Completing the Square

Earlier we solved some quadratic equations by finding the square root of each side. We can solve any quadratic equation which has real number solutions the same way. We just have to be able to find an equivalent equation which has the square of a binomial on one side and a number on the other.

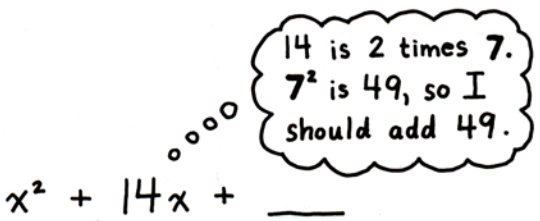
Look at what happens when we square the binomial $x + 5$.

	x	+	5
x	x^2		$5x$
+			
5	$5x$		25

$$(x + 5)^2 = x^2 + 5x + 5x + 25$$
$$= x^2 + 10x + 25$$

2 times 5 — 5 squared

When we square any binomial, $x + a$, the coefficient of x is always $2a$ and the constant term is always a^2. Knowing this, we can do some detective work to find out what to add to a binomial to make it a square.

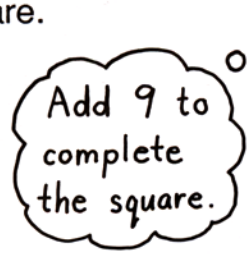


$x^2 + 14x +$ ____

Decide what must be added to each expression to make it a square.

$x^2 + 20x$ Add: _____	$x^2 + 2x$ Add: _____	$x^2 - 6x$ Add: _____
$x^2 - 10x$ Add: _____	$x^2 + 4x$ Add: _____	$x^2 - 12x$ Add: _____
$x^2 + 18x$ Add: _____	$x^2 - 16x$ Add: _____	$x^2 + 8x$ Add: _____
$x^2 - 24x$ Add: _____	$x^2 - 2x$ Add: _____	$x^2 + 30x$ Add: _____

To **complete the square** means to add a number which makes an expression into the square of a binomial. Here is how we can solve a quadratic equation by completing the square.



$$x^2 + 6x = 7$$
$$x^2 + 6x + 9 = 16$$
$$(x + 3)^2 = 16$$
$$x + 3 = 4 \text{ or } x + 3 = -4$$
$$x = 1 \text{ or } x = -7$$

Here are some for you to try.

$x^2 + 2x = 24$

$x^2 - 6x = -5$

$x^2 - 10x = 11$

$x^2 + 8x = -15$