math MAMMOTH

Grade 5-B Worktext

Light Blue Series

- Graphing and statistics
- Fractions: add and subtract
- Fractions: multiply and divide
- Geometry

By Maria Miller

Contents

Chapter 7: Fractions: Multiply and Divide

Chapter 8: Geometry

Foreword

Math Mammoth Grade 5-B Worktext comprises a complete math curriculum for the second half of fifth grade mathematics. Fifth grade focuses on fractions and decimals, in particular. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, and decimal arithmetic. In this part, 5-B, we study graphing, fraction arithmetic, and geometry.

This book starts with chapter 5, where we study graphing in a coordinate grid, line and bar graphs, and average and mode. Today's world has become increasingly complex, with lots of data in the media, so our children need a good grasp of graphs to be able to make sense of all that information.

Chapter 6 is about the addition and subtraction of fractions—another focus topic for 5th grade, besides decimals. Students learn to add and subtract unlike fractions, using the technique of first converting them to equivalent like fractions. In chapter 7, we study the multiplication and division of fractions (division only in special cases), relying first on visual models, and then proceeding to the abstract shortcuts.

Chapter 8 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success in your math teaching!

Maria Miller, the author

Chapter 5: Statistics and Graphing
Introduction

This chapter starts out with a study of the coordinate grid, but only in the first quadrant. Besides learning how to plot points, students also plot ordered pairs (points) from number patterns or rules. This is actually the beginning of the study of *functions*.

Practicing the use of the coordinate grid is a natural "prelude" to the study of line graphs, which follows next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories. The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study average (also called the *mean*) and mode, and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is plenty of time to learn that concept in subsequent grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all that many students are able to grasp from that is only the calculation procedures. I feel it is better to introduce and contrast initially only the two concepts, the mean and the mode, in order to give the student a solid foundation. We can introduce the median later, and then compare and contrast it with the other two.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 5

Helpful Resources on the Internet

Coordinate Grid

Billy Bug Returns at Primary Games
Move Billy Bug to the feeding place with given coordinates.
http://www.primarygames.co.uk/pg2/bug2/bug2.html

Co-ordinate Game
You will see a red circle on the grid. Enter the co-ordinates and click "check."
http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/maths/coordinate_game/game1.htm

Graphit
A graphing tool that plots both functions and ordered pairs.
http://www.shodor.org/interactivate/activities/graphit/index.html

Graph Mole
A fun game about plotting points on a coordinate plane. Plot points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Coordinate Grid Quiz from ThatQuiz.org
This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0

Graphing and Graphs

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool:
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness, and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Data Grapher
This basic data grapher can be used to create bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=204

Histogram Tool
Create a histogram from your data, or analyze histograms from pre-made data.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Create a Graph
Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Graphs Quiz from That Quiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, box plot, scatter graph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
http://www.thatquiz.org/tq-5/math/graphs

Math Goodies Interactive Data and Graphs Lessons
Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs, and exercises.
http://www.mathgoodies.com/lessons/toc_vol11.html

Data Analysis Gizmos from Explorelearning.com
Interactive online simulations or activities, with lesson plans. Topics include creating a bar graph or aline graph, pictographs, mean and median, and a reaction time gizmo. This is an excellent resource. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=383

Statistics Interactive Activities
(scroll down to Statistics and Probability concepts)
A set of interactive tools for exploring histograms, pie charts, box plots, stem-leaf plots, and mean, median, variance, and standard deviation of data. You can enter your own data or explore the examples.
http://www.shodor.org/interactivate/activities/tools.html

Mean, Median, Mode, Range

Using and Handling Data
Simple explanations for finding mean, median, or mode.
http://www.mathsisfun.com/data/index.html#stats

Math Goodies Interactive Statistics Lessons
Clear lessons with examples and interactive quiz questions, including range, arithmetic mean, non-routine mean, median, and mode, practice exercises, and challenge exercises.
http://www.mathgoodies.com/lessons/toc_vol8.htm

Mean, Median, and Mode
How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.
http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median Lessons
Explanations with simple examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/data/measuresofaveragerev1.shtml

Measures Activity
Enter you own data and the program will calculate mean, median, mode, range and some other statistical measures.
http://www.shodor.org/interactivate/activities/Measures

Mean/Mode Quiz
A 10-question quiz about calculating the mode and mean.
http://www.thatquiz.org/tq-p-z1/?-j6g00-l5-p0

Landmark Shark Game
You are dealt five number cards, and using that as your data set you need to choose which of the range, median, or mode is the largest number.
http://media.emgames.com/emgames/demosite/playdemo.html?activity=M5A006&activitytype=dcr&level=3

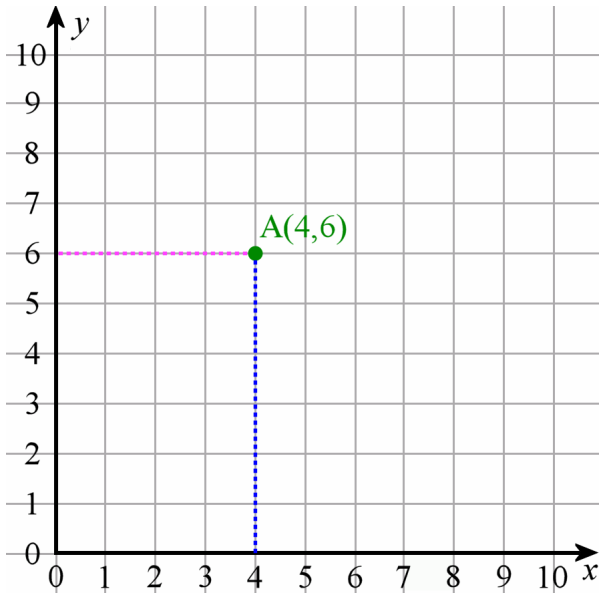
Coordinate Grid

This is a **coordinate grid**.

The long black line across the bottom, with the "*x*" near its end, is called the ***x*-axis**.

The vertical line that has "*y*" near the top is called the ***y*-axis**.

You can see one point, called "A," that is drawn or *plotted* on the grid.



It has two numbers *associated*, or matched, with it. Those two numbers are called the **coordinates** of the point A.

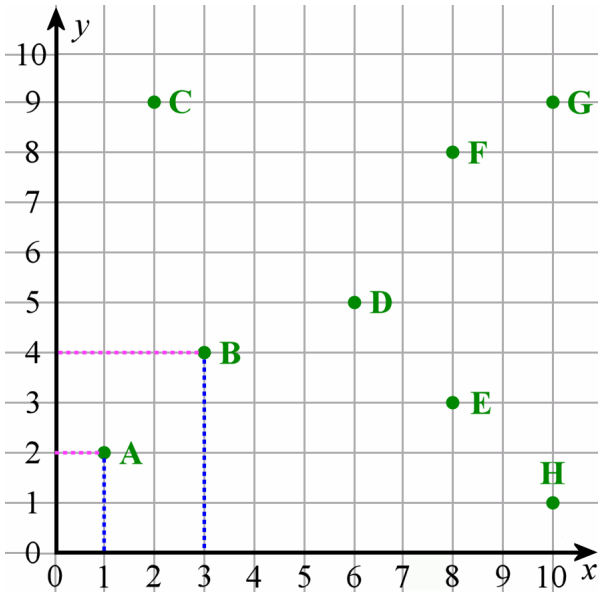
The first number is called the **x-coordinate** of the point A, and the second number is called the **y-coordinate** of the point A.

The x-coordinate of the point A is 4 because if you drew a line straight down from A, it would *intersect*, or "hit," the *x*-axis at 4. The y-coordinate of the point A is 6 because if you drew a line straight left from A, it would intersect the *y*-axis at 6.

We write the two coordinates of a point inside parentheses, separated by a comma.

Note: The order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is ALWAYS the *y*-coordinate, not the other way around.
So (5, 8) means the *x*-coordinate is 5 and the *y*-coordinate is 8.

1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in.



A (___, ___) B (___, ___)

C (___, ___) D (___, ___)

E (___, ___) F (___, ___)

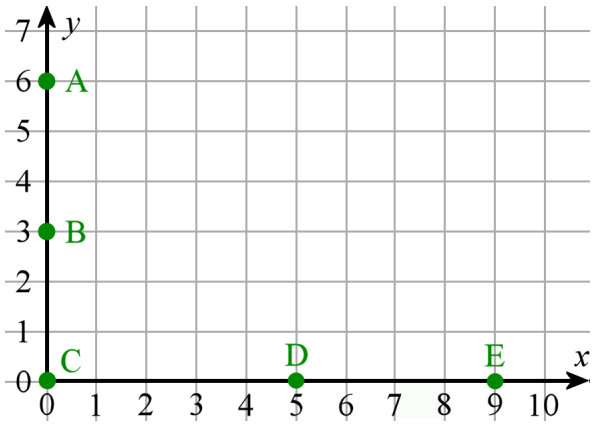
G (___, ___) H (___, ___)

Notice especially the points that are located on the two axes.

If a point lies on the *y*-axis, its *x*-coordinate is zero. A is (0, 6), and B is (0, 3) .

If the point lies on the *x*-axis, its *y*-coordinate is zero. D is (5, 0) and E is (9, 0).

The point C has the coordinates (0, 0). This point (0, 0) is called the *origin*.

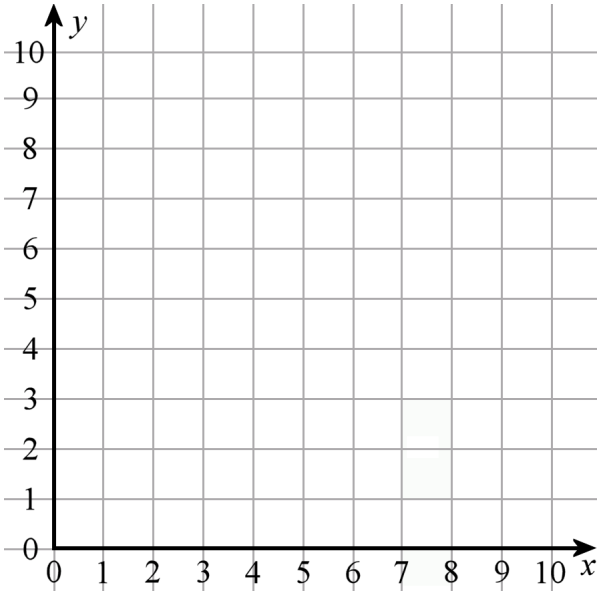


2. Plot and label the following points on the coordinate grid.

A (2, 8) B (0, 5) C (4, 0)

D (9, 10) E (8, 5) F (1, 4)

G (1, 0) H (0, 8) I (3, 7)



3. The coordinate grid is very useful for many things. For example, computer drawing programs use it frequently. Let's say "LINE (5,6) - (2,7)" means a straight line segment that is drawn from the point (5, 6) to the point (2, 7).

Draw the following line segments.
What figure is formed?

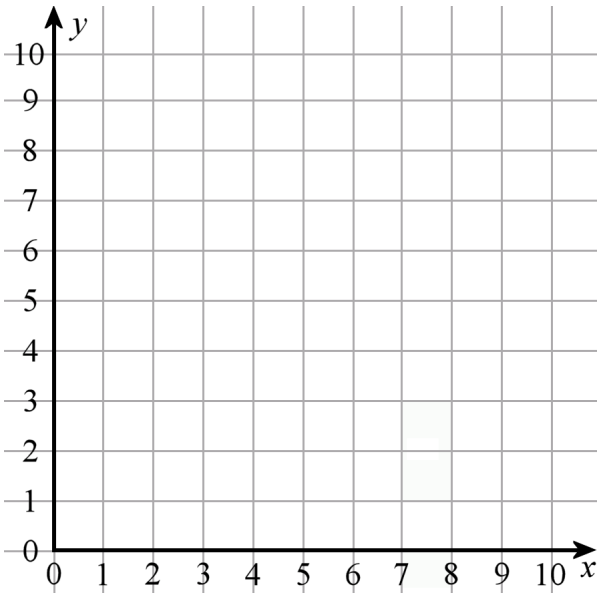
LINE (1, 0) - (7, 0) LINE (7, 0) - (7, 5)

LINE (1, 0) - (1, 5) LINE (1, 5) - (0, 5)

LINE (0, 5) - (4, 7) LINE (4, 7) - (8, 5)

LINE (8, 5) - (7, 5) LINE (3, 0) - (3, 3)

LINE (5, 0) - (5, 3) LINE (3, 3) - (5, 3)

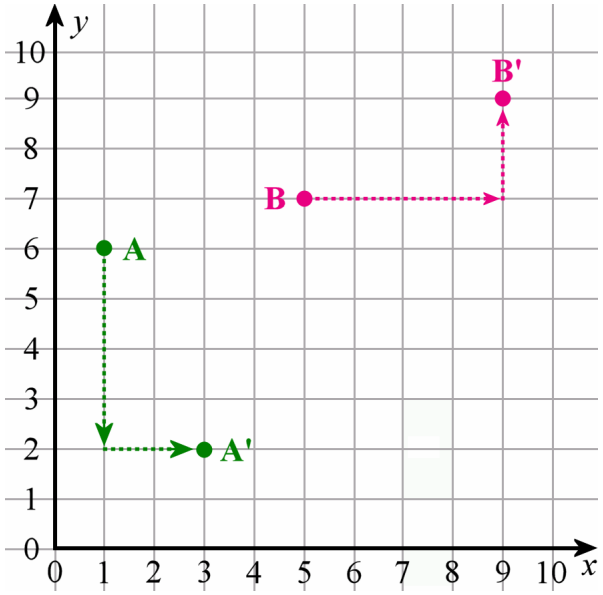


This example shows point A moving four units down and then two units to the right. The new location is called point A′ (read "A prime").

Originally A's coordinates were (1, 6).
After the movement, the coordinates are (3, 2)

Notice how you can just subtract four units from the *y*-coordinate (the movement four units straight down) and add two units to the *x*-coordinate (movement two units to the right).

Point B is originally at (5, 7). It moves four units to the right and two up. You add four to the *x*-coordinate, and two to the *y*-coordinate. Its new coordinates are (9, 9).

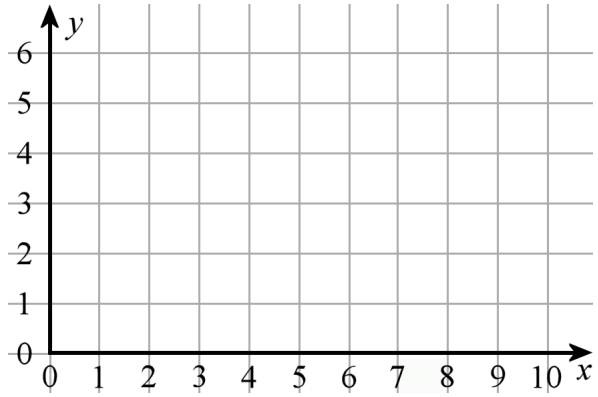


Movement up or down affects the *y*-coordinate.
Movement right or left affects the *x*-coordinate.
In other words, movement *parallel* to an axis affects that same coordinate.

4. The three vertices of a triangle are (2, 0), (5, 1) and (3, 4). The triangle is moved three units to the right and two up.

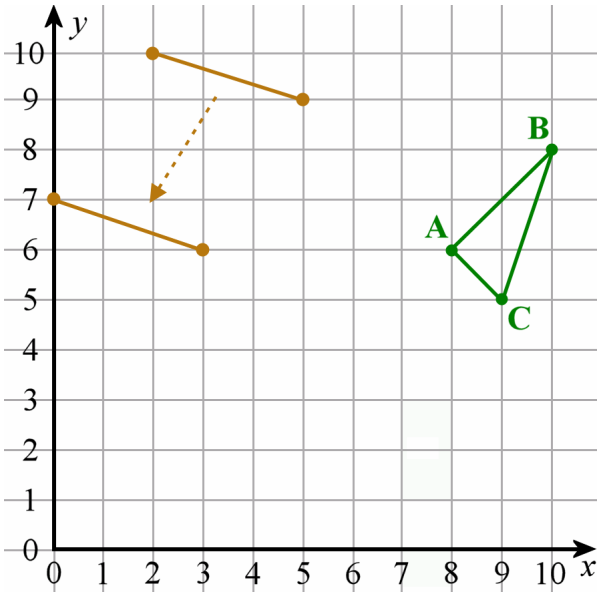
   a. Plot the vertices of the triangle before and after the movement.

   b. Write the coordinates of the vertices after the movement.



5. **a.** Determine how the line segment has been moved, and move the triangle ABC the same way. Let's call the new triangle A′B′C′. Write down the coordinates of the vertices of the triangle A′B′C′ after the movement.

   b. Let's say the point (3, 5) moves to (2, 7). Move the triangle ABC in a similar way. Write down the coordinates of the triangle's vertices after the movement.



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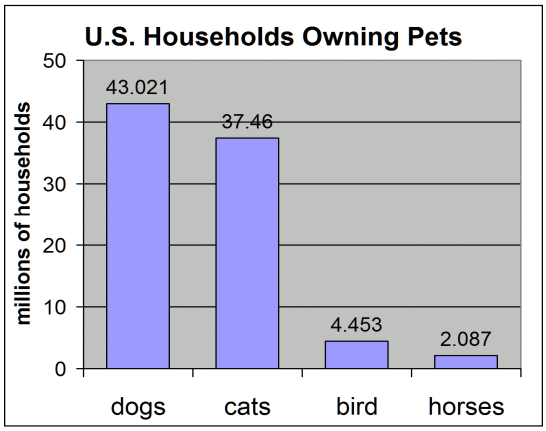
Mean, Mode, and Bar Graphs



Do you think you could calculate the average from the data shown in the bar graph? After all, there are numbers involved.

Actually, we cannot. To see why, you need to think *what kind of original data* produced this graph. What was asked of the people in the study? What did they respond?

The people were asked something like, "What pets do you have?" The people would have answered, "cat," or "dog," and so on.

The original data set consists simply of the words "cat," "dog," "bird," and "horse"—each one listed many times, because each mention of a "cat" would mean the answer of one particular household.

cat, cat, dog, dog, dog, bird, dog, dog, bird, cat, dog, horse, dog, cat, dog, ...

We cannot calculate anything from this kind of data set because it is not numerical data! However, we CAN find the most commonly occurring item, and that is called the ***mode***.

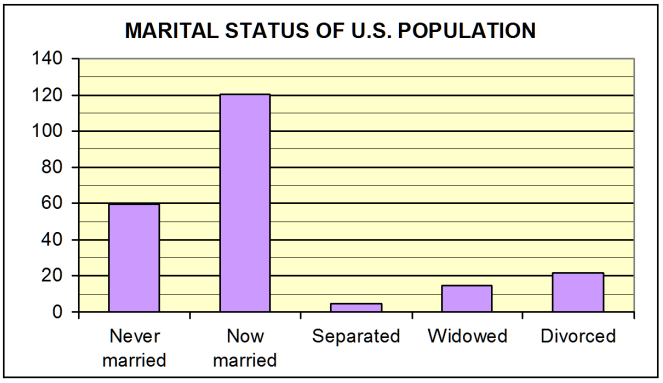
In this case, the mode is *dog.* It made the highest bar on the graph.

The *mode* is the most commonly occurring item in a data set.

- Sometimes a set of data has two or more modes. For example, the data set *green, green, blue, blue, black, brown, hazel* has two modes: both green and blue are equally common.

- If none of the items occurs twice or more, there is no mode. For example, this data: *green, blue, pink, red, black, brown, purple* has no mode.

1. Find the mode of the data set shown in the bar graph on the right.



2. **a.** Find the mode of this data:

   water, pop, juice, pop, juice, water, milk, water, pop, pop, juice, pop

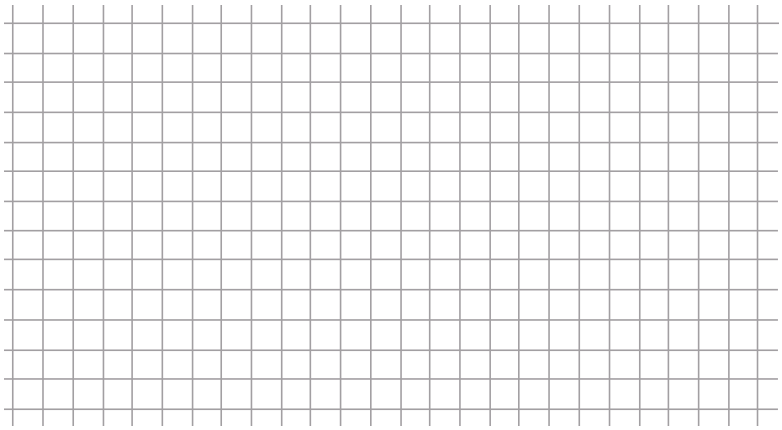
   b. If the above are the answers of 12 people to some question, what could have been the question?

3. Nineteen children were asked about their favorite ice cream flavor. Here are their responses:
strawberry, vanilla, chocolate, vanilla, chocolate chip, chocolate, pecan, pecan, vanilla, vanilla, strawberry, chocolate chip, vanilla, chocolate, chocolate, vanilla, strawberry, chocolate chip, vanilla.

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.



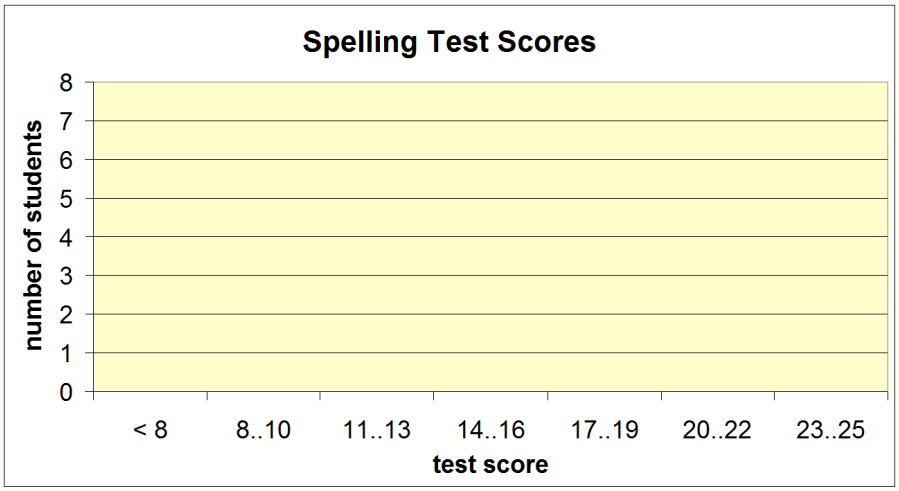
4. These are the spelling test scores of a fifth grade class:
4 5 7 9 9 10 10 11 11 12 12 12 13 14 17 18 18 18 19 19 19 20 24 25

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.

Test Score	Frequency
< 8	
8..10	
11..13	
14..16	
17..19	
20..22	
23..25	



5. **a.** Find the mode.

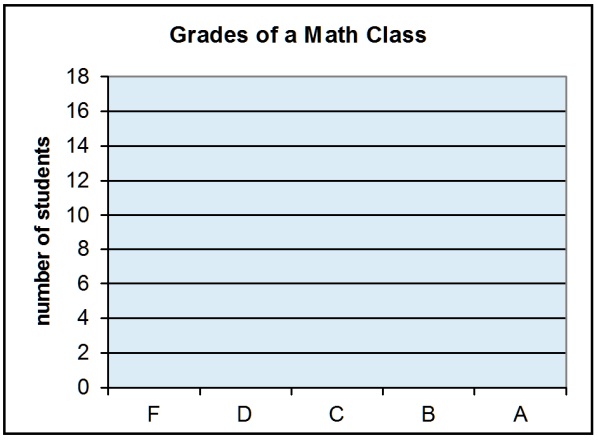
b. Draw a bar graph.

c. If possible, calculate the average.

d. There were ____ students in all. What *fraction* of the students got grade B?

Grades of a math class

Grade	Frequency
F	3
D	8
C	12
B	17
A	10



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Chapter 6: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade. I hope that students have already built a solid conceptual understanding in their minds in previous years, so we can build on that foundation.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a "stumbling block." Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions.

First, we review how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions. This chapter ends with a lesson on measuring in inches, using units as small as 1/16 of an inch.

The Lessons in Chapter 6

Helpful Resources on the Internet

General

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages. The activity includes several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the representations of the fractions and the models.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software
Games for addition and subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.
http://www.sheppardsoftware.com/math.htm#fractions

Who Wants Pizza?
This tutorial explains fraction addition and multiplication using a pizza, and then includes some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Fractioncity
Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Equivalent Fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Equivalent Fractions with Visual Models
Create custom-made worksheets for equivalent fractions. Choose to include pie images or not.
http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

Addition and Subtraction

Fraction Videos 1: Addition and Subtraction
My own videos that cover equivalent fractions, addition and subtraction of like and unlike fractions, and of mixed numbers.
http://www.mathmammoth.com/videos/fractions_1.php

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding Fractions
Illustrates how to find the common denominator when adding two unlike fractions using interactive pie models.
http://nlvm.usu.edu/en/nav/frames_asid_106_g_3_t_1.html

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure of adding two unlike fractions. A free registration required.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
The computer gives you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction.php

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Comparing Fractions Tool at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/app/tool/comparing-fractions

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Comparing/Ordering Fractions Worksheets
Create free worksheets for comparing two fractions or ordering 3-8 fractions. Compare fractions with the same denominator, fractions with the same numerator, or you compare a fraction to 1/2, or to 1, and so on. You can also include images (fraction pies).
http://www.homeschoolmath.net/worksheets/comparing_fractions.php

Measure It!
Practice measuring lines in inches. Multiple choice questions.
http://onlineintervention.funbrain.com/measure/index.html

Fraction Terminology

As we study fractions and their operations, it is important that you understand the terms, or words, that we use. This page is a reference. You can even post it on your wall or make your own fraction poster based on it.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.
The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

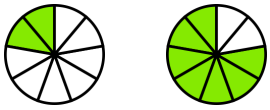
A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\frac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

Like fractions have the same denominator. They have the same kind of parts.

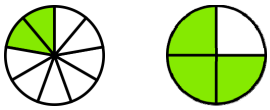
It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts.

It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

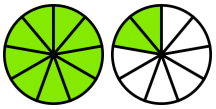


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

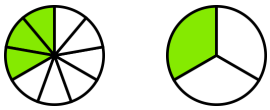
$\frac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). Since it is called a *fraction*, it is written as a fraction and *not* as a mixed number.



$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

Simplifying a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

$\frac{9}{12}$ simplifies to $\frac{3}{4}$.

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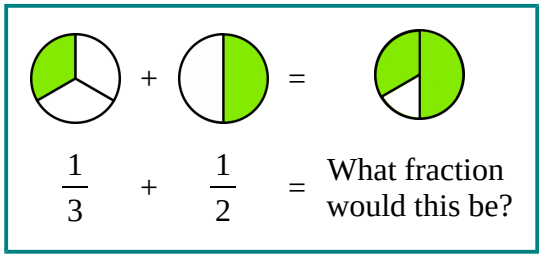
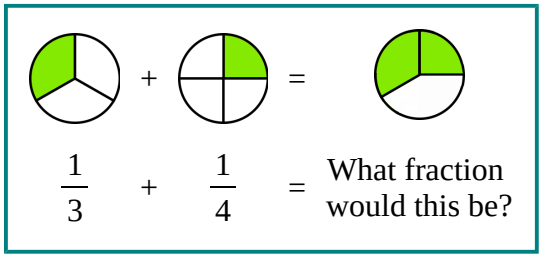
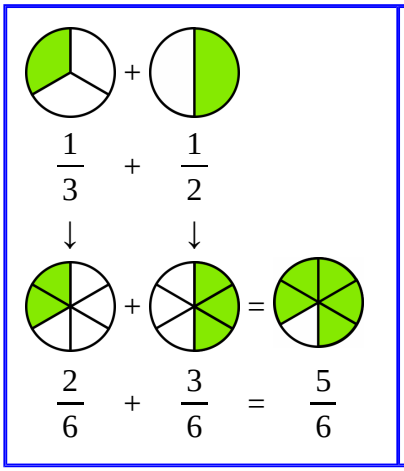
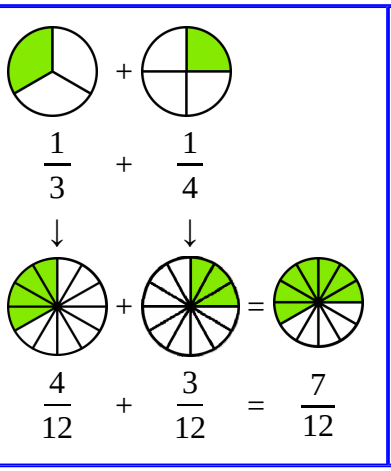
Adding and Subtracting Unlike Fractions

Cover the page below the black line. Then try to figure out the addition problems below.









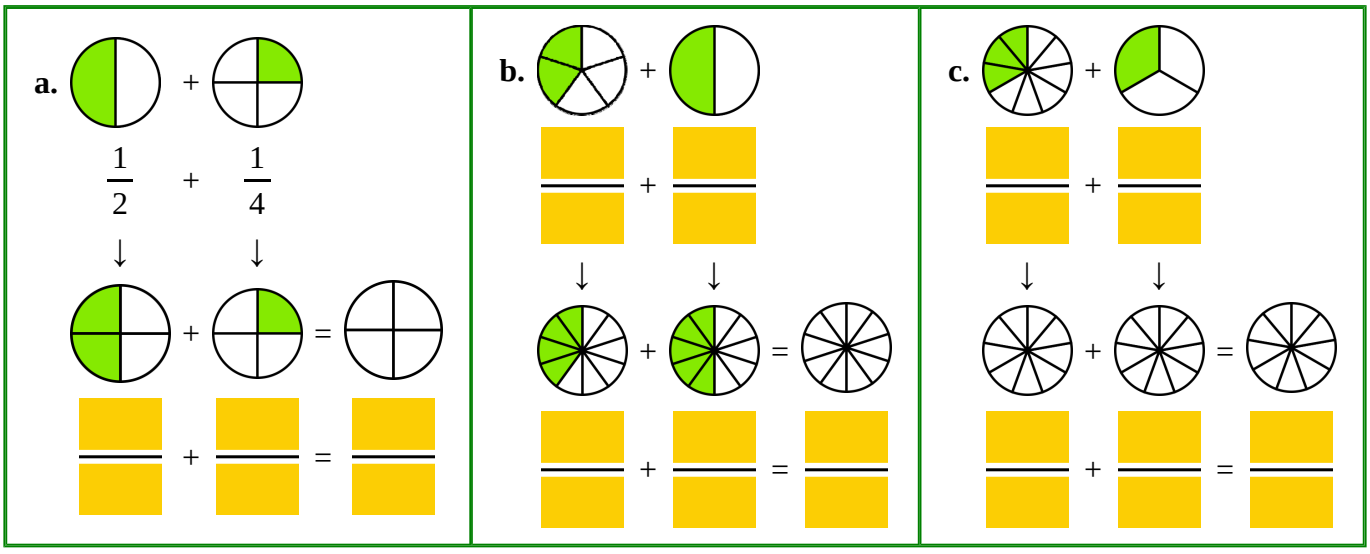
Did you solve the problems above?

The solution is this:

We convert the fractions so that they become *like* fractions (the *same* denominator), using equivalent fractions.

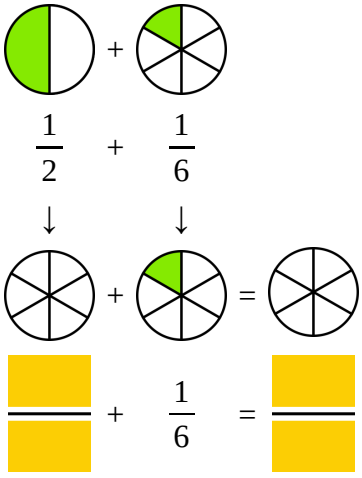
Then we can add or subtract.

1. Write the fractions shown by the pie images. Convert them into *equivalent fractions with the same denominator* (like fractions), and then add them. Color the missing parts.



2. Convert the fractions to like fractions first, and then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!

a.

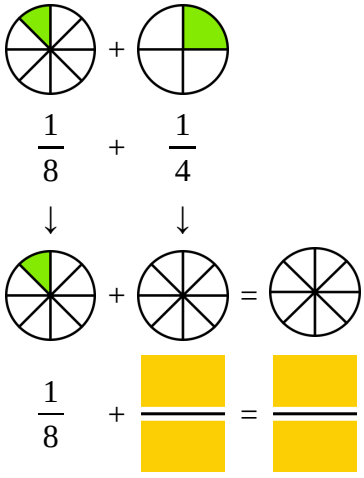


$\frac{1}{2} + \frac{1}{6}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{1}{6} = \frac{\square}{\square}$

b.

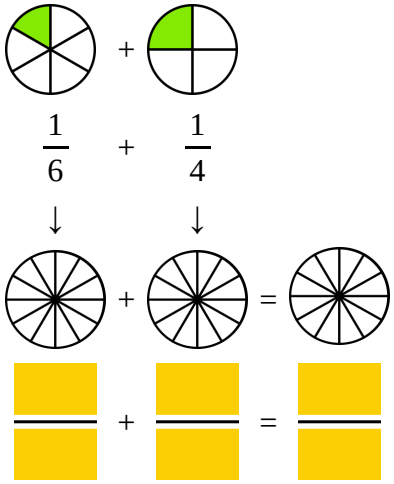


$\frac{1}{8} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{1}{8} + \frac{\square}{\square} = \frac{\square}{\square}$

c.



$\frac{1}{6} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

d. $\frac{5}{6} - \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{5}{6} - \frac{\square}{\square} = \frac{\square}{\square}$

e. $\frac{5}{8} - \frac{1}{4}$

$\downarrow \quad \downarrow$

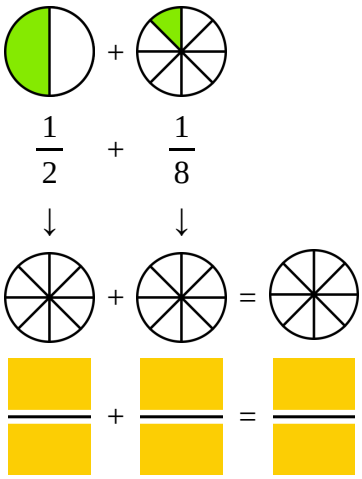
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

f. $\frac{5}{6} - \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

g.

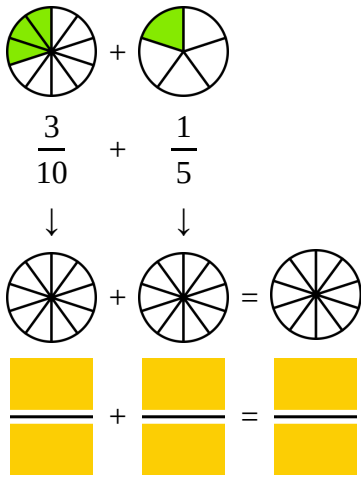


$\frac{1}{2} + \frac{1}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

h.

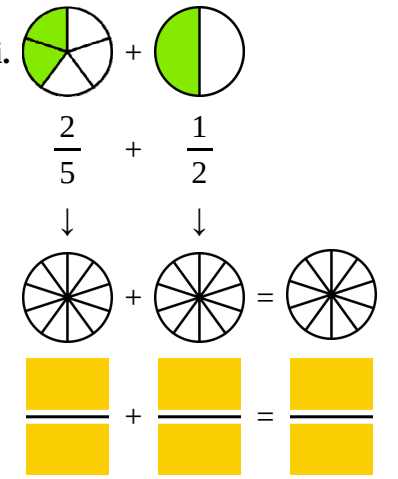


$\frac{3}{10} + \frac{1}{5}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

i.



$\frac{2}{5} + \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

j. $\frac{1}{2} + \frac{3}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

k. $\frac{9}{10} - \frac{2}{5}$

$\downarrow \quad \downarrow$

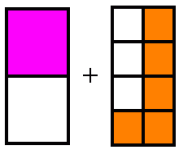
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

l. $\frac{4}{5} - \frac{1}{2}$

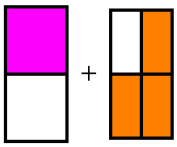
$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

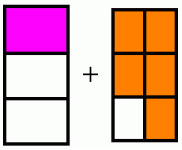
3. Split the parts only in the *first* fraction so that both fractions will have the same kind of parts. Add.



a. ___ + $\frac{5}{8}$ =

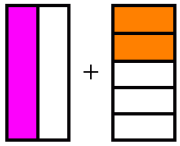


b. ___ + $\frac{3}{4}$ =

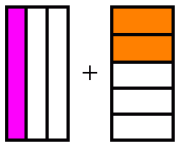


c. ___ + $\frac{5}{6}$ =

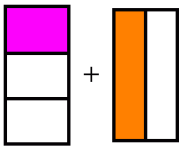
Now split the parts in *both* fractions so that they will have the same kind of parts. Add.



d. $\frac{\quad}{10}$ + $\frac{\quad}{10}$ =



e. $\frac{\quad}{15}$ + ___ =



f. ___ + ___ =

4. Fill in the table based on the problems above. What kind of parts did the two fractions have at first? What kind of parts did you use in the final addition?

	Types of parts:	Converted to:
a.	2nd parts and 8th parts	*8th* parts
b.	2nd parts and 4th parts	_____ parts
c.	3rd parts and 6th parts	_____ parts

	Types of parts:	Converted to:
d.	2nd parts and 5th parts	_____ parts
e.	3rd parts and 5th parts	_____ parts
f.	3rd parts and 2nd parts	_____ parts

5. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table above?

6. **Challenge:** If you think you know what kind of parts to convert these fractions into, then try these problems. Do not worry if you do not know how to do them—we will study this in the next lesson.

a. $\frac{1}{2}$ + $\frac{2}{3}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

b. $\frac{2}{3}$ − $\frac{2}{5}$

↓ ↓

$\frac{\quad}{\quad}$ − $\frac{\quad}{\quad}$ =

c. $\frac{1}{3}$ + $\frac{3}{4}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

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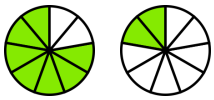
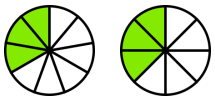
Comparing Fractions

Sometimes it is easy to know which fraction is the greater of the two. Study the examples below!



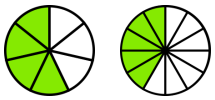
$\frac{7}{9} > \frac{2}{9}$

With **like fractions**, all you need to do is to check which fraction has more "slices," and that fraction is greater.



$\frac{3}{9} < \frac{3}{8}$

If both fractions have the **same number of pieces**, then the one with bigger pieces is greater.

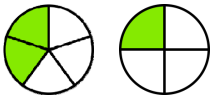


$\frac{4}{7} > \frac{5}{12}$

Sometimes you can **compare to 1/2**. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.

$\frac{6}{5} > \frac{9}{10}$

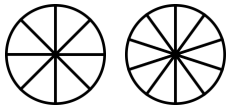
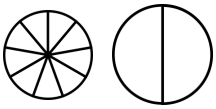
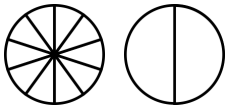
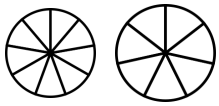
Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here, 6/5 is more than 1, and 9/10 is less than 1.



$\frac{2}{5} > \frac{1}{4}$

If you can imagine the pie pictures in your mind, you can sometimes "see" which fraction is bigger. For example, it is easy to see that 2/5 is more than 1/4.

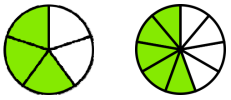
1. Compare the fractions, and write > , < , or = .

 **a.** $\frac{1}{8}$ $\frac{1}{10}$	 **b.** $\frac{4}{9}$ $\frac{1}{2}$	 **c.** $\frac{6}{10}$ $\frac{1}{2}$	 **d.** $\frac{3}{9}$ $\frac{3}{7}$
e. $\frac{8}{11}$ $\frac{4}{11}$	**f.** $\frac{7}{4}$ $\frac{7}{6}$	**g.** $\frac{5}{14}$ $\frac{5}{9}$	**h.** $\frac{4}{20}$ $\frac{2}{20}$
i. $\frac{2}{11}$ $\frac{2}{5}$	**j.** $\frac{1}{2}$ $\frac{5}{8}$	**k.** $\frac{3}{6}$ $\frac{1}{2}$	**l.** $\frac{1}{20}$ $\frac{1}{8}$
m. $\frac{1}{2}$ $\frac{3}{4}$	**n.** $\frac{8}{7}$ $\frac{3}{3}$	**o.** $\frac{49}{100}$ $\frac{61}{100}$	**p.** $\frac{7}{8}$ $\frac{8}{7}$
q. $\frac{9}{10}$ $\frac{3}{4}$	**r.** $\frac{6}{5}$ $\frac{3}{4}$	**s.** $\frac{4}{4}$ $\frac{9}{11}$	**t.** $\frac{1}{3}$ $\frac{3}{9}$

Sometimes none of the "tricks" explained in the previous page work, but we do have one more up our sleeve!

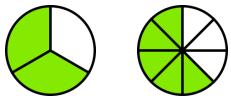
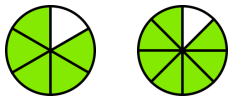
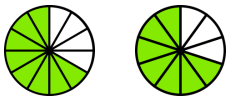
Convert both fractions into like fractions. Then compare.

In the picture on the right, it is hard to be sure if 3/5 is really more than 5/9. Convert both into 45th parts, and then it is easy to see that 27/45 is more than 25/45. Not by much, though!



$\frac{3}{5}$ ↓ $\frac{27}{45}$ > $\frac{5}{9}$ ↓ $\frac{25}{45}$

2. Convert the fractions into like fractions, and then compare them.

a.  $\frac{2}{3}$ ↓ $\frac{5}{8}$ ↓	**b.**  $\frac{5}{6}$ ↓ $\frac{7}{8}$ ↓	**c.**  $\frac{1}{3}$ ↓ $\frac{3}{10}$ ↓	**d.**  $\frac{8}{12}$ ↓ $\frac{7}{10}$ ↓
e. $\frac{5}{8}$ ↓ $\frac{7}{12}$ ↓	**f.** $\frac{11}{8}$ ↓ $\frac{14}{10}$ ↓	**g.** $\frac{6}{10}$ ↓ $\frac{58}{100}$ ↓	**h.** $\frac{6}{5}$ ↓ $\frac{11}{9}$ ↓
i. $\frac{7}{10}$ ↓ $\frac{5}{7}$ ↓	**j.** $\frac{43}{100}$ ↓ $\frac{3}{10}$ ↓	**k.** $\frac{9}{8}$ ↓ $\frac{8}{7}$ ↓	**l.** $\frac{7}{10}$ ↓ $\frac{2}{3}$ ↓

3. One cookie recipe calls for 1/2 cup of sugar. Another one calls for 2/3 cup of sugar. Which uses more sugar, a triple batch of the first recipe, or a double batch of the second?

How much more?

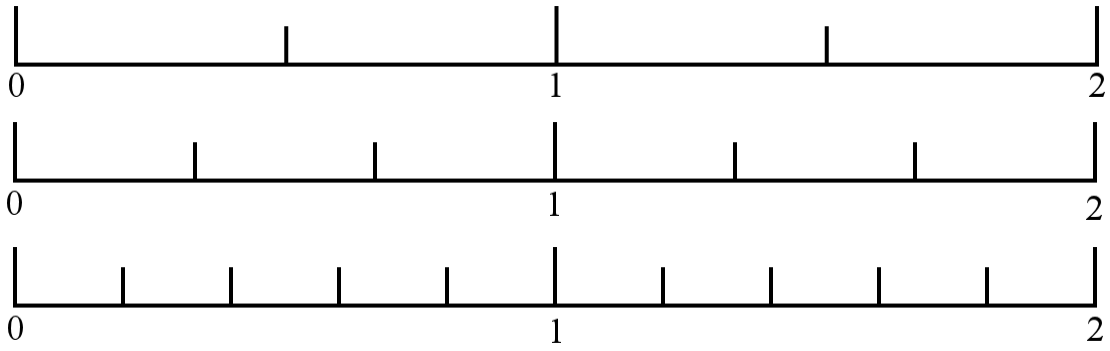
4. Compare the fractions using any method.

a. $\frac{5}{12}$ $\frac{3}{8}$	**b.** $\frac{5}{12}$ $\frac{4}{11}$	**c.** $\frac{3}{10}$ $\frac{1}{5}$	**d.** $\frac{3}{8}$ $\frac{4}{7}$
e. $\frac{4}{15}$ $\frac{1}{3}$	**f.** $\frac{5}{6}$ $\frac{11}{16}$	**g.** $\frac{7}{6}$ $\frac{10}{8}$	**h.** $\frac{5}{12}$ $\frac{5}{8}$
i. $\frac{3}{4}$ $\frac{4}{11}$	**j.** $\frac{13}{10}$ $\frac{9}{8}$	**k.** $\frac{2}{13}$ $\frac{1}{5}$	**l.** $\frac{1}{10}$ $\frac{1}{11}$

5. A coat costs $40. Which is a bigger discount:
1/4 off the normal price, or 3/10 off the normal price?

Does your answer change if the original price of the coat was $60 instead? Why or why not?

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



a. $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

___ < ___ < ___ < ___ < ___

b. $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

___ < ___ < ___ < ___ < ___

7. Write the three fractions in order.

a. $\frac{7}{8}, \frac{9}{10}, \frac{7}{9}$ ____ < ____ < ____	**b.** $\frac{1}{3}, \frac{4}{10}, \frac{2}{9}$ ____ < ____ < ____

8. Rebecca made a survey of a group of 600 women. She found that 1/3 of them never exercised, that 22/100 of them swam regularly, 1/5 of them jogged regularly, and the rest were involved in other sports.

a. Which was a bigger group, the women who jogged or the women who swam?

b. What fraction of this group of women exercise?

c. *How many women* in this group exercise?

d. How many women in this group swim?

Puzzle Corner

The seven dwarfs could not divide a pizza into seven equal slices. The oldest suggested, "Let's cut it into eight slices, let each dwarf have one piece, and give the last piece to the dog."

Then another dwarf said, "No! Let's cut it into 12 slices instead, and give each of us 1 ½ of those pieces, and the dog gets the 1 ½ pieces left over."

Which suggestion would give more pizza to the dog?

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Chapter 7: Fractions: Multiply and Divide

Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 8 on geometry in between chapters 6 and 7.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 6, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication (2/3) × 18 km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as 5 ÷ 3. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, 20 ÷ 6 = 3 2/6).

Then students encounter sharing divisions with fractions. For example, if two people share equally 4/5 of a pizza, how much will each person get? This is represented by the division (4/5) ÷ 2 = 2/5. Another case we study is dividing unit fractions by whole numbers (such as (1/2) ÷ 4).

We also divide whole numbers by unit fractions, such as 6 ÷ (1/3). Students will solve these thinking how many times the divisor "fits into" the dividend.

The last lesson is an introduction to ratios, and is optional. Ratios will be studied a lot in 6th and 7th grades, especially in connection with proportions. We are laying the groundwork for that here.

The Lessons in Chapter 7

Helpful Resources on the Internet

General

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
http://www.mathmammoth.com/videos/fractions_2.php

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games; for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Simplifying & Equivalent Fractions

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
http://www.homeschoolmath.net/worksheets/fraction.php

Multiplication and Division

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiply and Reduce Fractions Battleship Game
When you hit the enemy's battleship, you need to solve a fraction multiplication problem.
http://www.quia.com/ba/57713.htm

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Number line bars
Fraction bars that illustrate visually how many times a fraction "fits into" another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

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Multiplying Fractions by Fractions

We have studied how to find a fractional part of a whole number using multiplication.

For example, $\frac{3}{5}$ of 80 is written as a multiplication: $\frac{3}{5} \times 80 = \frac{240}{5} = 48$.

NOTE: The word *OF* translates here into **MULTIPLICATION**.

We can use the same idea to find *a fractional part* ***of*** *a fraction*!

One-half of  is  .

As a multiplication, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

One-fourth of  is  .

As a multiplication, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

a. Find $\frac{1}{2}$ of 

$\frac{1}{2} \times \frac{1}{4} =$

b. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

c. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

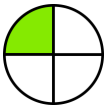
d. Find $\frac{1}{3}$ of 

e. Find $\frac{1}{3}$ of 

f. Find $\frac{1}{3}$ of 

g. Find $\frac{1}{4}$ of 

h. Find $\frac{1}{4}$ of 

i. Find $\frac{1}{4}$ of 

Did you notice a shortcut? If so, calculate $\frac{1}{5} \times \frac{1}{6} = \frac{\square}{\square}$

Shortcut: multiplying fractions of the type 1/*n*

To multiply fractions of the form 1/*n* where *n* is a whole number, simply multiply the denominators to get the new denominator →

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ or $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

2. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$	**b.** $\frac{1}{13} \times \frac{1}{3}$	**c.** $\frac{1}{5} \times \frac{1}{20}$

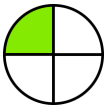
We have now studied how to find 1/2 or 1/3 or 1/5 of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

Review: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part *of* another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First, we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$. Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.

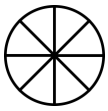
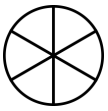
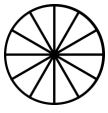
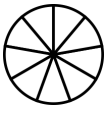
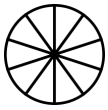
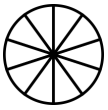
$\frac{2}{3}$ of  =

Example. Find $\frac{4}{5}$ of $\frac{1}{7}$.

First, we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction. It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Color in a picture to show the answer.

a. $\frac{3}{4} \times$  = 	**b.** $\frac{2}{3} \times$  = 
c. $\frac{3}{4} \times$  = 	**d.** $\frac{2}{3} \times$  = 
e. $\frac{2}{5} \times$  = 	**f.** $\frac{4}{5} \times$  = 

4. Solve the multiplications by using two helping multiplications. Lastly, simplify if possible.

a. $\frac{2}{3} \times \frac{1}{8} =$

First find 1/3 of 1/8, then multiply the result by 2.

$\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$ and $\frac{1}{24} \times 2 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

b. $\frac{3}{4} \times \frac{1}{10} =$

First find 1/4 of 1/10, then multiply the result by 3.

$\frac{1}{4} \times \frac{1}{10} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad}$

c. $\frac{3}{5} \times \frac{1}{6} =$

First find 1/5 of 1/6, then multiply the result by 3.

$\frac{1}{5} \times \frac{1}{6} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

d. $\frac{5}{6} \times \frac{1}{9} =$

First find 1/6 of 1/9, then multiply the result by 5.

$\frac{1}{6} \times \frac{1}{9} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 5 = \frac{\quad}{\quad}$

e. $\frac{2}{3} \times \frac{1}{7} =$

f. $\frac{3}{8} \times \frac{1}{4} =$

A shortcut for multiplying fractions

Multiply the numerators to get the numerator for the answer.
Multiply the denominators to get the denominator for the answer.

Study the examples on the right.

Remember always to give your final answer as a mixed number and in lowest terms (simplified).

$\frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$

$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1\frac{1}{10}$

5. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$

b. $\frac{11}{12} \times \frac{1}{6}$

c. $\frac{1}{3} \times \frac{3}{13}$

d. $9 \times \frac{2}{3}$

e. $\frac{2}{9} \times \frac{6}{7}$

f. $10 \times \frac{5}{7}$

COMPARE	
The roundabout way	**The shortcut**
$\frac{5}{6} \times \frac{1}{2} = ?$ First find 1/6 of 1/2, then multiply the result by 5. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ Find 1/8 of 3/5, then multiply that result by 2. And to find 1/8 of 3/5, first find 1/8 of 1/5, and then multiply that by 3. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$. That multiplied by 3 is $\frac{1}{40} \times 3 = \frac{3}{40}$. Then, that multiplied by 2 is $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$.	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.	

6. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8} =$	**b.** $\frac{7}{10} \times \frac{8}{5} =$
c. $\frac{9}{20} \times \frac{4}{5} =$	**d.** $\frac{2}{5} \times \frac{1}{3} =$
e. $\frac{1}{4} \times \frac{2}{7} =$	**f.** $\frac{9}{4} \times \frac{1}{3} =$
g. $\frac{2}{3} \times \frac{11}{8} =$	**h.** $\frac{2}{9} \times \frac{3}{10} =$

7. There was 1/4 of the pizza left. Marie ate 2/3 of that.

a. What part of the *original* pizza did she eat?

b. What part of the *original* pizza is left now?

8. Theresa has painted 5/8 of the room.

a. What part is still left to paint?

b. Now, Theresa has painted half of what was still left.
Draw a bar model of the situation.
What part of the room is still left to paint?

9. Ted has completed 2/3 of a job that his boss gave him.

a. What part is still left to do?

b. Now Ted has completed a third of what was still left to do.
Draw a bar model of the situation.
What (fractional) part of the original job is still left undone?

What part is completed?

10. Sally wants to make 1/3 of the recipe on the right.
How much does she need of each ingredient?

Carob Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
1/2 cup honey
1 teaspoon vanilla
3/4 cup whole wheat flour
3/4 teaspoon baking powder
1 cup walnuts or other nuts

11. For an upcoming get-together, Alison needs to multiply the coffee recipe. Assume that half of the guests drink one serving, and the other half drink two servings. Find how much coffee she will need, if she has:

a. 30 guests

b. 50 guests

c. 80 guests.

Coffee (5 servings)

3 1/2 cups water
1/4 cup coffee

Puzzle Corner Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$	**b.** $\times \frac{1}{4} = \frac{5}{16}$	**c.** $\times \frac{3}{8} = \frac{1}{16}$	**d.** $\times \frac{2}{5} = \frac{3}{10}$

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Chapter 8: Geometry
Introduction

The problems in this chapter involve lots of drawing. Geometry is a hands-on subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol “  ” are meant to be done in a notebook or on blank paper.

This chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and how to draw a perpendicular line through a given point on a line. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

The Lessons in Chapter 8

Helpful Resources on the Internet

General/Review/Fun things

Turtle Pond
Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Shape Explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Interactive Tangram Puzzle
Place the tangram pieces so that they form a given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation! Requires Java.
http://www.shodor.org/interactivate/activities/Tessellate

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry
A collection of interactive activities: fractals, geoboard activities, golden rectangle, ladybug leaf, ladybug mazes, tangrams, tessellations, transformations, and more.
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Quadrilaterals

Interactive Quadrilaterals
See all the different kinds of quadrilateral "in action". You can drag the corners, see how the angles change, and observe what properties do not change.
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Complete the Quadrilateral
Join the dots to complete these quadrilaterals. Where there are options, try to find the one on the grid with the largest possible area. The author, Fawn Nguyen, wishes, "If appropriate for your class, I hope you will consider doing this lesson — it will make the hours I spent recreating the handout and key worth it! :)" Great for 5th grader or whenever you study classifying quadrilaterals.
http://fawnnguyen.com/2013/02/08/don-stewards-complete-the-quadrilateral.aspx

Polygon Matching Game
Many of the polygons included are quadrilaterals.
http://www.mathplayground.com/matching_shapes.html

Classify Quadrilaterals Worksheets
Make free printable worksheets for classifying (idenfitying, naming) quadrilaterals. There are seven special types of quadrilaterals: square, rectangle, rhombus, parallelogram, trapezoid, kite, scalene, and these worksheets ask students to name the quadrilaterals among these seven types.
http://www.homeschoolmath.net/worksheets/classify_quadrilaterals.php

Quadrilaterals Quest
First, the quest asks you to choose all the quadrilaterals with the given properties. After several of those types of activities follows a quiz.
http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html

Quadrilateral Properties
Investigate the properties of square, rectangle, rhombus, an isosceles trapezoid, and a non-isosceles trapezoid in this dynamic, online activity.
http://www.glencoe.com/sites/texas/student/mathematics/assets/interactive_lab/geometry/G_08/G_08

Quadrilateral Classification Game
A virtual manipulative that challenges students to "draw" quadrilaterals with specific characteristics by moving vertices on a coordinate grid. Includes some challenging vocabulary, which is explained below the activity, such as orthodiagonal quadrilateral, cyclic, or convex quadrilateral.
http://www.uff.br/cdme/jcq/jcq-html/jcq-en.html

Triangles

Classify Triangles Worksheets
Make free printable worksheets for classifying triangles by their sides, angles, or both.
http://www.homeschoolmath.net/worksheets/classify_triangles.php

Triangle Classification at Cut The Knot
A tutorial and an applet about classifying triangles by their sides and angles. In the applet, you can drag any of the vertices of the triangle, and the applet tells you whether your triangle is acute, obtuse, or right, or equilateral, isosceles, or scalene.
http://www.cut-the-knot.org/triangle/Triangles.shtml

Rags to Riches: Classify Triangles by Sides and Angles
Answer multiple-choice questions about the angles of a triangle and classification of triangles in a quest for fame and fortune.
http://www.quia.com/rr/457498.html

Identify Triangles Quiz
A simple multiple-choice quiz about identifying (classifying) triangles either by their sides or angles. You can modify some of the quiz parameters, such as the number of problems in it.
http://www.thatquiz.org/tq-A/?-j1-l34-p0

Triangles & Quadrilaterals Classification Game
Look at the shapes as they go past, and drag them into the right groups (equilateral, isosceles, or scalene triangles, and quadrilaterals with 4 congruent sides, 2 congruent sides, or no congruent sides).
http://www.bbc.co.uk/bitesize/ks2/maths/shape_space/shapes/play/

Classifying Triangles Game
A fast-paced game where you drag triangles into the correct basket as fast as you can (acute, obtuse, right).
http://www.math-play.com/classifying-triangles/classifying-triangles.html

Triangle Classification Exploration Tool
Line segment AB is drawn in the plane. Where should point C be placed so that ABC is a right triangle? …so that it is an isosceles triangle? …so that it is obtuse? This activity will allow you to explore these questions.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=142

Volume

Geometric Solids
Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

Cuboid Exploder and **Isometric Shape Exploder**
These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes, and then "explode" them to the unit cubes, illustrating volume.
www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html and
www.teacherled.com/resources/isoexplode/isoexplodeload.html

Geometry Volume/Surface Area Quiz from ThatQuiz.org
An online quiz, asking either the volume or surface area of cubes, prisms, spheres, cylinders, or cones. You can modify the quiz parameters to your liking, for example to omit some shapes, solve only for volume or surface area, or instead of solving for volume/surface area, you solve for an unknown dimension (side or radius) when the volume or surface area is given.
www.thatquiz.org/tq-4/?-j3vu0-lc-m2kc0-na-p0

Cubes
An online tool where you can explore filling a rectangular prism (a box) with unit cubes, rows of cubes, or layers of cubes. You can use this to let the student find the rule for finding the volume of a box if you know its width, depth, and height. Requires Java.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

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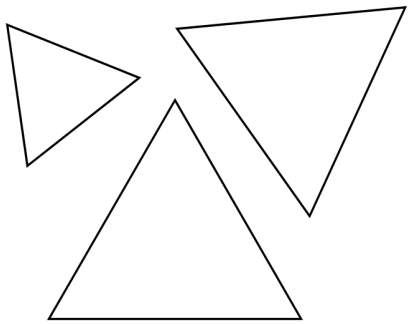
Equilateral, Isosceles, and Scalene Triangles

If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

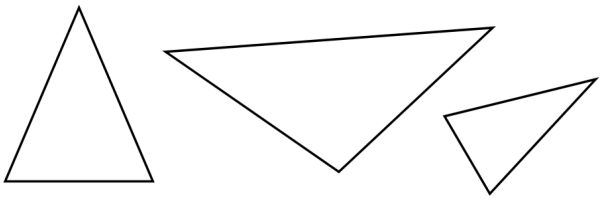
Equi- refers to things that are the "same" or "equal", and *lateral* means "sided." Think of it as a "same-sided" triangle.



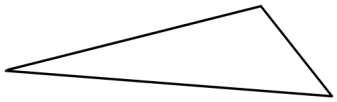
If just *two* of a triangle's sides are congruent, then it is called an **isosceles triangle**.

Think of it as a "same-legged" triangle, the "legs" being the two sides that are the same length.

Mark the two congruent sides of each isosceles triangle:

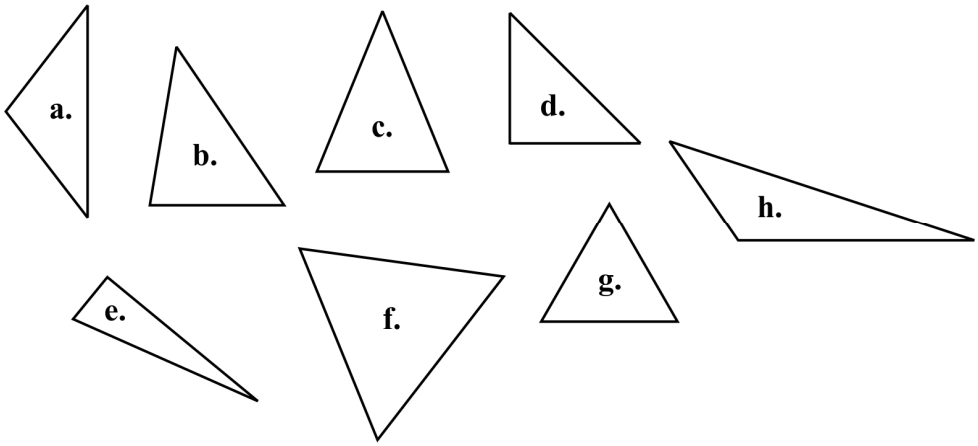


Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.



1. Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

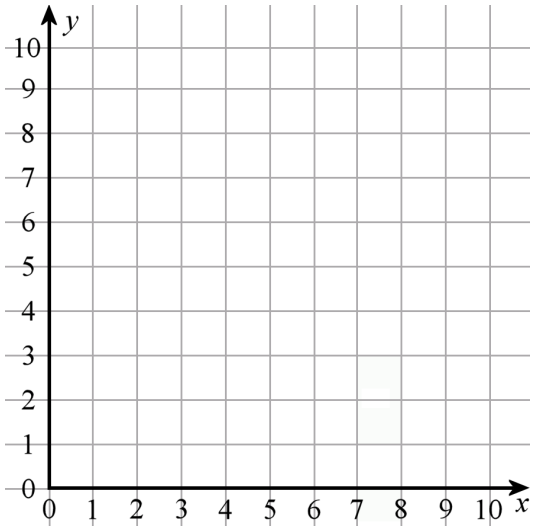
   You can mark each triangle with an "*e*," "*i*," or "*s*" correspondingly.



2. Fill in the table by classifying the triangles labeled as (a), (d), (e), and (g) above as "acute," "right," or "obtuse" (by their angles), and also as "equilateral," "isosceles," or "scalene" (by their sides).

Triangle	Classification by the sides	Classification by the angles
a		
d		
e		
g		

3. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles and sides.



Triangle 1: (0, 0), (4, 0), (0, 4)

_____ and

_____

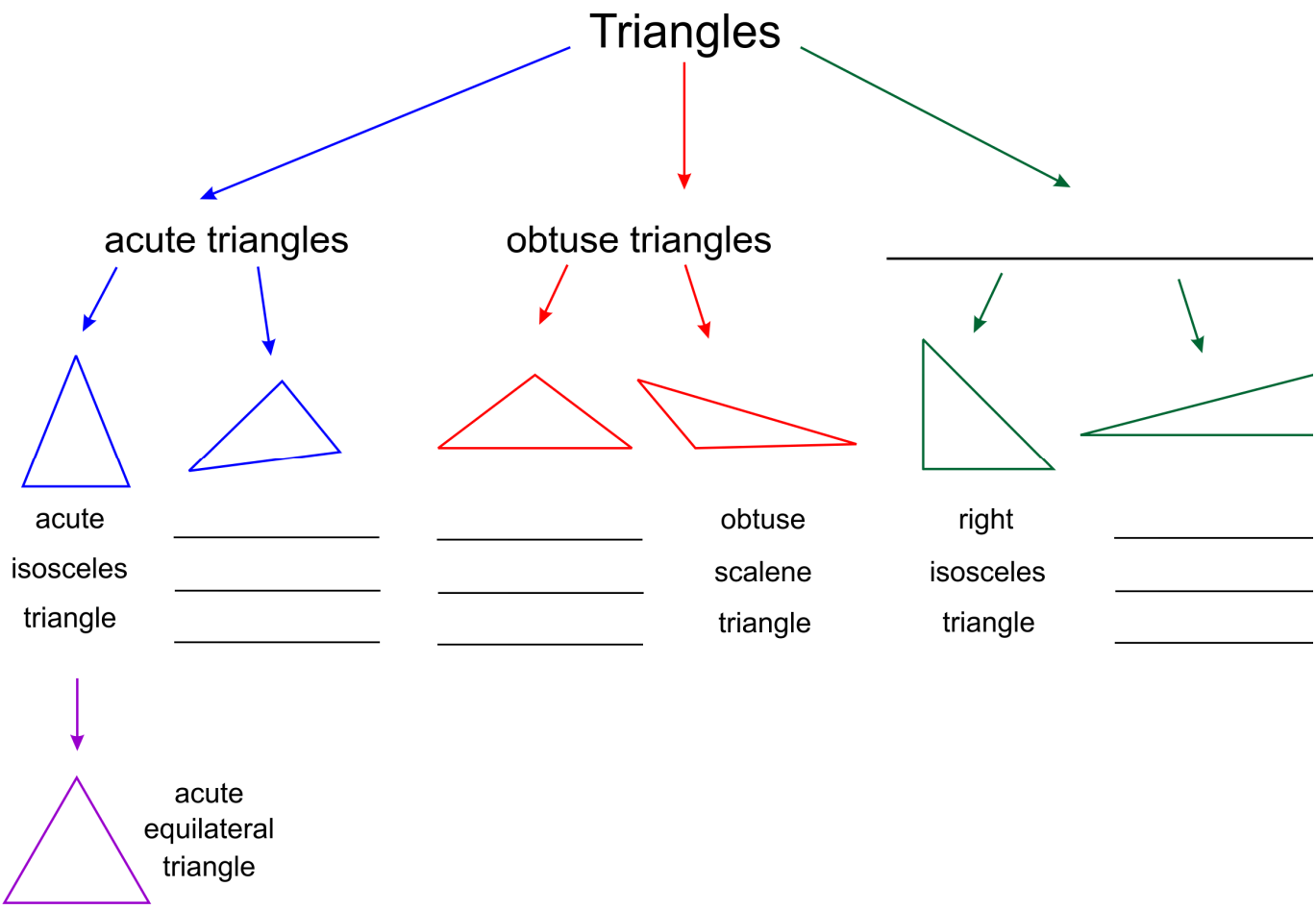
Triangle 2: (5, 5), (1, 8), (9, 4)

_____ and

_____

4. Plot in the coordinate grid an acute scalene triangle.

5. Fill in the missing parts in this tree diagram classification for triangles.



6. **a.** Draw a scalene obtuse triangle where one side is 3 cm and another is 7 cm.
Hint: Draw the 7-cm side first, then the 3-cm side forming any obtuse angle with the first side.

b. Measure the third side.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

7. **a.** Draw an isosceles right triangle whose two sides measure 5 cm.
Hint: Draw a right angle first. Then, measure off the 5-cm sides. Then draw in the last side.

b. Measure the third side. It is _____ cm.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

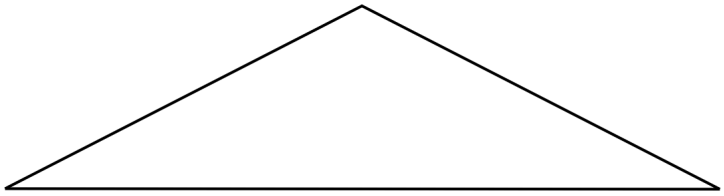
8. **a.** Draw any isosceles triangle.
Hint: Draw any angle. Then, measure off the two congruent sides, making sure they have the same length. Then draw the last side.

b. Measure the angles of your triangle. They measure _____ °, _____ °, and _____ ° .

The angle sum is _____ ° .

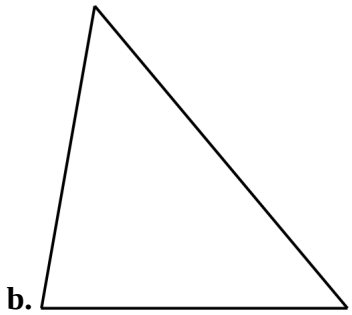
9. Measure all the angles in the isosceles triangles (a) and (b).
Continue their sides, if necessary.

a.



_____ °, _____ °, and _____ ° .

The angle sum is _____ ° .



b.

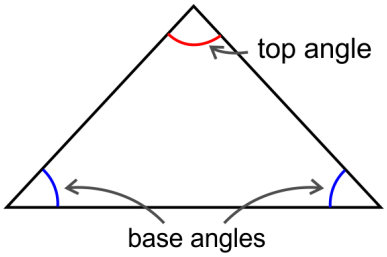
_____ °, _____ °, and _____ °.

The angle sum is _____ ° .

What do you notice?

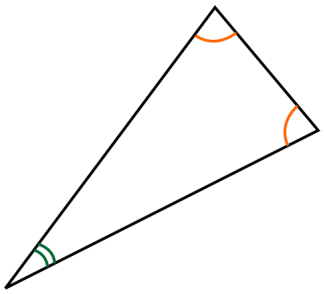
_____

_____



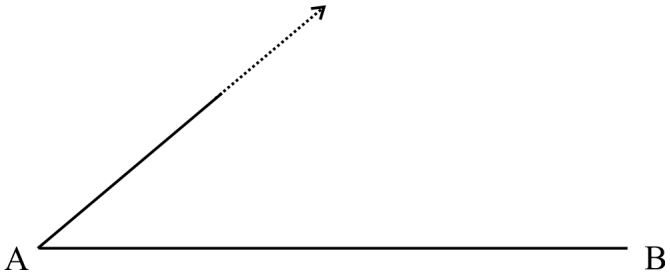
There are two angles in an isosceles triangle that have the same angle measure. They are called the **base angles.**

The remaining angle is called the **top angle**.



Can you find the top angle and the base angles in this isosceles triangle?

10. The angle at A measures 40°. Draw another angle of 40° at B, and then continue its side so that you get an isosceles triangle with 40° base angles.



Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

11. **a.** Draw an isosceles triangle with 75° base angles. (The length of the sides can be anything.)
Hint: start by drawing the base side (of any length). Then, draw the 75° angles.

b. Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

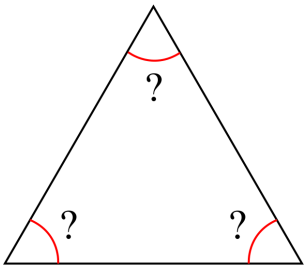
c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

12. **a.** Draw an isosceles triangle with a 50° top angle.
Hint: start by drawing a 50° angle. The two sides of the angle you drew are the two congruent sides of the triangle, so choose how long those sides should be, measure, and mark them. Then draw in the third side.

b. The base angles are _____ ° each. The three angle measures add up to _____ ° .

c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

13. Make a guess about the angle measures in an equilateral triangle: _____°
Measure to check.



14. **a.** Could an equilateral triangle be a right triangle?
If yes, sketch an example. If not, explain why not.

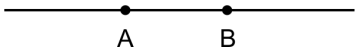
b. Could a scalene triangle be obtuse?
If yes, sketch an example. If not, explain why not.

c. Could an acute triangle be scalene?
If yes, sketch an example. If not, explain why not.

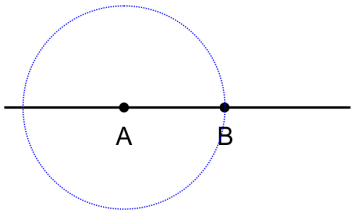
d. Could a right triangle be scalene?
If yes, sketch an example. If not, explain why not.

e. Could an obtuse triangle be equilateral?
If yes, sketch an example. If not, explain why not.

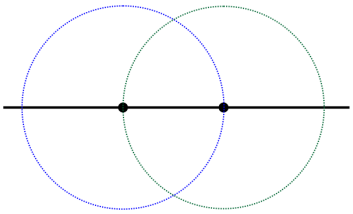
Draw an equilateral triangle with a compass and a ruler (optional)



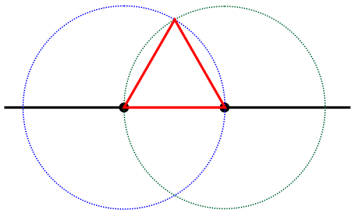
Draw a line segment and mark two points on it. These points mark one of the sides of the triangle.



Draw a circle using point A as the center point and the length AB as the radius. The third vertex of the triangle MUST lie on this circle. Can you see why?



Can you see what was done in this picture?



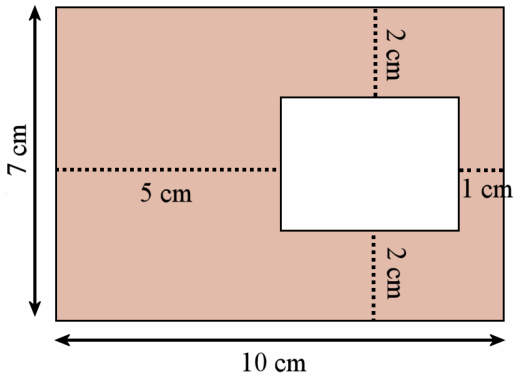
The triangle is finished!

15. Draw at least three different equilateral triangles in your notebook using the method above.
Can you explain why this method works?

Important Terms
• *equilateral triangle* • *isosceles triangle* • *scalene triangle*

Area and Perimeter Problems

Find the area of the shaded figure.



The easiest way to do this is:
(1) Find the area of the larger outer rectangle,
(2) find the area of the white inner rectangle, and
(3) subtract.

1. The area of the large rectangle is 7 cm × 10 cm = 70 cm^2.

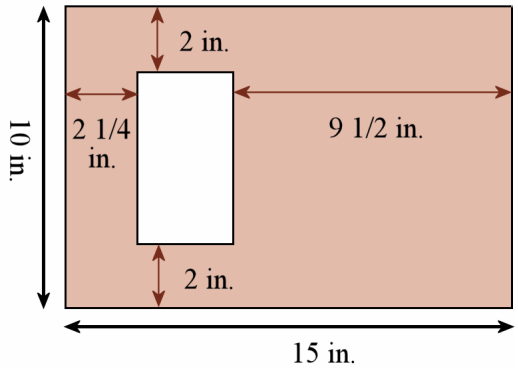
2. We find the *sides* of the white rectangle by subtracting.

   The longer side of the white rectangle is
   10 cm − 5 cm − 1 cm = 4 cm.
   The shorter side is 7 cm − 2 cm − 2 cm = 3 cm.

   So, the area of the white rectangle is 4 cm × 3 cm = 12 cm^2.

3. Now we subtract to find the shaded area: 70 cm^2 − 12 cm^2 = 58 cm^2.

1. **a.** Find the area of the white rectangle.
   All lines meet at right angles.



b. Find the area of the shaded figure.

2. The image on the right shows a picture frame.
   Find the area of the actual frame (that is, of the shaded part).
   All lines meet at right angles.