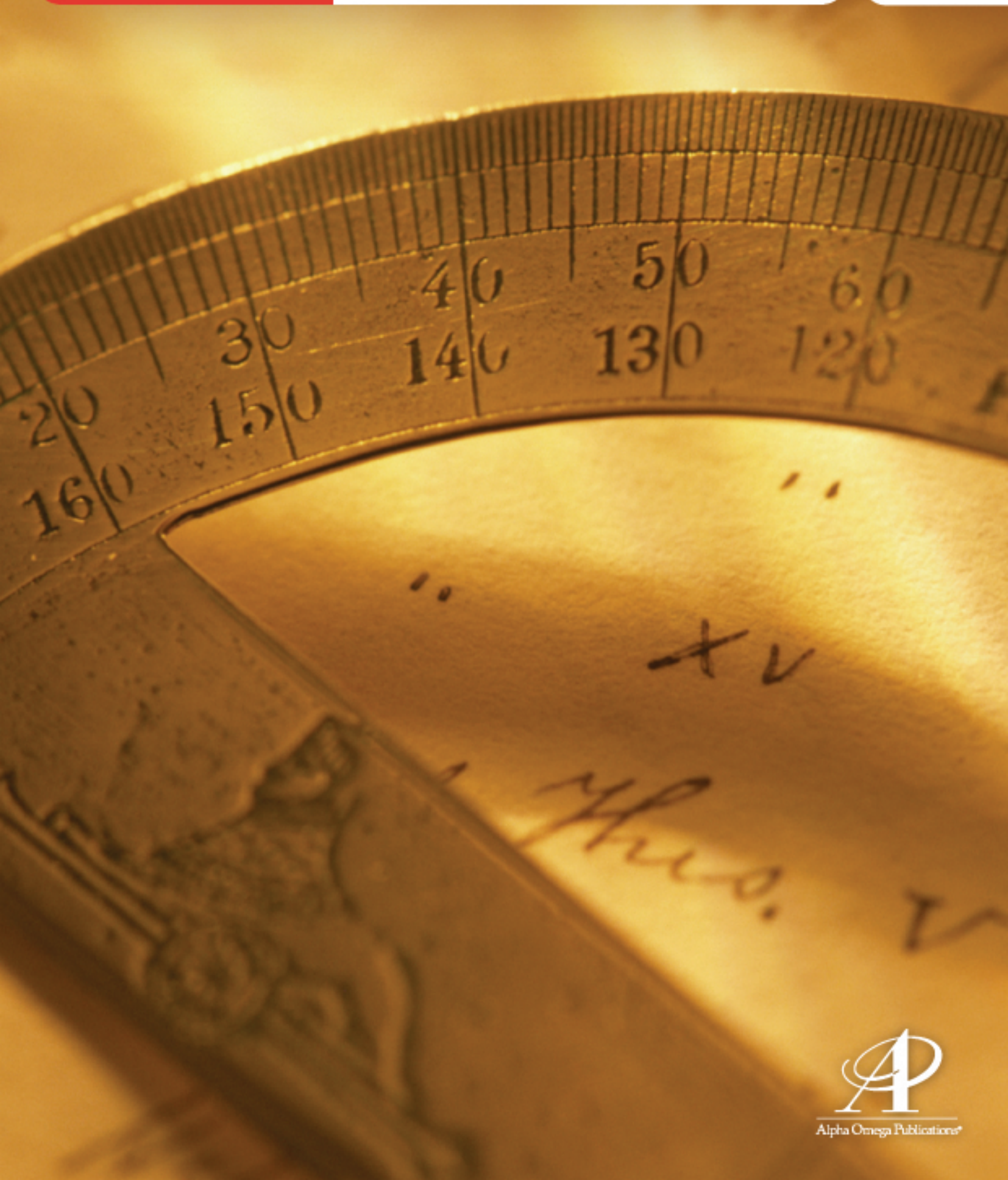
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1107
QUADRATIC RELATIONS AND SYSTEMS

CONTENTS

Author: **Floyd Vest, Ed.D.**
Editor-in-Chief: Richard W. Wheeler, M.A.Ed.
Editor: Robin Hintze Kreutzberg, M.B.A.
Consulting Editor: Robert L. Zenor, M.A., M.S.
Illustrator: Thomas R. Rush

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804 N. 2nd Ave. E., Rock Rapids, IA 51246-1759

MATHEMATICS 1107
QUADRATIC RELATIONS AND SYSTEMS

Millions of physical scientists, managerial specialists, social scientists, engineers, and mathematicians have studied and used the quadratic relations known as conic sections. In this LIFEPAC you will learn to describe by equation and graph the circle, ellipse, parabola, and hyperbola. You will also examine some of their applications. The study will begin with the formula for the distance between two points, which is basic to understanding these quadratic relations.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. **Use the distance formula to calculate the distance between two points in a coordinate plane.**
2. **Define this circle and ellipse and use the distance formula to derive their equations.**
3. **Graph the circle and ellipse and describe essential elements from their equations.**
4. **Write the equations of the circle and ellipse from the graphs and from descriptions of the essential elements.**
5. **Define the parabola and hyperbola and use the distance formula to derive their equations.**
6. **Graph the parabola and hyperbola and describe essential elements from their equations.**
7. **Write the equations of the parabola and hyperbola from the graphs and from descriptions of the essential elements.**
8. **Identify each conic section from its equation.**
9. **Solve and graph systems of first- and second-degree equations.**
10. **Solve and graph second-degree inequalities.**
11. **Solve application problems involving conic sections.**

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

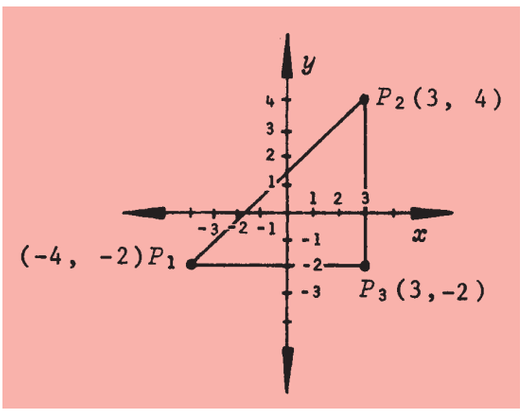
I. DISTANCE FORMULA AND CONIC SECTIONS

SECTION OBJECTIVES

1. Use the distance formula to calculate the distance between two points in a coordinate plane.
2. Define the circle and ellipse and use the distance formula to derive their equations.
3. Graph the circle and ellipse and describe essential elements from their equations.
4. Write the equations of the circle and ellipse from the graphs and from descriptions of the essential elements.

The distance formula is needed in deriving the general equations for the four conic sections we shall study in this LIFEPAC. The use of the distance formula will become clearer to you as you study the circle and ellipse.

Suppose that $P_2(3, 4)$ and $P_1(-4, -2)$ are two points in the coordinate plane.



You know that some specific distance exists between P_1 and P_2. If we locate an extra point P_3 making a right triangle, we can write by the Pythagorean Theorem:

$$\text{The distance between } P_1 \text{ and } P_2\text{, denoted by } |P_1P_2| \text{ or } d\text{, is}$$

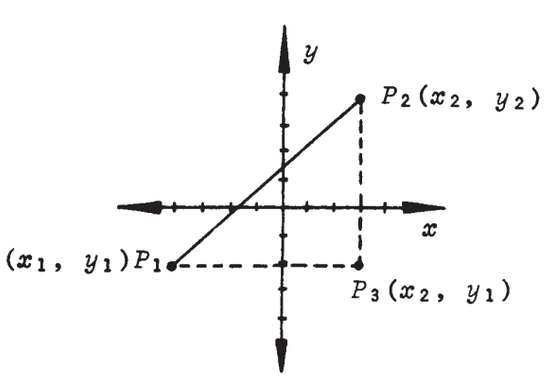
$$= \sqrt{(\text{distance between } P_1 \text{ and } P_3)^2 + (\text{distance between } P_3 \text{ and } P_2)^2}$$

$$= \sqrt{[3 - (-4)]^2 + [4 - (-2)]^2} = \sqrt{(7)^2 + (6)^2} = \sqrt{85} \approx 9.2$$

Thinking along the same lines, we can obtain a more useful and general formula for the distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

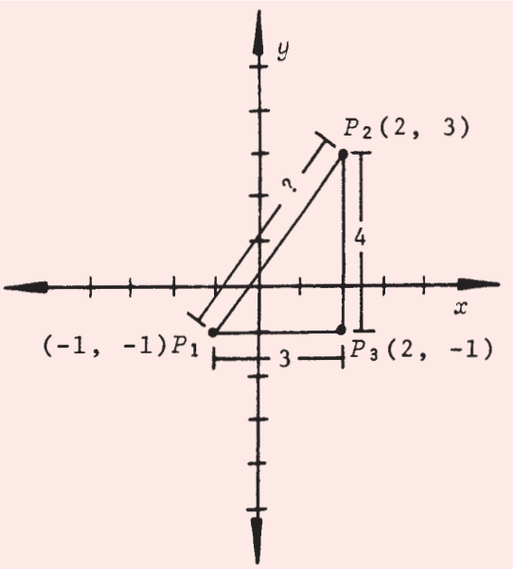
DISTANCE FORMULA

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



See if the results from the distance formula are reasonable by applying the formula to the familiar 3-4-5 right triangle in the model.

Model 1:

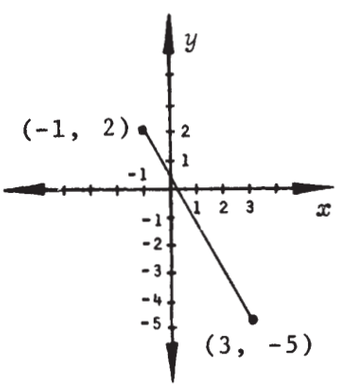


Calculate: $|P_1P_2| = \sqrt{[2 - (-1)]^2 + [3 - (-1)]^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Should $|P_1P_2|$ be 5? Yes.

Model 2: Graph and find the distance between (-1, 2) and (3, -5).

$$d = \sqrt{[3 - (-1)]^2 + (-5 - 2)^2}$$
$$= \sqrt{16 + 49} = \sqrt{65} \cong 8.1$$



Model 3: Graph and find the distance between (-1, 2) and (-1, -5).

We could use the distance formula; however, in cases of horizontal or vertical lines, use the absolute value of the difference between the x-coordinates or the y-coordinates:

$d = |2 - (-5)| = 7$