \div \left(-3\right) =$

$\left(-12\right) \div \left(-3\right) =$

3 Solve, following the rules of signed numbers.

(+57) + (+73) =   (-3)(7)(2) =

(+57) + (-73) =   (8)(-7)(1) =

(-57) + (+73) =   (-9)(-7)(-1) =

(-57) + (-73) =   (-7)(8)(2) =

(+242) − (+397) =   (-4)(-9)(3) =

(+242) + (-397) =   (12)(5)(-2) =

(-242) + (+397) =   (-11)(2)(-4) =

(-242) − (-397) =   (-9)(-4)(-3) =

4 Solve.

The Passer Rating of a college football quarterback is calculated using the formula
NCAA QB Passer Rating = $[(8.4y) + (330t) − (200i) + (100c)] ÷ a$, where y is the number of
passing yards, t is the number of touchdowns thrown, i is the number of interceptions thrown, c is
the number of completed passes, and a is the number of pass attempts.

Calculate the passer rating of a quarterback that had 220
passing yards, 1 touchdown thrown, no interceptions, 13
completed passes, and 17 pass attempts in his last game.
Round answer to the nearest hundredth.

NCAA QB Passer Rating

$[(8.4y) + (330t) − (200i) + (100c)] ÷ a$

ACTIVITIES

2 Find the prime numbers in the list below by following the directions.

1. Cross out the number 1.
2. Circle the number 2. Cross out every other number after two (the multiples of 2).
3. Circle the number 3. Cross out every third number after three (the multiples of 3).
4. Circle the number 5. Cross out every fifth number after five (the multiples of 5).
5. Circle the number 7. Cross out every seventh number after seven (the multiples of 7).
6. Circle all remaining numbers. The circled numbers are the prime numbers less than 100.

```
 1    2    3    4    5    6    7    8    9   10
11   12   13   14   15   16   17   18   19   20
21   22   23   24   25   26   27   28   29   30
31   32   33   34   35   36   37   38   39   40
41   42   43   44   45   46   47   48   49   50
51   52   53   54   55   56   57   58   59   60
61   62   63   64   65   66   67   68   69   70
71   72   73   74   75   76   77   78   79   80
81   82   83   84   85   86   87   88   89   90
91   92   93   94   95   96   97   98   99  100
```
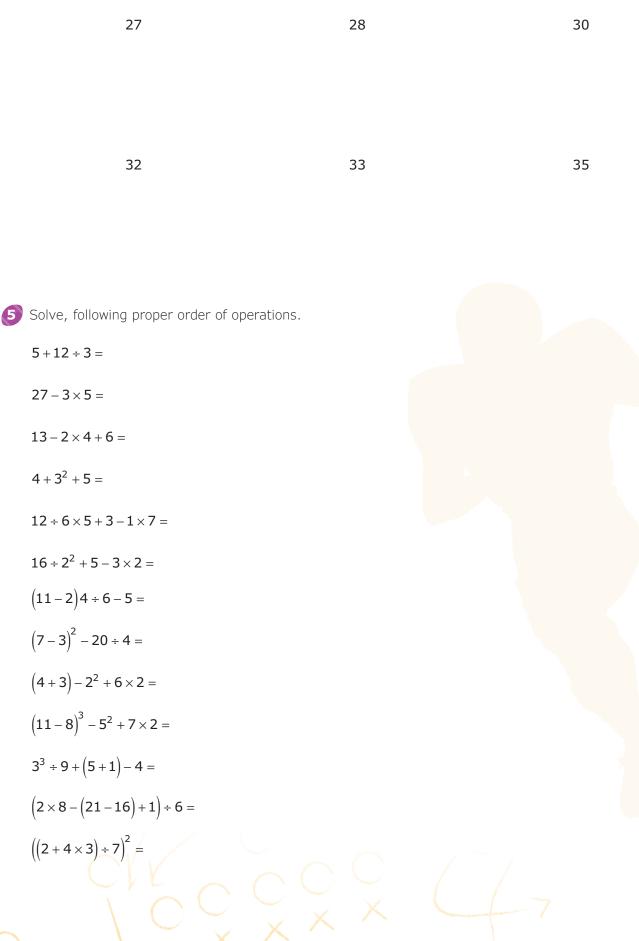
Write the prime numbers less than 100.

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____,

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____,

3 Find the prime factorization of each number. Use exponents where appropriate.

| 12 | 14 | 15 | 20 |

| 21 | 22 | 24 | 25 |

4 Find the prime factorization of each number. Use exponents where appropriate.

27   28   30

32   33   35

5 Solve, following proper order of operations.

$5 + 12 \div 3 =$

$27 - 3 \times 5 =$

$13 - 2 \times 4 + 6 =$

$4 + 3^2 + 5 =$

$12 \div 6 \times 5 + 3 - 1 \times 7 =$

$16 \div 2^2 + 5 - 3 \times 2 =$

$\left(11 - 2\right)4 \div 6 - 5 =$

$\left(7 - 3\right)^2 - 20 \div 4 =$

$\left(4 + 3\right) - 2^2 + 6 \times 2 =$

$\left(11 - 8\right)^3 - 5^2 + 7 \times 2 =$

$3^3 \div 9 + \left(5 + 1\right) - 4 =$

$\left(2 \times 8 - \left(21 - 16\right) + 1\right) \div 6 =$

$\left(\left(2 + 4 \times 3\right) \div 7\right)^2 =$

The opposite of raising a number to an exponent is taking the **root** of a number. The root is represented by the symbol $\sqrt{}$, called the **radical**. The number under the radical is called the **radicand** (or **argument**), and the number that indicates the root is called the **index** and corresponds to the exponent.

For example, $2^3 = 8$. To express this as a root, write $\sqrt[3]{8} = 2$, where 8 is the radicand, 3 is the index, and 2 is the root. In this case, 3 is the cube root of 8.

To find the **square root** of a number, find a number that, when multiplied by itself, gives the radicand.

For example, $\sqrt{16} = \sqrt{4 \times 4} = 4$

For larger numbers, write the radicand as the product of perfect square factors and find the square roots.

$\sqrt{128} = \sqrt{8 \times 8 \times 2} = 8\sqrt{2}$

To add or subtract roots, the radicands and indexes must be equal. Add the numbers immediately to the left of the radical. If there is no number, treat it as a 1.

For example, $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$ and $2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}$. If the radicands or indexes are not equal, the roots cannot be added or subtracted.

To multiply or divide roots with the same index, multiply or divide the radicands and write the answer under one radical. Multiply or divide the numbers outside the radical and write outside the radical in the answer. Simplify if necessary.

For example, $\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = 6$

1 **CLASSWORK**

Rewrite the following expressions as roots.

$2^4 = 16$

$3^2 = 9$

$5^2 = 25$

$5^3 = 125$

$6^3 = 216$

Solve the following roots.

$\sqrt{16} =$

$\sqrt[3]{27} =$

$\sqrt{32} =$

$\sqrt[3]{16} =$

$\sqrt{2} + \sqrt{2} =$

$\sqrt{5} + 2\sqrt{5} =$

$\sqrt[3]{10} + 5\sqrt[3]{10} =$

$6\sqrt{7} - 4\sqrt{7} =$

$5\sqrt[3]{5} - 4\sqrt[3]{5} =$

$\left(\sqrt{10}\right)\left(\sqrt{2}\right) =$

$\left(3\sqrt{5}\right)\left(2\sqrt{2}\right) =$

$\sqrt{27} \div \sqrt{3} =$

$10\sqrt[3]{16} \div 5\sqrt[3]{4} =$

$3 \div \sqrt{3} =$

ACTIVITIES

2 Rewrite the following expressions as roots.

$2^6 = 64$ $8^2 = 64$

$5^2 = 25$ $3^4 = 81$

$4^3 = 64$ $7^2 = 49$

A **polynomial** is an algebraic expression. If that expression contains two or more terms, the terms must be separated by a plus or minus sign. All variables must have a positive integer as an exponent, and no variable may appear in a denominator.

A **constant** is a term that has a number but no variable.

A **coefficient** is a number that is multiplied by a variable.

A **monomial** is an expression containing one term, such as x^2, $3x$, or 5. A constant is a monomial.

A **binomial** is a polynomial containing two terms, such as $3x + 5$ or $x^2 - 4x$.

A **trinomial** is a polynomial containing three terms, such as $x^2 - 4x + 3$.

Identify whether or not each expression is a polynomial. For each polynomial, identify it as a constant, monomial, binomial, or trinomial.

$x^2 + 2x - 1$
This is a polynomial and a trinomial.

$4x^{-2} - 3x + 7$
This is not a polynomial because there is a -2 as an exponent.

1 CLASSWORK

Identify whether or not each expression is a polynomial. For each polynomial, identify it as a constant, monomial, binomial, or trinomial.

$6x - 4$

17

$4x^2 + \dfrac{5}{x} - 3$

$3x^{-2} - 5$

$3x^2 - 4x + 2$

ACTIVITIES

2 Identify whether or not each expression is a polynomial. For each polynomial, identify it as a constant, monomial, binomial, or trinomial.

$9x - 4$

$7x^2 + \dfrac{3}{x} - 4$

$8x^{-2} + 9$

31

$10x^2 - 13x + 6$

-3x

The **Distributive Property** allows another method of working with parenthetical expressions that are multiplied by a single factor.

In some cases, it is easier to multiply each term in the parentheses by the factor outside the parentheses and then simplify.

$2(15+13) = 2(15) + 2(13) = 30 + 26 = 56$ rather than $2(15+13) = 2(28) = 56$

① CLASSWORK

Simplify the expressions, using the distributive property.

$4(10+9) =$

$5(12+7) =$

$9(20-3) =$

Something to Think About...
Two parentheses next to each other with no symbol between them means multiply.
(5)(4) = 20   (-5)(4) = −20

Commutative Property of Multiplication: You can change the order of the terms and still get the same product.
$2 \times 3 = 6$ and $3 \times 2 = 6$

Associative Property of Multiplication: You can group the terms in different ways and still get the same product.
$2 \times (3 \times 4) = 2 \times 12 = 24$ and
$(2 \times 3) \times 4 = 6 \times 4 = 24$

Identity Property of Multiplication: You can multiply any number by one and the product is always the number.
$0 \times 4 = 0$ and $4 \times 0 = 0$

ACTIVITIES

② Use the distributive property to simplify each expression.

$2(35+7) =$

$4(9+5) =$

$7(1+40) =$

$7(30+9) =$

$8(20+9) =$

$4(25+9+15) =$

③ Solve the following roots.

$\sqrt[3]{375} - 2\sqrt[3]{24} =$

$(\sqrt{10})(\sqrt{5}) =$

$(4\sqrt{5})(3\sqrt{15}) =$

$\sqrt{27} \div 3 =$

$12\sqrt[3]{54} \div 3\sqrt[3]{2} =$

1 P = the set of positive integer factors of 16

Q = the set of positive integer factors of 20

R = the set of positive integer factors of 24

P, Q, and R represent three sets of numbers, as defined above. Which set of numbers below belongs to all three sets?

A. {1, 2, 4}

B. {1, 2, 3, 4}

C. {1, 2, 4, 16}

D. {1, 2, 3, 4, 16}

E. {1, 2, 3, 4, 16, 24}

2 Given $4(e-f) - 5 = 3$, what is the value of $e - f$?

A. $-\frac{1}{2}$

B. 2

C. 4

D. 8

E. 32

3 Given $(3+a)(7-b) = 0$ and a is a natural number, what is the value of b?

A. -7

B. -3

C. 0

D. 3

E. 7

4 If $13^7 \times 13^x = 13^{21}$, what is the value of x?

A. 3

B. 7

C. 14

D. 21

E. 147