



Index

- α , 29
- \setminus , 6
- β , 29
- \cap , 6
- \cup , 5
- ∞ , 2
- ω , 29
- \perp , 309
- ϕ , 179
- ρ , 179
- θ , 29
- 30-60-90 triangle, 32
- 45-45-90 triangle, 32
- Agnesi, Maria Gaetana, 191
- Airy, Sir George, 180
- AMC, vii
- American Mathematics Competitions, *see* AMC
- American Regions Math League, *see* ARML
- amplitude, 56
- angle
 - dihedral, 155
 - inscribed, 141
- Angle Bisector Theorem, 155
- angle difference identity, 91–99
- angle of depression, 129, 130
- angle of elevation, 129, 130
- angle sum identity, 91–99
- angular coordinate, 167
- \arccos , 68
- arccot , 68
- arccsc , 68
- arcsec , 68
- arcsin , 68
- arctan , 68
- argument, 219
- ARML, vii
- Art of Problem Solving, v, 512
- associative property
 - complex numbers, 195
- asymptote, 50
- bilinear, 307, 383
- box product, 451
- Brahmagupta's formula, 157
- Bretschneider's formula, 157
- Cartesian space, 177
- Cauchy-Schwarz Inequality, 310
- centroid, 271, 470
 - of a tetrahedron, 475
 - vector representation, 470
- Chebyshev polynomial, 223
- circle
 - complex number representation, 201, 273
- circle of Apollonius, 274
- circular reasoning, 230
- circumcircle, 144, 148
- circumradius, 144, 148, 474
- cis, 219
- cofactor, 405

INDEX

- collinearity, 265, 269
 - complex numbers and, 265, 269
- column vector, 342, 409
- commutative property
 - complex numbers, 195
- complex number, 192–296
 - argument, 219
 - arithmetic, 192–197
 - associative property, 195
 - commutative property, 195
 - complex plane, 198–202
 - conjugate, 195
 - distributive property, 195
 - equilateral triangle, 279
 - exponential form, 224–232
 - geometry, 255–296
 - graphing, 198–202
 - imaginary part, 192, 202–208
 - magnitude, 200, 219
 - multiplicative inverse, 196
 - norm, 200
 - parallelogram law, 216
 - polar form, 218–224
 - real part, 192, 202–208
 - rectangular form, 219
 - root of unity, 232–248
 - square root, 193, 206
- complex number geometry, 255–296
 - circle, 201, 273
 - collinearity, 265, 269
 - midpoint, 263
 - parallel lines, 266, 269
 - perpendicular lines, 267, 269
 - regular polygon, 277–285
- complex plane, 198–202
- component, 297
- composition (of functions), 16–19
- conjugate, 195
 - conjugate pair, 195
- converse, 141
- conversion factor, 41
- coordinates
 - cylindrical, 178–187
 - polar, 167–176
 - rectangular, 178
 - spherical, 179, 187
- \cos , 30
- \cos^{-1} , 68
- cosecant, 30
 - graph of, 52
- cosine, 30
 - angle difference identity, 95
 - angle sum identity, 95
 - difference-to-product identity, 109
 - double angle identity, 101
 - graph of, 48
 - half angle identity, 104
 - in terms of e , 230
 - period of, 47
 - sum-to-product identity, 109
 - triple angle identity, 106
- \cot , 30
- cotangent, 30
- Cramer's rule, 356, 397
- cross product, 442–449
- \csc , 30
- cyclic quadrilateral, 157
- cycloid, 165
- cyclotomic polynomial, 252
- cylindrical coordinates, 178–187
- de Moivre's Theorem, 222, 226
- degree, 212
- determinant, 356
 - geometric interpretation (2×2), 363–368
 - geometric interpretation (3×3), 450–454
 - of a 2×2 matrix, 354–368
 - of a 3×3 matrix, 395–408, 450–454
- diagonal matrix, 347
- difference-to-product identity, 106–110
- dihedral angle, 155
- dilation, 256
 - center of, 256
 - scale factor, 256
- dimensionless unit, 40
- direction vector, 315, 423
- distributive property
 - complex numbers, 195
- domain, 1
- dot product
 - in three dimensions, 382–384
 - in two dimensions, 304–311
- double angle identity, 100–106
- double root, 216
- dummy variable, 1
- e , 226–227
- Edwards, Miles Dillon, 484
- eigenvalue, 419
- eigenvector, 419
- ellipse, 175
- equation
 - parametric, 158–167
- equator, 180

equilateral triangle
complex numbers and, 279
Euler line, 471, 472
even function, 50
excircle, 474
expansion by minors, 404
exponential form, 224–232
Extended Law of Sines, 144
extraneous solution, 85
 f^{-1} (functions), 20
Factor Theorem, 209
Feuerbach's Theorem, 474
fractals, xiv
frequency, 61, 111
function, 1–28
 composition, 16–19
 domain, 1
 even, 50
 graphing, 9–16
 inverse, 19–24
 invertible, 22
 linear, 307, 310
 odd, 49
 periodic, 46
 range, 1
 real, 2
 real-valued, 2
 transformation, 12–16
 trigonometric, 29
Fundamental Theorem of Algebra, 209
generalization, 138
generated, 426, 427
geometric transformation, 255
 complex numbers and, 255–264
geometry
 complex number, 255–296
great circle, 180
Greek letter, 29
guitar, 111

half angle identity, 100–106
Harvard-MIT Math Tournament, *see* HMMT
head, of a vector, 302
head-to-tail addition, 302
Heron's formula, 152
 proof with complex numbers, 485
HMMT, viii
horizontal line test, 23
hyperbolic cosine, 251
hyperbolic sine, 251

hyperbolic tangent, 251
 i , 192
identity (trigonometric), 38, 83–127
 angle difference, 91–99
 angle sum, 91–99
 difference-to-product, 106–110
 double angle, 100–106
 half angle, 100–106
 product-to-difference, 106–110
 product-to-sum, 106–110
 sum-to-product, 106–110
 triple angle, 106
identity matrix, 349, 393
 2×2 , 349
 3×3 , 393
if and only if, 204
 i^i , 254
Im, 202
imaginary axis, 198
imaginary number, 192
imaginary part, 192, 202–208
incircle, 148, 474
inner product, 305
inradius, 148, 474
inscribed angle, 141
instrument tuning, 111
intercept, 9
intersection, of intervals, 6
interval
 intersection, 6
 union, 6
interval notation, 2
inverse cosine, 68
inverse function, 19–24
 graph of, 23
inverse matrix (2×2), 369–374
inverse matrix (3×3), 454–457
inverse sine, 68
inverse tangent, 68
inverse trigonometric function, 64–76
invertible, 372
irreducible polynomial, 241
isometry, 256

Jacobi Identity, 449

kite, 469

Lagrange Identity, 449
latitude, 180
lattice point, 368
Law of Cosines, 136–141

INDEX

- Law of Sines, 141–148
Law of Tangents, 152
length
 of a vector, 381
line
 definition of, two dimensions, 315
 distance from a point to, 329
linear combination, 318, 421
linear function, 307, 310
linearly dependent, 321, 441
linearly independent, 321, 441
links, vi
logarithm
 base e , 254
longitude, 180
magnitude, 219
 of a complex number, 200
 of a vector, 381
Mandelbrot Competition, vii
Mandelbrot set, xv
matrices, 334
matrix
 2×2 , 334–379
 3×3 , 384–408
 addition, 337
 cofactor, 405
 determinant, 354–368, 395–408, 450–454
 diagonal, 347
 dimensions, 409
 eigenvalue, 419
 eigenvector, 419
 identity, 349, 393
 inverse (2×2), 369–374
 inverse (3×3), 454–457
 multiplication, 341
 nonsingular, 361
 permutation, 394
 rotation, 351
 singular, 361
 skew-symmetric, 419
 square, 414
 transpose, 377, 399
 zero, 335
median, 271
 of a tetrahedron, 475
midpoint
 in the complex plane, 263
minor, 404
Monge's Theorem, 288
monic, 212
multiplicative inverse, 196
New York State Math League, viii
nine-point circle, 471
 complex numbers, 479
nonreal, 192
nonsingular matrix, 361, 397
norm
 in three dimensions, 381
 in two dimensions, 299
 of a complex number, 200
normal, 322, 438
number
 complex, 192
 imaginary, 192
 nonreal, 192
 real, 192
NYSML, viii
odd function, 49
orientation, 255
origin, 198
orthocenter, 471
 complex numbers, 479
 vector representation, 471
orthogonal, 309, 384
Outer Napoleon Triangle, 287
parallel lines
 complex numbers and, 266, 269
parallel planes, 427
parallelepiped, 450
parallelogram
 area of, 449
 parallelogram law, 216
parameter, 158
parameterization, 163
parameterize an equation, 164
parametric equation, 158–167
period, 46
periodic function, 46
permutation matrix, 394
perpendicular lines
 complex numbers and, 267, 269
phase shift, 58
Pigeonhole Principle, 119
pirate, 262
pitch, 111
plane, 427
 normal, 438
 parallel, 427
 through the origin, 426
polar coordinates, 167–176
polar form, 218–224

- polynomial
complex roots of, 208–213
cyclotomic, 252
degree, 212
Factor Theorem, 209
irreducible, 241
monic, 212
Rational Root Theorem, 209
- prime meridian, 180
primitive root of unity, 237
problem solving, iii
product-to-difference identity, 106–110
product-to-sum identity, 106–110
proj, 322, 327
projection, 322–330
prosthesis, 124
Ptolemy's Inequality, 286
Pythagorean Theorem, 39
- quadrant, 35
- \mathbb{R} , 2
 $\mathbb{R} \setminus 6$
radial coordinate, 167
radian, 40–44
range, 1
Rational Root Theorem, 209
Re, 202
real axis, 198
real function, 2
real number, 192
real part, 192, 202–208
rectangular coordinates
three dimensions, 178
two dimensions, 167
rectangular form, 219
reflection, 256
regular polygon
complex numbers and, 277–285
resources, v
right triangle trigonometry, 35, 128–136
right-hand rule, 445
root of unity, 232–248
primitive, 237
rotation, 255
angle, 255
center, 255
orientation, 255
rotation matrix, 351
row vector, 342, 409
Rule of Sarrus, 397
- scalar, 297
- scalar triple product, 451
scale factor, 256
sec, 30
secant, 30
semiperimeter, 148
Shoelace Theorem, 378
signed area, 366
signed volume, 451
Simson Line, 290
sin, 30
 \sin^{-1} , 68
sine, 30
angle difference identity, 95
angle sum identity, 95
difference-to-product identity, 109
double angle identity, 101
graph of, 47
half angle identity, 104
in terms of e , 230
sum-to-product identity, 109
triple angle identity, 106
- sine curve, 48
singular matrix, 361, 397
sinusoid, 48
sinusoidal variation, 61
skew-symmetric matrix, 419
slope, 30, 131
SOHCAHTOA, 128
sound, 111
sound waves, 111
spherical coordinates, 179, 187
square matrix, 414
Stewart's Theorem, 151
sum-to-product identity, 106–110
- tail, of a vector, 302
tan, 30
 \tan^{-1} , 68
tangent, 30
and slope, 131
angle difference identity, 95
angle sum identity, 95
double angle identity, 101
graph of, 51
half angle identity, 104
Tchebychev polynomial, 223
telescoping series, 116
terminal point, 30
tetrahedron
centroid, 475
median, 475
three-dimensional space, 177

INDEX

- transformation, 255
- translation, 255
- transpose, 377, 399, 419
 - of a 2×2 matrix, 377
 - of a 3×3 matrix, 399
 - of an $r \times c$ matrix, 419
- triangle area, 133
- Triangle Inequality, 274, 276
- trig, 30
- trigonometric function, 29
 - graph of, 45–64
 - in a right triangle, 35
 - inverse of, 64–76
- trigonometric identity, 38, 83–127
- trigonometry, 29–157
 - Extended Law of Sines, 144
 - Law of Cosines, 136–141
 - Law of Sines, 141–148
 - Law of Tangents, 152
 - right triangle, 128–136
- triple angle identity, 106
- union, of intervals, 6
- unit circle, 30
- unit vector, 298, 381
- upper triangular matrix, 408, 457
- USA Mathematical Talent Search, *see* USAMTS
- USAMTS, vii
- variable
 - dummy, 1
- variation
 - sinusoidal, 61
- vector, 297
 - centroid of a tetrahedron, 475
 - centroid of a triangle, 470
 - column, 342
 - component, 297
 - cross product, 442–449
 - direction, 301, 381
 - dot product, 304–311, 382–384
 - head, 302
 - head-to-tail addition, 302
 - in three dimensions, 380–384
 - in two dimensions, 297–333
 - inner product, 305
 - length, 299, 381
 - linear dependence, 312–321
 - magnitude, 299, 381
 - norm, 299, 381
 - normal, 322, 438
 - orthocenter, 471
- projection, 322–330
- row, 342
- tail, 302
- unit, 298, 381
- zero, 298
- vertical line test, 12
- Vieta's formulas, 245
- witch of Agnesi, 191
- x -intercept, 9
- xy -plane, 177
- xz -plane, 177
- y -intercept, 9
- yz -plane, 177
- \bar{z} , 195
- zero matrix, 335
- zero vector, 298
- zero-product property, 210