

$$\overbrace{1000}^{999} \times \overbrace{1000}^{999} - 1 = \left(\overbrace{1000}^{999} - 1 \right) \times \left(\overbrace{1000}^{999} + 1 \right)$$

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