math MAMMOTH

Grade 5-A Worktext

Light Blue Series

- The four operations
- Large numbers and the calculator
- Problem solving
- Decimals

By Maria Miller

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Chapter 3: Problem Solving

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Foreword

Math Mammoth Grade 5-A and Grade 5-B worktexts comprise a complete math curriculum for the fifth grade mathematics studies, aligned to the Common Core Standards.

The fifth grade is time for fractions and decimals, in particular. In part 5-A, we study decimals in depth and with substantial detail. Students also deepen their understanding of whole numbers, learn much more problem solving, and get introduced to the calculator.

The year starts out with a study of whole numbers and their operations. Students get to review multi-digit multiplication and learn long division with two-digit divisors. We also review divisibility and prime factorization from fourth grade.

In the second chapter, the focus is on large numbers and using a calculator. This is the first time a calculator is introduced in Math Mammoth complete curriculum—thus far, all calculations have been done mentally, or with paper and pencil. I want students to learn to be critical in their use of the calculator—use it with good judgment. Every exercise where calculator use is to be allowed is marked with a little calculator symbol.

The third chapter is about equations and problem solving. We study simple equations with the help of a balance and bar models. The main idea is to get students used to the idea of an equation and what it means to solve an equation. Students also do a fair amount of problem solving using the visual bar model.

The fourth chapter is about decimals and their operations. It is a long chapter because now is the time to learn decimal operations well. It is assumed that the student already has a solid foundation for decimal place value, as taught in Math Mammoth 4th grade curriculum. That is the true means of preventing common misconceptions, or students resorting to rote memorization of the decimal operations.

In part 5-B, students study graphing, fractions, and geometry.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 1: The Four Operations
Introduction

We start fifth grade by studying the four basic operations. This includes studying the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought throughout this chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students write simple expressions for problems they solve. They study the correct order of operations in an expression.

An *equation* in mathematics consists of an expression that equals another expression (expression = expression). We study simple equations, both with the help of visual bar models and also without. Bar models are also used for simple multiplication and division equations.

Next, we review multi-digit multiplication (multiplying in columns), starting with multiplying in parts (partial products) and how that can be visualized geometrically. Then it is time for long division, especially practicing long division with two-digit divisors. We also study why long division works, in the lesson *Long Division and Repeated Subtraction*. All along there are also word problems to solve.

Lastly, we study the topics of divisibility, primes, and factoring. Students learn the common divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10. In prime factorization, we use factor trees.

Although the chapter is named "The Four Operations," please notice that the idea is not to practice each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are trying to develop students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, "undo" operations, and so on. Many of the ideas in this chapter are preparing them for algebra in advance.

The Lessons in Chapter 1

Helpful Resources on the Internet

Long division & multiplication

Rectangle Multiplication
An interactive tool that illustrates multiplying in parts using the area model. Choose the "common" option for this grade level, to show multiplying in parts.
http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html

Snork's Long Division Game
Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems. In the beginning, choose the highest number you want to work with (the divisor) to be a two-digit number, in order to practice with two-digit divisors.
http://www.kidsnumbers.com/long-division.php

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Short Division
A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.
http://www.themathpage.com/ARITH/divide-whole-numbers.htm

All four operations

Math Mahjong
A Mahjong game where you need to match tiles with the same value. It uses all four operations and has three levels.
http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

Pop the Balloons
Pop the balloons in the order of their value. You need to use all four operations.
http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/index.html

Order of operations

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

Connect-the-Four
Solve very simple math problems about the order of operations and get to play connect-the-four game. Requires Java.
http://www.shodor.org/interactivate/activities/OrderOfOperationsFou/

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parentheses in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, —, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.htm

Order of Operations Practice
A simple online quiz of 10 questions. Uses parentheses and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate
Practice your arithmetic of all four operations plus the order of operations.
http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Factors and primes

Factor Game
Interactive game to practice divisibility among numbers 1-100. Play against the computer or a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

Factor Feeder
Eat factors of the given number, and avoid numbers that are not factors of the given number in this Pacman-style game. Use Arrow Keys to move.
http://hoodamath.com/games/factorfeeder.php

Primes, Factors and Divisibility - Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes/

Sliding Tile Factorization Game
Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.
http://www.visualmathlearning.com/Games/sliding_factors.html

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

Octopus Factors
Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.
http://www.counton.org/games/map-numbers/octopus/

Factors Millionaire Game
A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.
http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Not a Factor
Choose a number that is NOT a factor of the given number.
http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

MathGoodies Interactive Factor Tree Game
Type in a missing number to the factor tree, and the program will find the other factor, and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.pompuzzle.com/Snake

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Primes, Factors and Divisibility—Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes

Prime Number Calculator
This calculator tests if a number is a prime, and tells you its smallest divisor if it is not prime.
http://www.basic-mathematics.com/prime-number-calculator.html

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many are there, and more.
http://primes.utm.edu/

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips, and historical information how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.
http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20

Primality of 1 from Wikipedia
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and Against the Primality of 1
http://primefan.tripod.com/Prime1ProCon.html

Unique Prime Factorization
A video explaining the fundamental theorem of arithmetic: that each composite number has a unique prime factorization.
http://www.youtube.com/watch?v=5kl28hmhin0

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Order of Operations and Equations

- ***First*** **solve whatever is inside parentheses.**
- ***Next,*** **solve multiplications and divisions "on the same level," from left to right.**
- ***Last,*** **solve additions and subtractions "on the same level," from left to right.**

1. Solve what is within parenthesis first. You can enclose the operation to be done first in a "bubble."

$(36 + 4) \div (5 + 5)$ $= 40 \div 10$ $= 4$	**a.** $(50 - 2) \div (3 + 5)$	**b.** $20 \times (1 + 7 + 5)$
	c. $2 \times (600 \div 60) + (19 - 8)$	**d.** $180 \div (13 - 7 + 3)$

2. Solve. When there are many multiplications and divisions, do them from left to right.

$24 \div 3 \times 2 \div 4$ $= 8 \times 2 \div 4$ $= 16 \div 4 = 4$	**a.** $36 \div 4 \div 3$	**b.** $1{,}200 \div 4 \times 5 \div 3$
	c. $7 \times 90 \div 2 \times 2 \div 10$	**d.** $5 \times 6 \div 3 \div 2 \times 20$

3. Solve in the right order. You can enclose the operation to be done first in a "bubble" or a "cloud."

a. $12 \times 5 + 8 =$ _____	**b.** $10 + 2 \times 9 + 8 =$ _____
c. $45 + 5 \times 7 =$ _____	**d.** $10 + 2 \times (9 + 8) =$ _____
e. $(8 + 16) \div 3 \div 2 =$ _____	**f.** $2 \times (100 - 80 + 20) =$ _____
g. $120 - 2 \times (11 - 5) =$ _____	**h.** $25 + 8 \times 5 \div 2 =$ _____

4. Division can also be written with a fraction line. Solve in the right order.

a. $6 + \frac{24}{2} =$ _____	**b.** $\frac{32}{2} - 6 =$ _____	**c.** $\frac{54}{6} - 6 - 2 =$ _____

An **equation** has numbers, letters, operation symbols, and one equal sign, "=".
It's called an *equa*tion because it contains an *equal* sign. For example, "5 = 1 + 4" is an equation.

An **expression** only has numbers, letters, and operation symbols—but no equal sign.
For example, "40 × 2 + 6 × 5" is an expression.

5. Equation or expression? (Do not solve these.)

a. $4t = 180$ **b.** $2 + 60 \times 345 \div 9$ **c.** $15 = x + y$

d. $\frac{5.4 - 2.12}{0.4} = 8.2$ **e.** $1{,}000 = 1{,}000$ **f.** $12 - \frac{24 \div 0.8}{189}$

6. Which expression matches each problem? Also, solve the problems.

a. Mark bought three light bulbs for $8 each, and paid with $50. What was his change?

(1) $3 \times \$8 - \$50$ **(2)** $\$50 - \$8 + \$8 + \$8$
(3) $\$50 - 3 \times \$8$ **(4)** $\$50 - (\$8 - \$8 - \$8)$

b. Shirts costing $16 each are discounted by $5, so mom buys six of them. What is her total cost?

(1) $\$16 - \$5 \times 6$ **(2)** $6 \times (\$16 - \$5)$
(3) $\$16 \times 6 - \$5$ **(4)** $(\$16 - 6) \times 5$

c. Andy buys a salad for $8 and a pizza for $13, and shares the cost evenly with his friend. How many dollars is Andy's share of the cost?

(1) $\$8 + \$13 \div 2$ **(2)** $\$2 \div (\$8 + \$13)$
(3) $2 \times \$8 + 2 \times \$13$ **(4)** $(\$8 + \$13) \div 2$

d. Melissa shares equally the cost of a meal with three other people and the cost of a taxi with two other people. The meal costs $48 and the taxi costs $30. How much does Melissa pay?

(1) $\$48 \div 4 + \$30 \div 3$ **(2)** $(\$48 + \$30) \div 3 \div 2$
(3) $\$48 \div 3 + \$30 \div 2$ **(4)** $(\$48 + \$30) \div 5$

$120 - 75 = 3 \times 15$ This is the left side of the equation. This is the right side of the equation. Do the left and right sides have the same value? Just calculate $120 - 75$, then calculate 3×15, and check. If yes, it's a **true equation**. If not, it's a **false equation**.	$2 = 5$ left side right side This is a very simple equation —but it is false! $4 + 5 = 21 - 3$ left side right side This is also a false equation!	$18 = x - 3$ left side right side Solving the equation means finding the value of x (the **unknown**) that makes it true. The value $x = 21$ makes this equation true, so we say $x = 21$ is the solution.

7. If the equation is false, change one number in it to make it true.

a. $6 + \frac{32}{8} = 5$	**b.** $(6 - 2) \times 3 = 5 + 5$	**c.** $5 \times 2 = 16 \div 2 + 2$

8. Place parenthesis into these equations to make them true.

a. $10 + 40 + 40 \times 2 = 180$	**b.** $144 = 3 \times 2 + 4 \times 8$	**c.** $40 \times 3 = 80 - 50 \times 4$

9. Find a number to fit in the box so the equation is true.

a. $40 = (\square + 9) \times 2$	**b.** $4 \times 8 = 5 \times 6 + \square$	**c.** $4 + 5 = (20 - \square) \div 2$
d. $81 = 9 \times (2 + \square)$	**e.** $\square \times 11 = 12 + 20 \times 6$	**f.** $(4 + 5) \times 3 = \square \div 2$

10. Solve these simple equations.

a. $s \times 2 = 660$ $s =$ _____	**b.** $\frac{x}{2} = 5$ $x =$ _____	**c.** $200 - y = 60$ $y =$ _____

11. Build at least three true equations using (only) the symbols and numbers given. You may use the same number or symbol many times.

11, 3, 1, −, +, ×, ( ), =

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A Two-Digit Divisor 1

Often, it is helpful to write the multiplication table of the divisor before you divide.

Example 1. The division is by 16. Here is the multiplication table of 16:

3 × 16 = 48
4 × 16 = 64
5 × 16 = 80
6 × 16 = 96
7 × 16 = 112
8 × 16 = 128
9 × 16 = 144

```
  0 3
16)5 5 6 8
```

16 goes into 5 zero times, so we look at 55.

How many times does 16 go into 55?

Check in the table on the left. We see it goes into 55 three times.

```
  0 3 4
16)5 5 6 8
  -4 8
     7 6
```

Now, how many times does 16 go into 76?

From the table we can see that it is four times.

```
  0 3 4 8
16)5 5 6 8
  -4 8
     7 6
   - 6 4
     1 2 8
   - 1 2 8
         0
```

Lastly, 16 goes into 128 exactly 8 times, and the division is over.

Example 2. We are dividing by 32. Here is the multiplication table of 32:

3 × 32 = 96
4 × 32 = 128
5 × 32 = 160
6 × 32 = 192
7 × 32 = 224
8 × 32 = 256
9 × 32 = 288

```
  0 1
32)4 7 0 7
  -3 2
   1 5
```

32 goes into 47 once.

```
  0 1 4
32)4 7 0 7
  -3 2
   1 5 0
  -1 2 8
     2 2
```

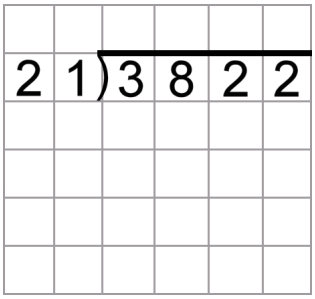
32 goes into 150 four times.

```
  0 1 4 7
32)4 7 0 7
  -3 2
   1 5 0
  -1 2 8
     2 2 7
   - 2 2 4
         3
```

32 goes into 224 seven times. Notice there is a remainder.

1. Divide. First write a multiplication table for the divisor. Check each answer by multiplying.

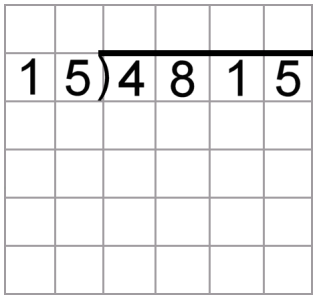
Table of 21:
2 × 21 =
3 × 21 =
4 × 21 =
5 × 21 =
6 × 21 =
7 × 21 =
8 × 21 =
9 × 21 =



2. Divide. First write a multiplication table for the divisor. Check each answer by multiplying.

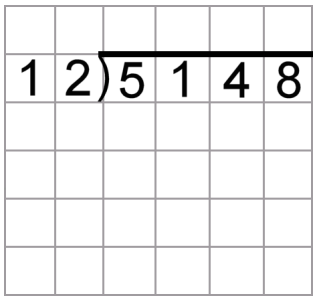
a.

Table of 15:
2 × 15 =
3 × 15 =
4 × 15 =
5 × 15 =
6 × 15 =
7 × 15 =
8 × 15 =
9 × 15 =



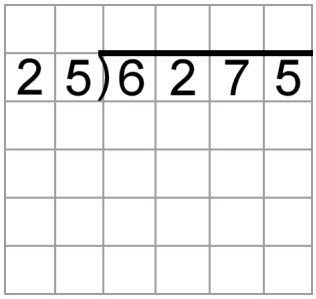
b.

Table of 12:
2 × 12 =
3 × 12 =
4 × 12 =
5 × 12 =
6 × 12 =
7 × 12 =
8 × 12 =
9 × 12 =



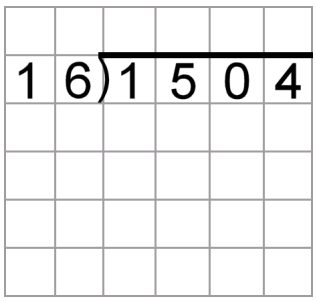
c.

Table of 25:
2 × 25 =
3 × 25 =
4 × 25 =
5 × 25 =
6 × 25 =
7 × 25 =
8 × 25 =
9 × 25 =



d.

Table of 16:
2 × 16 =
3 × 16 =
4 × 16 =
5 × 16 =
6 × 16 =
7 × 16 =
8 × 16 =
9 × 16 =



3. Divide. Check each answer by multiplying.

a.

Table of 12:
2 × 12 =
3 × 12 =
4 × 12 =
5 × 12 =
6 × 12 =
7 × 12 =
8 × 12 =
9 × 12 =

$12\overline{)888}$

b.

Table of 22:
2 × 22 =
3 × 22 =
4 × 22 =
5 × 22 =
6 × 22 =
7 × 22 =
8 × 22 =
9 × 22 =

$22\overline{)6710}$

c.

Table of 14:
2 × 14 =
3 × 14 =
4 × 14 =
5 × 14 =
6 × 14 =
7 × 14 =
8 × 14 =
9 × 14 =

$14\overline{)1736}$

d.

Table of 51:
2 × 51 =
3 × 51 =
4 × 51 =
5 × 51 =
6 × 51 =
7 × 51 =
8 × 51 =
9 × 51 =

$51\overline{)7548}$

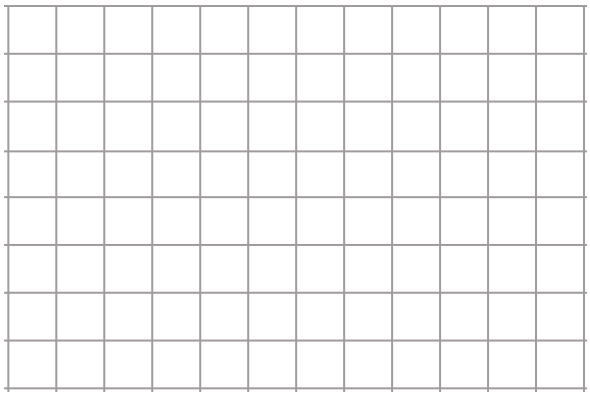
4. Mental math! If 20 goes into 800 forty times, then 20 goes into 820 one time more, or 41 times. In each box, use the top problem to help you solve the bottom problem.

a. 800 ÷ 20 = 820 ÷ 20 =	**b.** 700 ÷ 50 = 750 ÷ 50 =	**c.** 150 ÷ 15 = 300 ÷ 15 =
d. 480 ÷ 40 = 520 ÷ 40 =	**e.** 600 ÷ 30 = 690 ÷ 30 =	**f.** 1,200 ÷ 60 = 1,320 ÷ 60 =

5. **a.** How many inches are in one foot?

   b. Convert 245 inches into feet and inches.

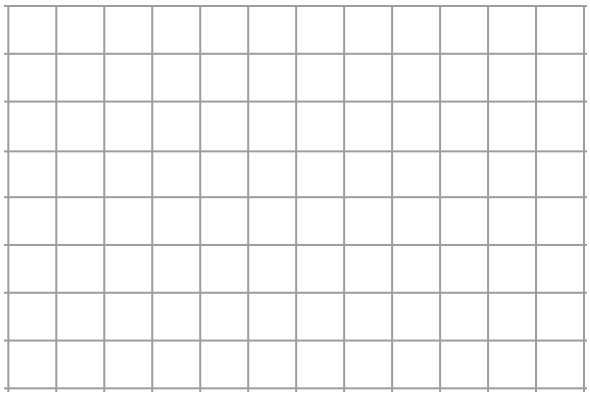
   c. Convert 387 inches into feet and inches.



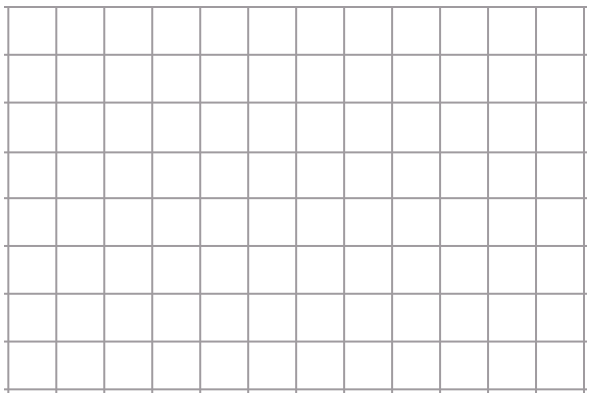
6. **a.** How many ounces are in one pound?

   b. Convert 163 ounces into pounds and ounces.

   c. Convert 473 ounces into pounds and ounces.



7. A newborn baby gains weight at approximately one ounce per day. Suppose that the baby gained weight at that rate for a FULL YEAR. (In reality, babies don't; their growth rate slows down.) How many pounds and ounces would the baby gain in a year?



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Chapter 2: Large Numbers and the Calculator

Introduction

In this chapter, we study large numbers and place value up to billions—that is, up to 12-digit numbers. We study adding, subtracting, rounding, exponents, and using a calculator.

This is the first time the calculator is introduced in Math Mammoth complete curriculum. I have delayed introducing the use of a calculator (as compared to many math curricula) for good reasons. I have received numerous comments on the harm that indiscriminate calculator usage can cause. In a nutshell, if children are allowed to use calculators freely, their minds get "lazy," and they will start relying on calculators even for simple things such as 6 × 7 or 320 + 50. It is just human nature!

As a result, students enter college without even knowing their multiplication tables by heart. Then they have tremendous trouble if they are required to use mental math to solve simple problems.

Therefore, we educators need to *limit* calculator usage until the students are much older. Children can *not* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is extremely useful, and students do need to learn to use it. In this curriculum, I strive to show the students not only *how* to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practice estimating the result before calculating it with a calculator. In the last lesson, students need to choose whether mental math or a calculator is the best "tool" for the calculation.

The Lessons in Chapter 2

Helpful Resources on the Internet

Naming Numbers
These pages teach number naming skills covered in K8 math courses. Each page has an explanation, interactive practice and challenge games about naming numbers.
http://www.aaamath.com/B/nam.htm

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Powers of Ten
A 9-minute movie that illustrates the dramatic changes of scale when zooming in or out by powers of ten (40 powers of ten), starting from a picnic blanket and ending in the universe, and then starting from a hand to the proton inside an atom.
http://www.youtube.com/watch?v=0fKBhvDjuy0

Cookie Dough
Practices naming big numbers.
www.funbrain.com/numwords/index.html

Keep My Place
Fill in the big numbers to this cross-number puzzle.
http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html

Estimation
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences.
http://www.aaamath.com/B/est.htm

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Place Value Game
Create the largest possible number from the digits the computer gives you.
Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be.
http://education.jlab.org/placevalue/index.html

Free Exponent Worksheets
Create a variety of customizable, printable worksheets to practice exponents.
http://www.homeschoolmath.net/worksheets/exponents.php

Baseball Exponents
Choose the right answer from three possibilities before the pitched ball comes.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=95

Exponents Quiz from ThatQuiz.org
Ten questions, fairly easy, and not timed. You can change the parameters as you like to include negative bases, square roots, and even logarithms.
http://www.thatquiz.org/tq-2/?-j1-l4-p0

Exponents Jeopardy
The question categories include evaluating exponents, equations with exponents, and exponents with fractional bases.
http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html

Pyramid Math
Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.
http://www.mathnook.com/math/pyramidmath.html

Exponents Battleship
A regular battleship game against the computer. Each time you "hit", you need to answer a math problem involving exponents (and multiplication).
http://www.quia.com/ba/1000.html

Exponent Battle
A card game to practice exponents. I would limit the cards to small numbers, instead of using the whole deck.
http://www.learn-with-math-games.com/exponent-game.html

Pirates Board Game
Steer your boat in pirate waters in this online board game, and evaluate powers.
http://mathgames4children.com/fun-board-games/6th-grade/pirate/exponents-pirate-waters-grade-6-game.html

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Adding and Subtracting Large Numbers

Just like 25 marbles + 54 marbles = 79 marbles, so will 25 million + 54 million = 79 million.

Just keep in mind: **a thousand thousands** makes a **million**, and **a thousand millions** makes a **billion**.

800,000 + 200,000	**Half a million**
Think of it as 800 thousand + 200 thousand. The answer is 1,000 thousand or 1,000,000.	Think of it as half of a thousand thousands, or 500 thousands = 500,000.
34,999,000 + 1,000	**2 billion − 300 million**
This is 34 million 999 thousand + 1 thousand, making 34 million 1000 thousand, or 35 million.	Think of it as 2,000 million − 300 million, which makes 1,700 million, or 1,700,000,000.

1. Add.

	a. 90,000	**b.** 99,000,000	**c.** 999,000
+ 1,000			
+ 10,000			
+ 100,000			
+ 1,000,000			

2. Match.

a.		**b.**	
1/2 million	750,000	1 million − 50,000	100,000,000
a hundred hundreds	100,000	1 million − 500,000	500,000
1/10 million	10^6	10^8	950,000,000
1/4 million	500,000	1 billion − 500 million	1/2 billion
3/4 million	10^4	1 billion − 50 million	950,000
a thousand thousands	200,000	1 million − 5,000	995,000
2/10 million	250,000	1 billion − 5 million	995,000,000

3. Add and subtract. Simply write the numbers under each other, lining up the place values. Use the usual addition or subtraction algorithm, regrouping the same way as you have learned before.

a. 329,145,000 + 2,809,125,093

b. 5,049 + 45,390,000 + 5,483,700

c. 45,700 + 90,567,000 + 2,560 + 2,300,560

d. 290,800 + 254,000,230 + 56,391 + 2,381

e. 480,560,000 − 23,980,000

f. 1,000,000 − 156,990

g. 22,300,000 − 4,431,190

h. 7,014,289,000 − 3,103,559,391

4. Subtract and compare.

a. 1 million − 100 thousand =

1 million − 10 thousand =

1 million − 1 thousand =

b. 7 million − 500 thousand =

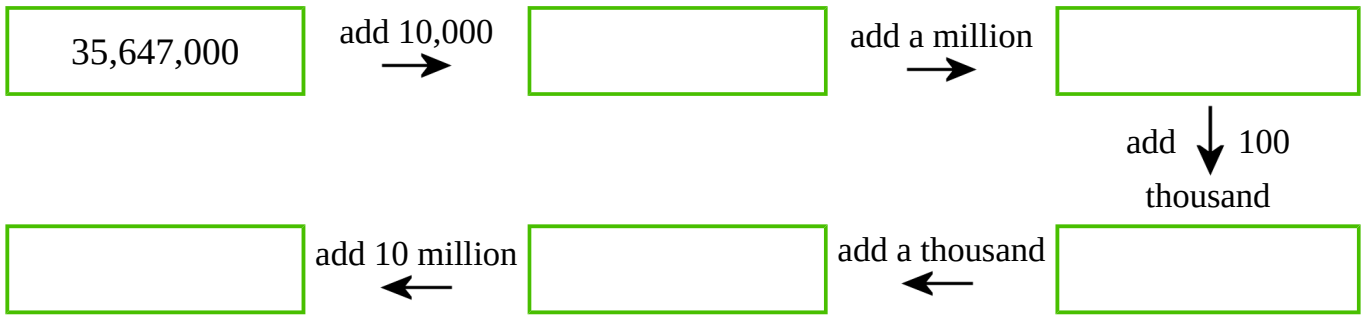
7 million − 50 thousand =

7 million − 5 thousand =

5. Continue counting for seven more numbers in each set:

a.	**b.**	**c.**
458,000,000 468,000,000 478,000,000	79,650,000 79,800,000 79,950,000	450,996,000 450,997,000 450,998,000
Each difference is _____	Each difference is _____	Each difference is _____

6. Complete the addition path.



7. Solve for x.

a. $x + 400{,}000 = 4{,}000{,}000$ $x =$ _____	**b.** $x - 350{,}000 = 2{,}000{,}000$ $x =$ _____
c. $200{,}000 + x + 600{,}000 = 7{,}000{,}000$ $x =$ _____	**d.** $2x = 3{,}000{,}000$ $x =$ _____

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Chapter 3: Problem Solving
Introduction

First in this chapter, students solve some equations, presented as pan balance puzzles. Then we study mixture equations, such as $4x + 38 = 128$, once again using the bar model as a visual model.

The bulk of this chapter is spent on problem solving. We use the bar model a lot. The problems include a fractional part of a whole, a fractional part more, the total is known, one part is more than the other, and so on.

Encourage the student to draw the bar model for the problems, as it is such a helpful tool. Some of the problems here could even be found in regular Algebra 1 textbooks where they would be solved with algebra. However, the bar model enables us to solve them without algebra; yet, it helps students' algebraic thinking! Essentially, one block in the bar model corresponds to the unknown x in an equation.

The Lessons in Chapter 3

Helpful Resources on the Internet

Pan Balance - Numbers
Enter a numerical expression in one pan and then in the other. The pans will move up and down depending on which expression is greater. When the expressions are equivalent, the pans will balance and the full equation will be entered into the *Balanced Equations* table. This tool strengthens understanding and computation of numerical expressions and equality. In understanding equality, one of the first things students must realize is that equality is a relationship, not an operation. Many students view "=" as "find the answer." For these students, it is difficult to understand equations such as $11 = 4 + 7$ or $3 \times 5 = 17 - 2$.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=26

Pan Balance - Shapes
An online balance that builds your algebraic thinking. Find the unknown weight of each shape by placing shapes on the two pans, and trying to find situations where the weights are equal. One square always weighs 1 unit.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=33

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Thinking Blocks
An interactive math tool developed to help students learn how to solve multi-step word problems. Using brightly colored blocks, students model the relationships among the components of each word problem. The website has addition/subtraction problems, multiplication/division problems, and ratio problems. This block model corresponds to the bar model used in this book.
http://www.thinkingblocks.com/

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/algebra_puzzle.html

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/index.html

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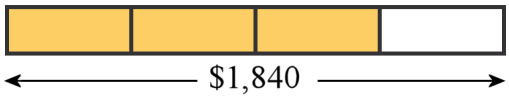
Problem Solving with Bar Models, Part 1

A fractional part of the whole

Jackie earns $1,840 monthly and Jessie earns 3/4 as much.
How much does Jessie earn?



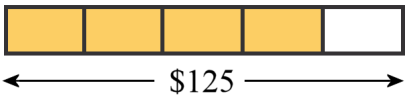
In the model, Jackie's salary is divided into four equal parts (blocks). To find 3/4 of it, *first find 1/4 of it*, which is **one block** in the model.

$$\$1{,}840 \div 4 = \$460$$

Then multiply that result by three: $3 \times \$460 = \$1{,}380$.
So, Jessie earns $1,380.

Solve. Draw a bar model. Write an expression (number sentence) for *each* calculation you do.

1. A $125 camera was discounted by 1/5 of its price.
   What is its new price?



_____ ÷ _____ = _____

_____

2. A pizza that weighs 680 g is divided into five equal pieces.
   How much do two pieces weigh?

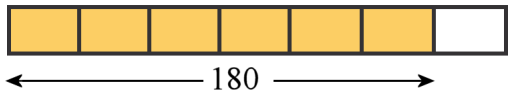
_____ ÷ _____ = _____

_____ × _____ = _____

3. A bottle of water costs 2/3 as much as a $1.50 juice.
   How much do *two* bottles of water and *two* juices cost?

A Fractional Part More

The school year in country A is 180 days long. In country B it is 1/6 part longer than that. How long is the school year in country B?



First, we divide the 180-day school year into 6 parts, to find how much one "block" is in the model:

180 ÷ 6 = 30. So, one block is 30 days.

Then we *add* one-sixth more to the whole bar model, and that is how long the school year is in country B.

180 + 30 = 210

So, the school year in country B is 210 days long.

Solve. Draw a bar model. Write an expression (number sentence) for each calculation you do.

4. The price of a $12 train ride went up by 1/6.
   What is the new price?

   _____ ÷ _____ = _____

   _____ + _____ = _____

5. A cafeteria lunch used to cost $4.50 but the price was increased by 1/5. What is the price now?

6. A one-way bus ride from Helen's home to town costs $1.
   The bus company will raise the price by 1/10 in June.

   a. How much will a one-way ride cost in June?

   b. How much more will a two-way ride (home-town-home) cost Helen in June than in May?

7. A T-shirt cost $10.50, but now it is discounted by 2/5 of its price.
Annie buys *ten* shirts with the discounted price. What is her total bill?

8. Duckville has 3,687 inhabitants, which is 3/5
of the number of inhabitants in Eagleby.
How many people *in total* live in Eagleby *and* Duckville?

9. A package of 10 small envelopes costs $2.50,
and a package of 10 large ones costs 2/5 more.
Find the total cost of buying 50 envelopes of each kind.

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Chapter 4: Decimals
Introduction

In this chapter, we study place value with decimals and learn to perform the four basic operations with decimal numbers.

The chapter starts with a short review of tenths and hundredths, after which, we study numbers with three decimal digits (thousandths). Students also compare and round numbers with up to three decimal digits.

The rest of the chapter is spent studying the four basic operations with decimals. We start with addition and subtraction, which we are familiar with from fourth grade, and then spend a considerable amount of time with multiplication and division of decimals.

I have tried to emphasize mental calculations based on the conceptual understanding of decimals. For that reason, the text often includes little "tricks" that can help with mental calculations. Along with that, the chapter has lessons on long multiplication and long division with decimals.

Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

We also study using decimal numbers in measuring units, the metric system, and conversions between the customary units of measurement. I have tried to emphasize sensible and intuitive methods for converting measuring units within the metric system, instead of relying on mechanical formulas.

You might wonder why *Math Mammoth Grade 5* presents decimals before fractions. The traditional way is to teach fractions first because fractions are more general, and then, to show that decimals are simply a specific type of fractions with denominators that are powers of ten.

There are several reasons I present decimals before fractions. First, students have studied some about both decimals and fractions in earlier grades, so they should have the necessary background to comprehend that decimals are fractions. Therefore, I see no need to study all fraction arithmetic in 5th grade before decimal arithmetic.

Secondly, I feel that decimal arithmetic is somewhat easier than fraction arithmetic and students already know more about it than they know about all the fraction arithmetic that is studied in 5th grade (in 5-B). Thus, studying decimal arithmetic first may be easier for some students.

The Lessons in Chapter 4

Helpful Resources on the Internet

Decimal Arithmetic
These are my videos that go through all of the important decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Great for grades 5, 6, and 7.
http://www.youtube.com/user/MathMammoth#grid/user/CCFD68119A0DA3E8

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers in ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

Smaller and Smaller Maze
Practice ordering decimal numbers to find your way through the maze.
http://www.counton.org/magnet/kaleidoscope/smaller/index.html

Decimal and Whole Number Jeopardy
Review place value and comparing and rounding numbers. Also, practice number patterns.
www.quia.com/cb/8142.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
http://themathgames.com/arithmetic-games/place-value/decimal-place-value-math-game.php

Sock
Push the green blocks into the holes to make the target number.
www.interactivestuff.org/sums4fun/sock.html

Decimal Squares Blackjack
Play cards with decimals, trying to get as close to 2 as possible without going over.
www.decimalsquares.com/dsGames/games/blackjack.html

A Decimal Puzzle
Make every circle add up to 3.
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.htmlsopen=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football
Simple games for addition, subtraction, multiplication, and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.
http://www.funbrain.com

Exploring Division of Decimals
Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.
www.hbschool.com/activity/elab2004/gr6/1.html

Decimal Speedway
Practice decimal multiplication in this fun car-racing game.
www.decimalsquares.com/dsGames/games/speedway.html

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More Long Division with Decimals

Fractions and division

Remember? The fraction line is *also* a division symbol. So $\frac{1}{8}$ can mean both one-eighth (a fraction) and a division problem $1 \div 8$. This gives us a means of writing fractions as decimals!

Example. Write $\frac{8}{9}$ as a decimal, to three decimal digits.

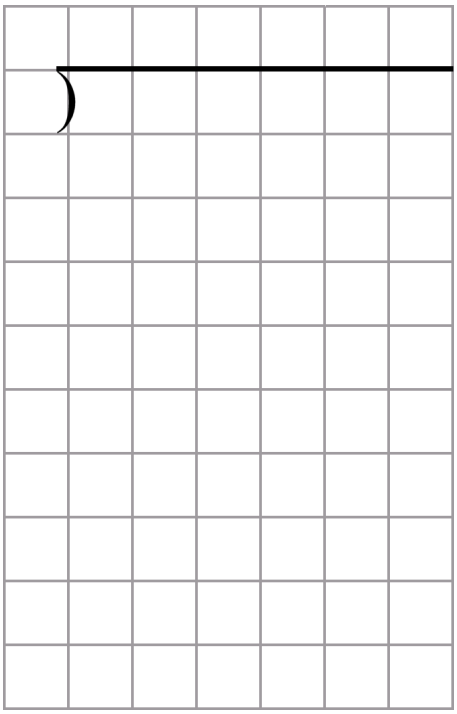
We simply divide 8 by 9, but writing 8 as 8.0000—with lots of decimal zeros. Look at the division on the right. We need to find *four* decimal digits for the quotient before we can round it to *three* decimal digits:

$\frac{8}{9} = 8 \div 9 \approx 0.889.$

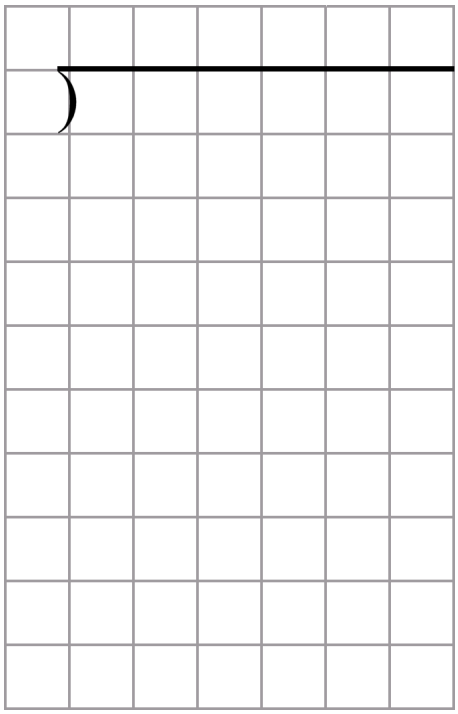
$$
\begin{array}{r}
0.8\,8\,8\,8 \\
9\overline{)8.0\,0\,0\,0} \\
\underline{7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8
\end{array}
$$

1. Write the fractions as decimals, to three decimal digits.

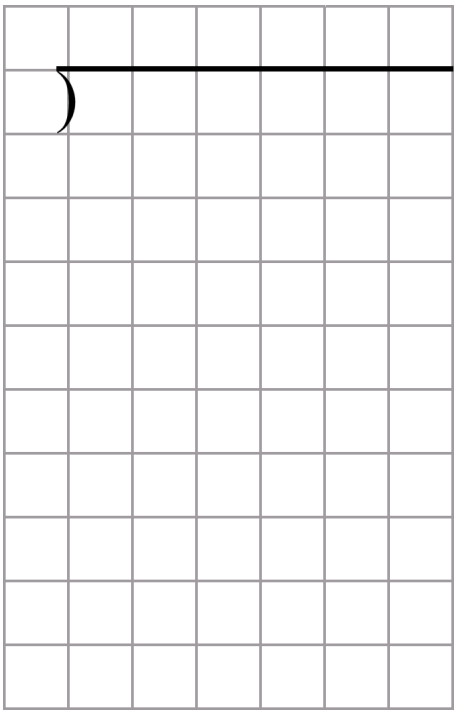
a. $\frac{5}{8} =$



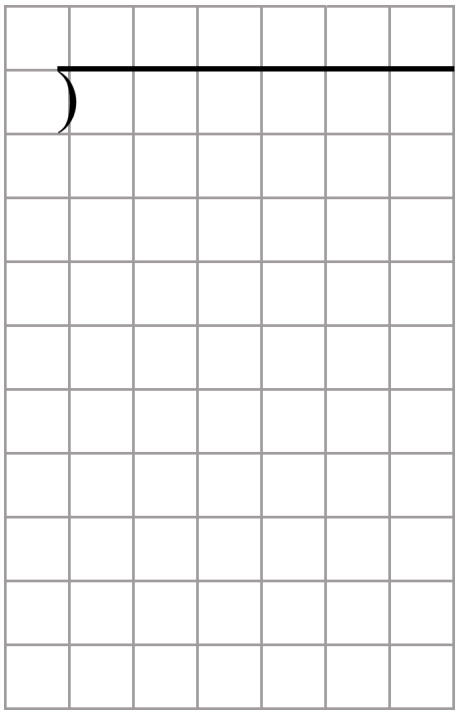
b. $\frac{6}{7} =$



c. $\frac{1}{6}$ =

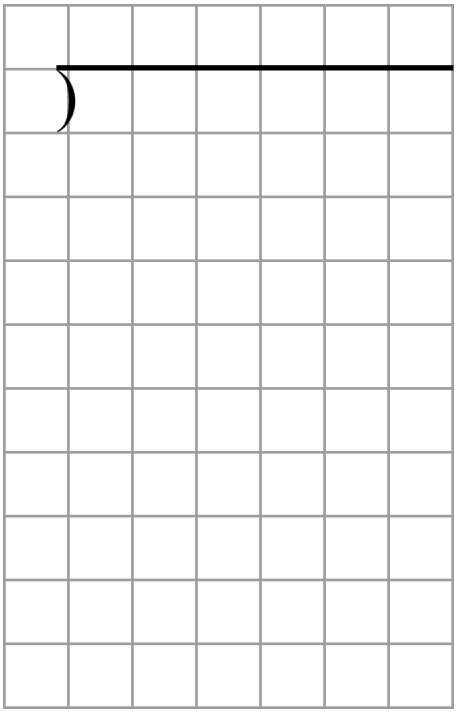


d. $\frac{7}{20}$ =

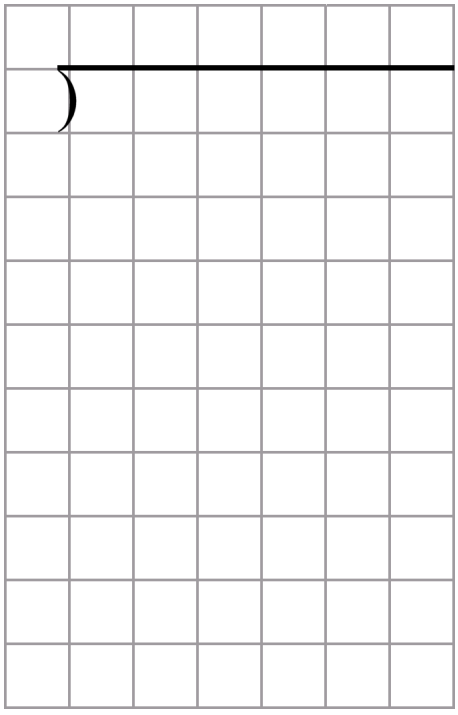


2. Calculate. You will need to add decimal zeros to the dividend.

a. 250 ÷ 6 to two decimal digits



b. 37.5 ÷ 11 to three decimal digits



3. **a.** Fill in the explanation as to how to solve the problem.

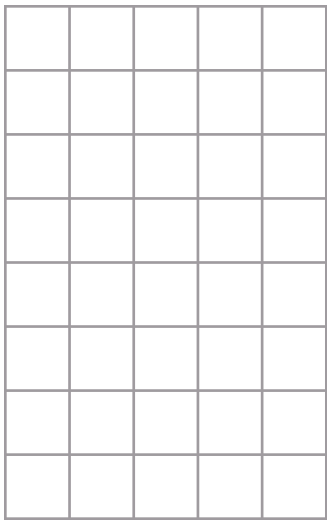
Three packs of transistors and seven packs of capacitors cost a total of $8.70. One capacitor pack costs $0.60. Find the cost of one transistor pack.

First _____ the cost of seven capacitor packs

from _____. Then divide that result by _____.

b. Write a *single* expression to match the explanation above.

c. Solve the problem.



4. Three friends equally shared the cost of a taxi fare, $35.40, and the cost of a meal, $128.95. How much did each person pay?

5. Write a word problem that matches each calculation below. *You do not have to calculate anything.*

a. ($50 − $26) ÷ 3 = $8

b. 25 × $1.40 ÷ 2 = $17.50

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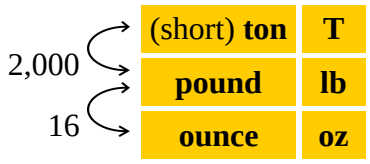
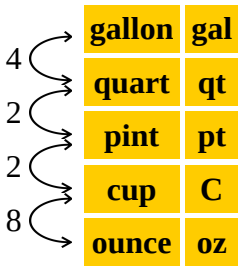
Converting Between Customary Units of Measurement

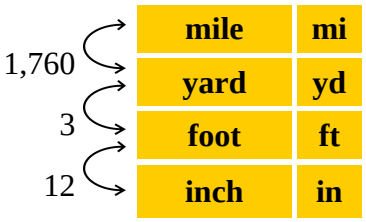
Units of weight



Units of volume



Units of length



To convert from one neighboring unit to another, either **multiply** or **divide** by the conversion factor.

If you do not know which, THINK if the result needs to be a smaller or bigger number.

Example. Convert 53 ounces into cups.

Ounces are smaller units than cups, so 53 ounces as cups will make *fewer* cups (you need fewer cups since they are the bigger units). So, we need to divide by the factor 8 (*since 8 ounces makes a cup*).

53 ÷ 8 = 6 R5. The results means 54 ounces is 6 cups and 5 (leftover) ounces.

You can also think of it this way: since 8 ounces makes a cup, we need to figure how many cups or how many "8 ounces" there are in 53 ounces... or how many 8s are in 53? The answer to that is solved by division.

1. Convert.

a. 6 ft = _____ in. 7 ft 5 in. = _____ in.	**b.** 25 in = _____ ft _____ in 45 in = _____ ft _____ in	**c.** 13 ft 7 in = _____ in 71 in. = _____ ft _____ in

2. Convert.

a. 2 lb 8 oz = _____ oz 45 oz = _____ lb _____ oz	**b.** 8 lb = _____ oz 56 oz = _____ lb _____ oz	**c.** 43 oz = _____ lb _____ oz 90 oz = _____ lb _____ oz

3. Convert.

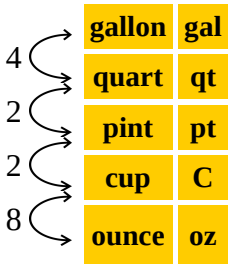
a. 3 C = _____ oz 55 oz = _____ C_____ oz	**b.** 4 C = _____ pt 3 pt = _____ C	**c.** 7 gal = _____ qt 45 qt = _____ gal _____ qt

Example. Convert 3 quarts into ounces.

We are going from bigger units (quarts) to smaller units (ounces), so there will be LOTS more of them. We need to multiply.

This time, quarts and ounces are not neighboring units in the chart. We need to **multiply 3 quarts by all of the factors** between them:

by 2, by 2, and by 8. So, we multiply 3 qt × 2 × 2 × 8 = 96 oz.



Example. Convert 742 inches into yards.

Between inches and yards we have the conversion factors 12 and 3. Also, yards are bigger units than inches... so we need *fewer* of them. That means we divide 742 by both 12 and 3—or we can divide it by 36.

742 ÷ 36 = 20 R 22. This means 742 in. = 20 yd 22 in. The remainder 22 is <u>in the same unit</u> as the dividend: in inches.

But, 22 inches is *also* 1 ft 10 in! So, in the end 742 in. = 20 yd 1 ft 10 in.

```
    0 2 0
36 )7 4 2
   7 2
     2 2
   -   0
     2 2
```

4. Convert. Use long division or multiplication.

a. 11 yd = _____ in.

b. 711 in. = _____ ft _____ in

c. 982 in. = _____ yd _____ ft _____ in

d. 254 oz. = _____ C _____ oz

Now, convert the cup-amount of your answer above into quarts and cups.

254 oz. = _____ qt _____ C _____ oz

Lastly, convert the quart-amount into gallons and quarts.

254 oz. = ____ gal _____ qt _____ C _____ oz

Example. Convert 4.52 lb into ounces.

We are going from bigger units (pounds) to smaller units (ounces), so there will be LOTS more of them. We need to multiply.

Using a calculator, multiply 4.52 × 16 = 72.32 oz.

2,000 →	(short) **ton**	**T**
	pound	**lb**
16 →	**ounce**	**oz**

Example. How many miles is 8,400 feet?

Since one mile is 5,280 feet, then 8,400 feet would be somewhere between 1 and 2 miles.
To find out exactly, use division, and round the answer: **8,400 ÷ 5,280 = 1.59090909... ≈ 1.59 miles.**

5. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 7.4 mi = _____ ft 16,000 ft = _____ mi	**b.** 1,500 ft = _____ yd 7,500 yd = _____ mi	1,760 → **mile** mi 3 → **yard** yd 12 → **foot** ft **inch** in 1 mile = 5,280 feet
c. 900 ft = _____ mi 2.56 mi = _____ yd	**d.** 12.54 mi = _____ ft 82,000 ft = _____ mi	

6. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 15.2 lb = _____ oz 655 oz = _____ lb	**b.** 4.78 T = _____ lb 7,550 lb = _____ T	**c.** 78 oz = _____ lb 0.702 T = _____ lb

7. Solve the riddle. Use the calculator for the problems that you feel cannot be solved mentally.

F 0.6 mi = _____ ft

A 5,632 yd = _____ mi

O 10 qt = _____ C

H 2 lb 11 oz = _____ oz

L 1.3 mi = _____ yd

G 7 C = _____ oz

R 6,200 lb = _____ T

S 3 lb 5 oz = _____ oz

E 5 ft 2 in = _____ in

O 40 oz = _____ lb

I 14,256 ft = _____ mi

W 6 ft 7 in = _____ in

L 732 in = _____ ft

D 42 in = _____ ft

P 3 gal = _____ pt

A 0.75 mi = _____ ft

What did one potato chip say to the other?

53	43	3960	61	2288		57	62		56	40

3168	2.5	3.1		3.2		3.5	2.7	24	
									?

8. Solve.

a. If you serve 1-cup servings of juice to 30 people, how many *whole* gallons of juice will you need?

b. Mom was making applesauce in 2-gallon batches and bottling it in 1-quart jars. After 9 batches, how many jars of applesauce had she made?

c. How many 8-inch pieces can you cut out of 9 3/4 ft of ribbon?

d. A 4-ounce serving of coffee costs $1.20. What would a 5-ounce serving cost?

e. A bottle of shampoo weighs 13 oz, and there are 20 of them in a box. The box itself weighs 8 oz. How much does the box with the bottles of shampoo weigh in total (in pounds and ounces)?

f. Mark drinks three 5-ounce servings of coffee a day. Find how much coffee he drinks in a month (30 days). Give your answer in units other than ounces.

g. Erica lost 5 lb of weight over 4 weeks of time. How much weight did she lose daily, on average?

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