Art of Problem Solving Textbooks

Are You Ready For

Introduction to Number Theory

If you've mastered division, remainders, integer exponents and basic equations as illustrated in the problems below, then you are ready for the Art of Problem Solving book **Introduction to Number Theory**. (Answers to these problems are on the following page.)

1. **Remainders**. Find the remainder in each division problem.
   (a) $248 \div 8$
   (b) $399 \div 13$
   (c) $1333 \div 109$
2. **Integer exponents**. Evaluate the following. Express your answer as an integer or as a fraction in lowest terms.
   (a) $(-1)^6$
   (b) 3^{2+1}
   (c) 12^{-1}
   (d) $2 \cdot \left(2^3\right)^2$
3. **Equations**. Solve each of the following for x.
   (a) $4x + 7 = 23$
   (b) $5x + 10 = 54$
   (c) $3x - 4 = x + 2$
   (d) $x^2 + 2x - 4 = 0$
4. **Algebraic expressions**.
   (a) Simplify $(3x + 2) + (5x + 7)$.
   (b) Expand the product $(4n + 1)(4n + 3)$.
   (c) Factor $49 - 16x^2$.
   (d) Factor $3y^2 + 7y + 2$.
5. **Problem solving**.
   (a) The sequence $5, 7, 8, 9, 5, 7, 8, 9, \ldots$ continues to repeat with the same pattern. What is the 79th number on the list?
   (b) When the integer n is divided by 12, the remainder is 2. What is the remainder when n is divided by 6?

The answers to Are You Ready for Introduction to Number Theory are below.

1. **Remainders**.
   (a) 0
   (b) 9
   (c) 25
2. **Integer exponents**.
   (a) 1
   (b) 27
   (c) $\frac{1}{12}$
   (d) 128
3. **Equations**.
   (a) 4
   (b) $\frac{44}{5}$
   (c) 3
   (d) $-1+\sqrt{5}, -1-\sqrt{5}$
4. **Algebraic expressions**.
   (a) $8x+9$
   (b) $16n^2+7n+3$
   (c) $(7+4x)(7-4x)$
   (d) $(3y+1)(y+2)$
5. **Problem solving**.
   (a) 8
   (b) 2