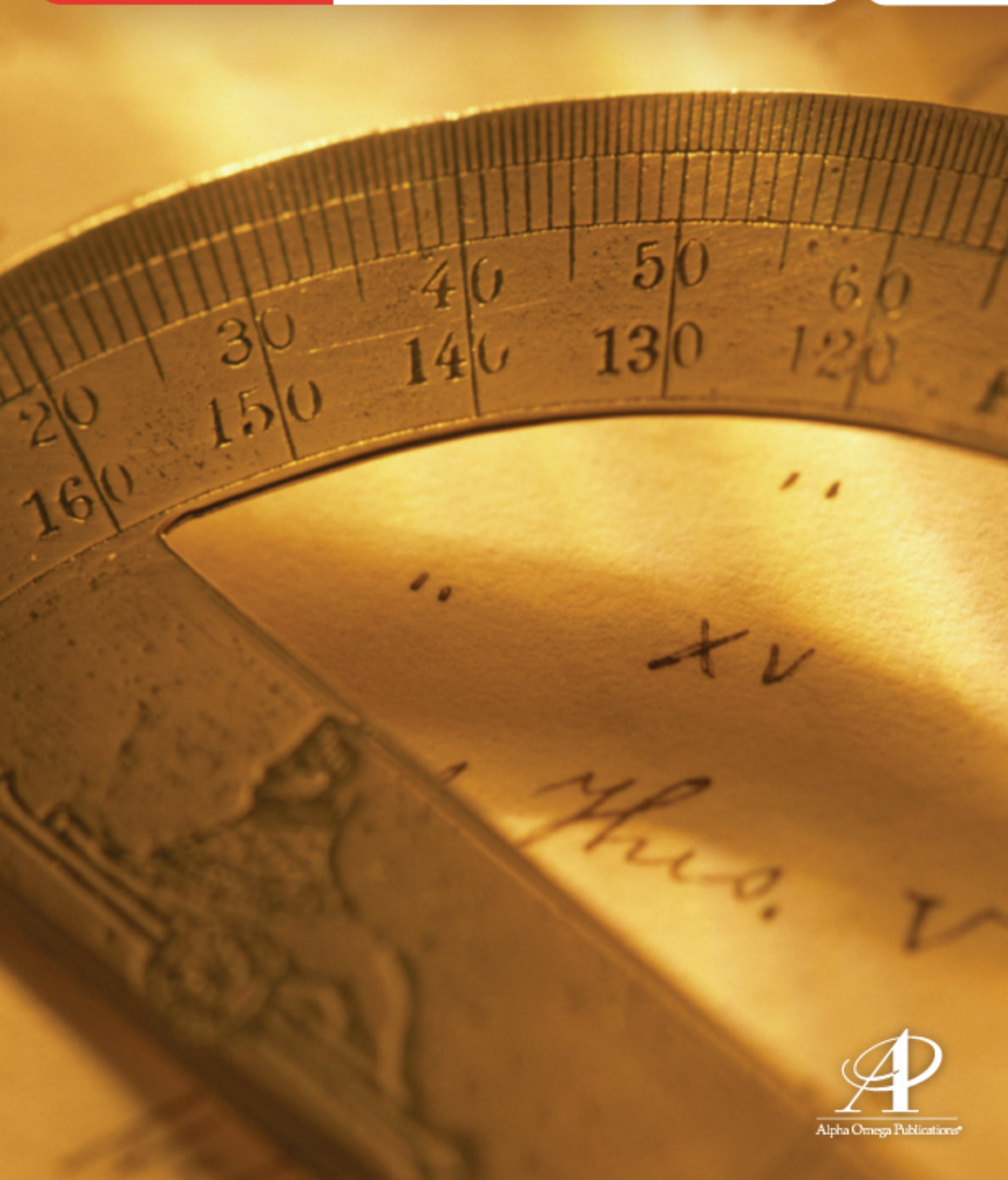
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1108
EXPONENTIAL FUNCTIONS

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EXPONENTIAL FUNCTIONS

In this LIFEPAC you will study three important concepts in mathematics: exponential functions, logarithmic functions, and matrices. You have perhaps heard of the exponential rate of growth that is associated with exponential functions. Closely related to exponential functions are logarithmic functions. Logarithms were developed by John Napier in the seventeenth century. At that time many discoveries in astronomy were being made with the aid of the telescope and logarithmic calculations. Today we have the computer to perform calculations with the aid of matrices, which are rectangular arrays of numbers. You will study both operations with matrices and applications of matrices.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. **Evaluate and simplify mathematical expressions using positive integer exponents.**
2. **Evaluate and simplify mathematical expressions and solve exponential equations using radicals and integral and rational exponents.**
3. **Graph exponential functions.**
4. **Express logarithmic relations as exponential relations and exponential relations as logarithmic relations.**
5. **Evaluate and simplify mathematical expressions using logarithms.**
6. **Find logarithms and antilogarithms in logarithm tables.**
7. **Use scientific notation in logarithmic calculations.**
8. **Perform calculations and solve equations using logarithms.**
9. **Graph logarithmic functions.**
10. **Solve selected problems using exponential functions, logarithmic functions, logarithm tables, and theorems about logarithms (OPTIONAL)**

11. **Solve systems of linear equations using matrices.**
12. **Add and multiply matrices.**
13. **Apply and interpret matrix operations with respect to business applications.**

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

I. EXPONENTIAL FUNCTIONS

SECTION OBJECTIVES

1. Evaluate and simplify mathematical expressions using positive integer exponents.
2. Evaluate and simplify mathematical expressions and solve exponential equations using radicals and integral and fractional exponents.
3. Graph exponential functions.

Exponential functions are used in a wide range of applications. Some of these applications occur in business, chemistry, biology, and statistics. In business the exponential function $y = e^x$ ($e \doteq 2.71828$, the *natural number*) is used in calculating the earnings of money invested at continuous compound interest. In chemistry, $y = e^{-x}$ is used in calculating the half-life of radioactive material. In biology, $y = e^x$ is used to calculate the amount of bacteria present after a period of growth. In statistical studies, used in almost every career area, $y = e^{x^2}$ is the equation of a *normal curve*, which is the standard arrangement of data for many experiments or surveys. For many years most areas of physical science have used the exponential function $y = 10^x$, which is the basis for common logarthims.

Your study of exponential functions will begin with a definition of exponents. You should then be ready to solve and graph exponential equations and observe some applications of exponents.

EXPONENTS

Rules for exponents will be helpful in your study of this LIFEPAC. Fractional exponents (exponents that are fractions) will also be presented.

RULES FOR EXPONENTS

You no doubt remember the following definition of exponents.

DEFINITION

When x is a positive integer such as 1, 2, 3, . . .

$$b^x = \underbrace{b \cdot b \cdot b \cdot \ . \ . \ . \cdot b}_{x}$$

Models: $b^1 = b$

$b^2 = b \cdot b$

$b^3 = b \cdot b \cdot b$

(In the expression b^x, b is called the *base* and x is called the *exponent*.) REMEMBER?

To expand the use of exponents, mathematicians have agreed upon the following definitions, which are also related to exponents.

DEFINITIONS

A. If $b \neq 0$, $b^0 = 1$.

B. If x is an integer and $b \neq 0$, then

$$b^{-x} = \frac{1}{b^x} \text{ and } \frac{1}{b^{-x}} = b^x.$$

A. Models: $2^0 = 1$

$5^0 = 1$

B. Models: $b^{-2} = \frac{1}{b^2}$ if $b \neq 0$

$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

$\frac{1}{2^{-4}} = 2^4$