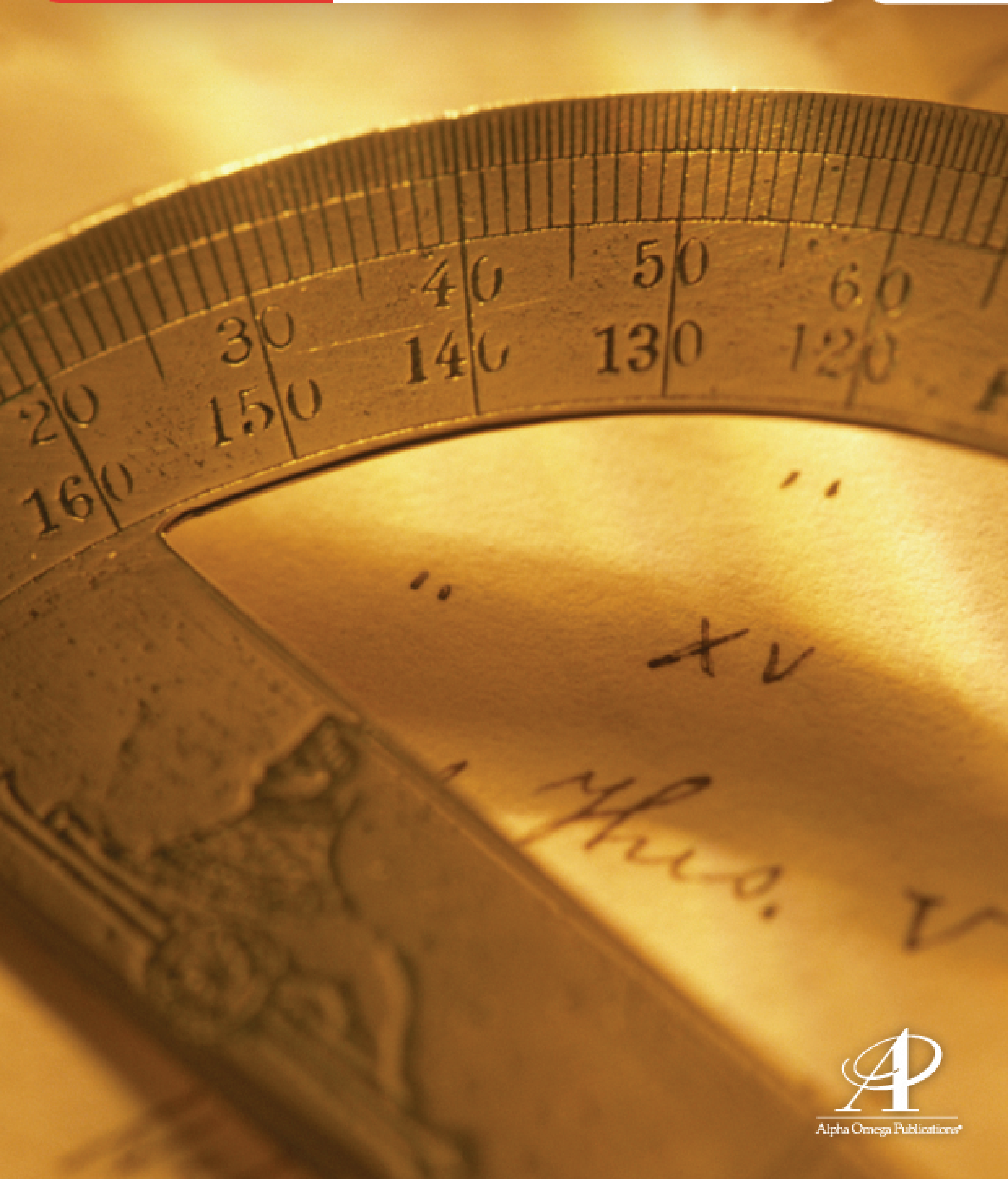
LIFEPAC® Math



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MATHEMATICS 1207
INVERSE TRIGONOMETRIC FUNCTIONS AND POLAR COORDINATES

CONTENTS

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INVERSE TRIGONOMETRIC FUNCTIONS AND POLAR COORDINATES

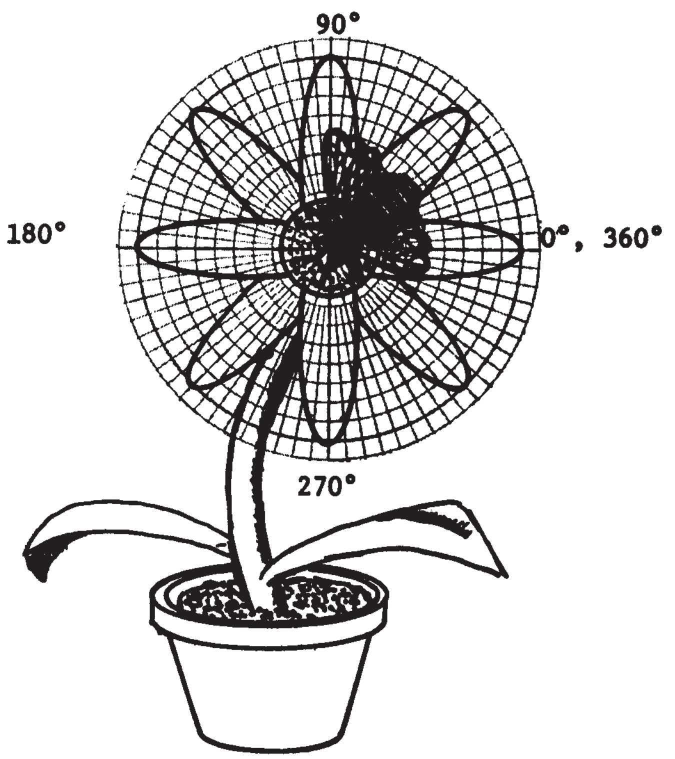
The inverse of a function has been defined as the function resulting from the interchanging of the range and domain sets. The trigonometric functions also have inverses. These functions will be defined and graphed in this LIFEPAC®, and a few applications will be observed.

A set of points (x, y) in the coordinate plane is called a set of Cartesian or rectangular coordinates. Each point and each set of points may have many other equivalent expressions. One such expression is the "polar" form or polar coordinate system. Polar coordinates consist of the length and direction of the ray that connects the Cartesian coordinate with the origin. This LIFEPAC will discuss the relationships that exist between the Cartesian coordinate system and its polar form.

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have completed this LIFEPAC, you should be able to:

1. Define the inverse trigonometric functions.
2. Graph the inverse functions.
3. Apply the inverse functions to problem situations.
4. Convert Cartesian coordinates to polar coordinates.
5. Convert polar coordinates to Cartesian coordinates.
6. Graph polar coordinates.
7. Convert equations from Cartesian form to polar form and conversely.
8. Graph polar equations.



I. THE INVERSE SIN FUNCTION

Consider a few first- and second-quadrant solutions to $y = \sin x$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

To form the inverse of a function, interchange the range and domain sets. The result is the following table.

x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
inverse $\sin x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

First, re name "inverse sin" as "arcsin." Then we have the notation: if $F(x) = \sin x$, then $F^{-1}(x) = \arcsin x$; or if $y = \sin x$, then the inverse is $y = \arcsin x$.

Next, observe from the preceding table of values of the arcsin, that when $x = \frac{1}{2}$, we have two values for arcsin, namely $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. Therefore, arcsin cannot be called a function unless the domain x is redefined so that the arcsin, the function value, is not double-valued. The range will accordingly be restricted to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, which will be shown later by the graph.

Now, observe that the domain also has a restriction in that x must be between negative one and positive one: $-1 \leq x \leq 1$. This domain was the range of the sin function before the range and domain were interchanged.

CONSIDER THESE GENERAL SOLUTIONS:

If y = arcsin x and $x = \frac{1}{2}$, then y = arcsin $\frac{1}{2}$; and $y = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \ldots$ The solution set is infinite and is called "a general solution." The solution may be written as $y = \frac{\pi}{6} \pm 2\pi K$ or $\frac{5\pi}{6} \pm 2\pi K$, K = 0, 1, 2, 3. . . .

STUDY THIS EXAMPLE:

If y = arcsin x and $x = \frac{\sqrt{2}}{2}$, then y = arcsin $\frac{\sqrt{2}}{2}$; and $y = \frac{\pi}{4} \pm 2\pi K$ or $\frac{3\pi}{4} \pm 2\pi K$, K = 0, 1, 2, 3. . . .

NOTE: The use of K in the general solution denotes the set of whole numbers.

COMPLETE THESE ACTIVITIES.

1.1 Write the general solution to y = arcsin 1.

____

1.2 Write the general solution to y = arcsin $\frac{1}{2}$.

____

1.3 Write the general solution to y = arcsin 0.

____

1.4 Write the general solution to y = arcsin(0.6428).

____

1.5 Write the general solution to y = arcsin(0.9659).

____

1.6 Write the general solution to y = arcsin 1.2.

____

GIVEN $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$*, FIND THE FOLLOWING SOLUTIONS.*

1.7 $y = \arcsin \frac{1}{2}$ ____________________

1.8 $y = \arcsin \frac{\sqrt{3}}{2}$ ____________________

1.9 $y = \arcsin(0.6947)$ ____________________

1.10 $y = \arcsin(0.7071)$ ____________________

COMPLETE THESE ACTIVITIES.

1.11 Simplify $\arcsin(\sin 40°)$. ____________________

1.12 Simplify $\arcsin(\sin 60°)$. ____________________

1.13 Simplify $\arcsin(\sin \frac{\pi}{2})$. ____________________

1.14 Simplify $\arcsin(\sin 6\pi)$. ____________________

1.15 Simplify $\arcsin\ (\sin -\frac{3\pi}{2})$. ____________________