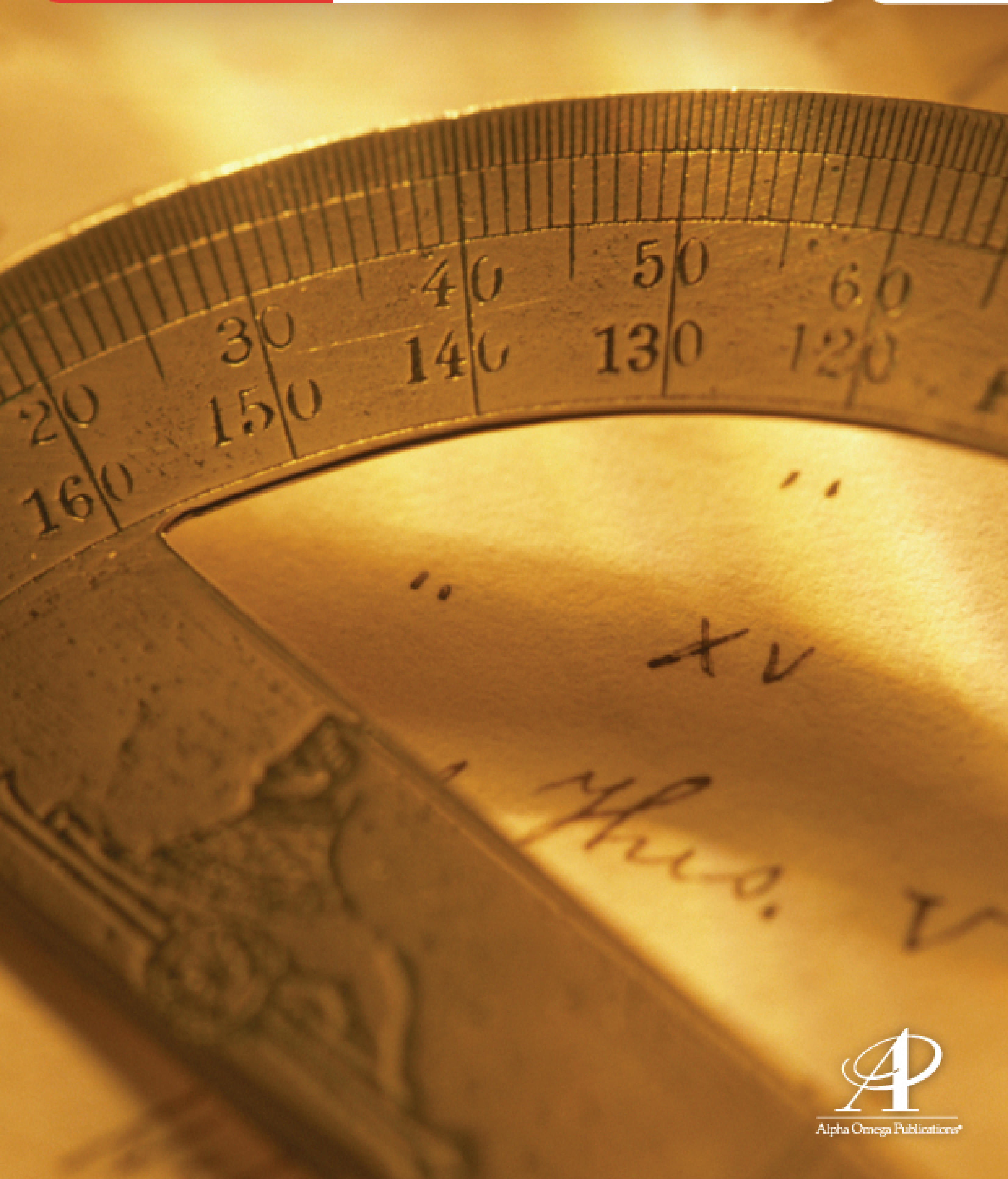
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1106
REAL NUMBERS

CONTENTS

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REAL NUMBERS

In this LIFEPAC® you will study the real-number system and will learn about radicals and imaginary numbers. All of the previously learned properties apply to radicals, because they are real numbers. However, you will learn some special properties of radicals.

In this LIFEPAC you will learn to solve radical equations and quadratic equations. Since much of this LIFEPAC is necessary review, you should also acquire a better understanding of previous material and increased development of your skill-type learning.

OBJECTIVES

Read these objectives: The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. Identify real numbers as rational or irrational.
2. Perform operations with radicals and simplify.
3. Solve radical equations.
4. Solve quadratic equations.
5. Write and use the quadratic formula to solve quadratic equations.
6. Define and use imaginary numbers.

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

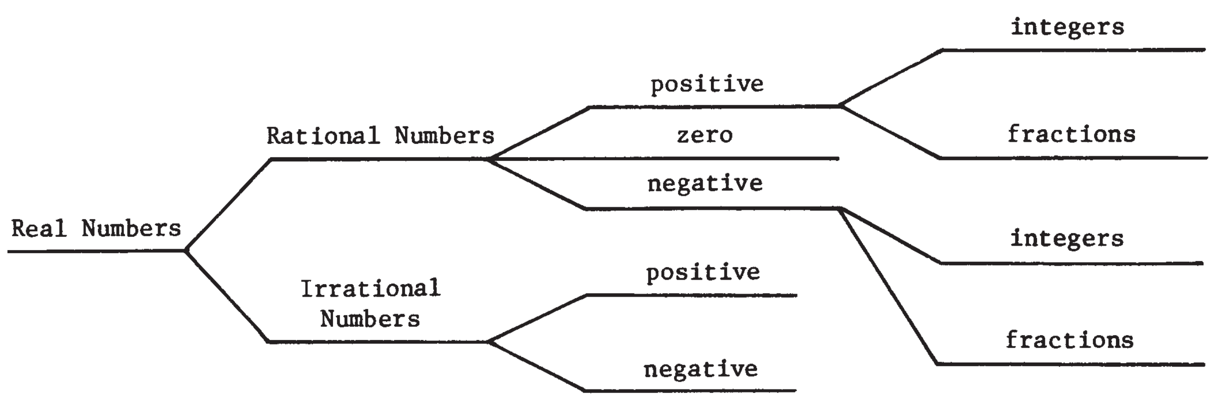
I. REAL NUMBERS

OBJECTIVES

1. Identify real numbers as rational or irrational.
2. Perform operations with radicals and simplify the resulting expressions.
3. Solve radical equations.

The simplest and earliest numbers were positive whole numbers. Soon man discovered a need for zero and fractions. Negative numbers also became useful; finally, the concept of irrational numbers was needed. To increase your understanding of our real-number system, you must learn more about radicals and how to solve equations that involve radicals.

RATIONAL AND IRRATIONAL NUMBERS



This chart shows the relationship of the real numbers.

Rational numbers are real numbers that can be represented in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. When considering rational numbers, remember that we are simply talking about all positive or negative whole numbers, zero, and the fractions you have always used in arithmetic.

Irrational numbers are the real numbers that are not rational numbers. They sometimes occur as special numbers, like π, which is the ratio of the circumference of a circle to its diameter. They may be indicated as nonrepeating nonterminating decimals.

DEFINITIONS

Rational number: A number that can be expressed as a quotient of two other numbers; a terminating or repeating decimal.

Irrational number: A number that cannot be fully expressed as a quotient of two other numbers; a nonterminating, nonrepeating decimal.

The most commonly occurring type of irrational number is a number that contains a radical that cannot be simplified to a rational number.

Models: $\sqrt{16} = 4$, $\frac{4}{6} = \frac{2}{3}$ — rational numbers

$\sqrt{3}$, $\frac{\sqrt{11}}{5}$ — irrational numbers

A *radical* is an indicated root of a number. We know that 9 has two square roots, +3 and -3. Any real number has two square roots. When we use the radical symbol, we are referring only to the principal, or positive, square root.

Radicals occur not only as second-degree radicals (square roots) but as third-degree or higher-degree radicals.

DEFINITIONS

Radical: An expression consisting of a number with a radical sign indicating some root of the number beneath the radical sign.

Index: A small number written over the radical sign indicating which root of the number is being sought.

Radicand: The number inside the radical sign.

Radical sign: A symbol indicating that an expression is a radical.

Model:

$\sqrt[5]{79}$

5 is the index.

79 is the radicand.

$\sqrt[5]{79}$ is the radical.

$\sqrt{}$ is the radical symbol.

Notice that if the radical has no index number, the root indicated is the square root. Therefore, $\sqrt{} = \sqrt[2]{}$.

Any decimal that does not terminate or repeat is an irrational number. If a decimal does terminate or repeat, it can be written as a common fraction (in the form $\frac{a}{b}$).

Model:

$$0.51 = \frac{51}{100}$$

$$0.25 = \frac{1}{4}$$

$$0.1666\ldots = \frac{1}{6}$$

A repeating decimal can be changed to a common fraction, as shown in the following model, by multiplying by the same power of 10 as the number of digits repeating.

Model: 0.2525...

Let 0.2525... be x. Multiplying by 10^2 or 100, because two digits are repeating, gives $100x = 25.25\ldots$

Subtract.

$$\begin{aligned} 100x &= 25.25\ldots \\ x &= 0.2525\ldots \\ \hline 99x &= 25 \\ x &= \frac{25}{99} \end{aligned}$$

Multiplying by a power of 10 that matches the number of digits repeating will allow use of this procedure for a decimal with any number of repeating digits.