Students beginning **Intermediate Counting & Probability** should be comfortable with geometric series, factoring and multiplying polynomials, and basic counting techniques. Examples of each are below.

Geometric Series.

1. Evaluate the sum: $1 + 2 + 2^2 + 2^3 + \cdots + 2^{10}$.

2. Assuming that $-1 < x < 1$, find a closed form expression for $1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$ by evaluating the sum as an infinite geometric series with common ratio $-x$.

Factoring and Multiplying Polynomials.

3. Find the polynomial $f(x)$ such that $(x - 1)f(x) = x^6 - 1$.

4. Find the five terms with smallest degree of the product $(1 + x + x^2 + x^3 + x^4 + \cdots)(1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots)$.

Counting Techniques. The following questions are from the "Do You Know Introduction to Counting & Probability" quiz. If you cannot easily solve most of them, you should consider taking our **Introduction to Counting & Probability** course before taking **Intermediate Counting & Probability**.

5. How many multiples of 7 are between 83 and 229?

6. How many distinct arrangements are there of the letters in the word MATHEMATICS?

7. A coin is flipped, a 6-sided die numbered 1 through 6 is rolled, and a 10-sided die numbered 0 through 9 is rolled. What is the probability that the coin comes up heads and the sum of the numbers that show on the dice is 8?

8. Find the coefficient of $x^3 y^8$ in the expansion of $(x - 2y^2)^7$.

9. Particle Man is at the origin in three-dimensional space. How many ways can Particle Man take a series of 12 unit-length steps, each step parallel to one of the coordinate axes, from the origin to $(3, 4, 5)$ without passing through the point $(2, 3, 2)$?

10. In poker, a hand is formed with 5 cards. The deck has 52 cards, separated into 4 suits. Each suit has 13 ranks which are the same in every suit. A full house occurs when a hand has 3 cards of one rank and 2 of another. How many different poker hands are full houses?

11. How many distinguishable ways can the faces of a regular hexagonal prism be painted 8 different colors (one color per face, no color used twice)?

12. There are $2n$ players in a chess tournament. The first round consists of pairing the players to participate in n matches with every player playing one match. In terms of n, how many ways can this pairing take place?

13. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?

The answers to Are You Ready for Intermediate Counting & Probability are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. 2047

2. $\frac{1}{1+x}$

3. $x^5 + x^4 + x^3 + x^2 + x + 1$

4. $1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots$

5. 21

6. 4989600

7. 1/20

8. 560

9. 23520

10. 3744

11. 3360

12. $\frac{(2n)!}{2^n n!}$

13. 35/128