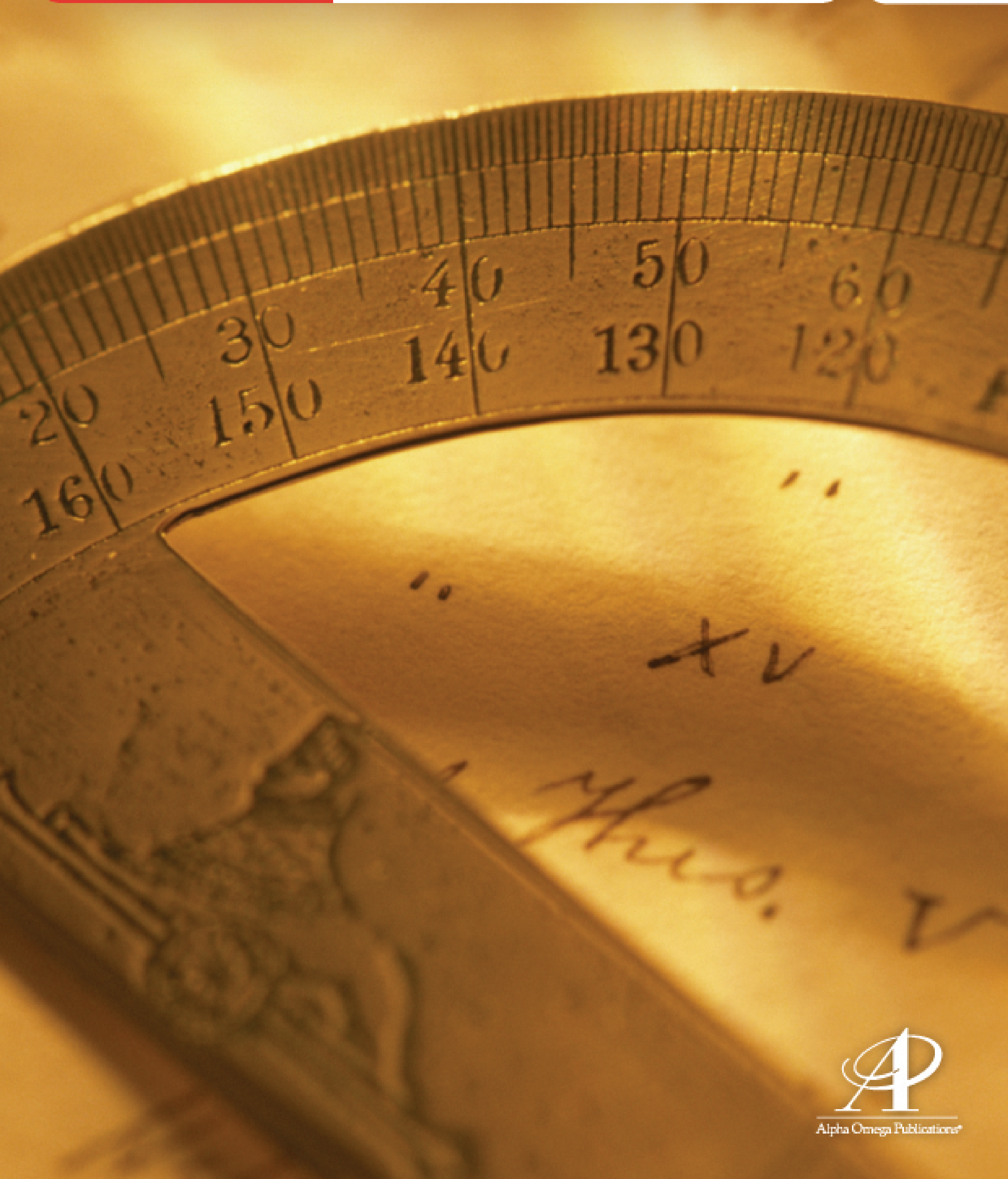
LIFEPAC® Math



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MATHEMATICS 1109
COUNTING PRINCIPLES

CONTENTS

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COUNTING PRINCIPLES

Counting is the oldest and most basic concept in mathematics. Many important developments in counting principles have occurred in recent years. Computer technology has allowed solutions to previously impractical or unsolvable problems through these principles.

We shall begin this LIFEPAC with a look at *progressions*, which represent real situations and are tools for more advanced mathematics. *Permutations* and *combinations* are used in solving many problems that ask, "How many ways are possible?" These two concepts also provide the necessary background for the fascinating study of *probability*. Probability is one of the most important counting principles used in business and science.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. **Write the general term of a sequence.**
2. **Identify arithmetic and geometric series.**
3. **Use factorial notation.**
4. **Find the number of ways items can be arranged.**
5. **Find the number of ways a group can be subdivided.**
6. **Define probability.**
7. **Compare probabilities.**

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

I. PROGRESSIONS

SECTION OBJECTIVES

1. Write the general term of a sequence.
2. Identify arithmetic and geometric sequences.

The natural numbers are those numbers you used when you first learned to count; that is, the positive integers. Notice that neither negative numbers nor zero nor fractions are included. The natural numbers begin with 1 but have no ending; hence, they are infinite.

The word progression can refer to either a *sequence* or a *series*. A sequence is an arrangement of quantities whose positions are based upon the natural numbers. A series is a summation of quantities based upon a sequence.

SEQUENCES

One type of progression is a sequence. After you have studied the definition of a sequence, the form for the general term of a sequence will be presented.

DEFINITIONS

Sequence: a group of numbers arranged in a definite order, with a specific first term.

Term: an individual quantity or number in a sequence.

Consider this sequence:

2, 4, 6, 8, 10,

This sequence is different from

2, 6, 4, 8, 10

because the order is different even though the same numerals appear in both sequences.

Each individual quantity or number is called a *term*, and the terms are separated by commas. The ordering of the terms is a one-to-one correspondence between the terms and the natural numbers.

Although the position of a term in a sequence is given by a natural number, the term itself may be any number. The following sequences are all valid sequences.

Models: $1,\ 1\frac{1}{2},\ 2,\ 2\frac{1}{2},\ 3$

$-3,\ 0,\ 3,\ -2,\ 0,\ 2,\ -1$

$2a + 1,\ 2a + 2,\ 2a + 3,\ 2a + 4$

$\pi,\ \frac{1}{2}\pi,\ \frac{1}{3}\pi,\ \frac{1}{4}\pi,\ \frac{1}{5}\pi,\ \frac{1}{6}\pi$

$0.31,\ 0.61,\ 0.91,\ 1.21$

All the models given have had a last term; therefore, the sequences were finite. We can represent an infinite sequence by writing the first few terms followed by three periods. The sequence of positive odd integers could be written

1, 3, 5, 7, . . .

We can also adapt this notation to finite sequences with many terms. For example, powers of 2 in order up to 256 are written

2, 4, 8, 16, . . . 256.

Complete these activities.

1.1 The natural numbers are ______________________________

1.2 A sequence is ______________________________

1.3 Give an example of a finite sequence. ______________________________

1.4 Give an example of an infinite sequence. ______________________________

1.5 Why are the following numbers not a sequence?

. . . -4, -2, 0, 2, 4, . . . ______________________________

GENERAL TERM

Consider the following sequence:

$$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$$

We would probably expect that the fourth term of this sequence would be $\frac{5}{6}$. We say probably because to indicate a pattern simply by listing a few terms is not mathematically precise. If we only wrote three terms, we could not distinguish between the following two sequences:

$$1, 2, 3, 4, 5, 6, \ldots$$

and

$$1, 2, 3, 1, 2, 3, \ldots$$

Naturally, we want to be more precise. We can be more precise by writing the general term of the sequence.

The *general term* of a sequence is a formula that yields the value of a term when that term's position is substituted into it. In the sequence

$$3, 5, 7, 9, \ldots 2n + 1, \ldots$$

the first term is $(2 \cdot 1) + 1 = 3$, the second term is $(2 \cdot 2) + 1 = 5$, the 103rd term is 207, and so on. Computations of any term given the general term is almost trivial. Often a sequence is simply referred to by its general term since the sequence is completely described by the general term. Thus

$$3, 6, 9, 12, \ldots 3n, \ldots$$

is referred to as the sequence $3n$.