



# MATHEMATICS 1104

## POLYNOMIALS

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# POLYNOMIALS

In this LIFE PAC® you will be studying products and factoring, as well as operations with polynomials and the patterns of variations.

You may have already studied multiplication of polynomials and factoring of polynomials. In this LIFE PAC you will review and learn some new factoring techniques.

Addition, subtraction, and division will be reviewed and a new synthetic division will be

introduced. Learning the operational aspects of polynomials will be useful in solving equations.

Direct variation and inverse variation are important relationships between variables that are found in many natural laws. These relationships are often used alone or in combination to solve practical problems through the equations they represent.

## OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFE PAC.

When you have finished this LIFE PAC, you will be able to:

1. Multiply polynomials.
2. Identify special products and factor completely.
3. Perform addition, subtraction, and division with polynomials.
4. Use direct, inverse, joint, and combined variations in writing equations and solving problems.

**Survey the LIFE PAC.** Ask yourself some questions about this study. Write your questions here.

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# I. PRODUCTS AND FACTORING

## OBJECTIVES

When you have completed this section you should be able to:

1. Multiply polynomials.
2. Identify special products and to factor completely.

This LIFEPAK section will deal with topics that you have studied before. Multiplying polynomials, recognizing special products, and factoring trinomials and special products were studied earlier.

New material included deals with a special product not previously covered and the factors of this product. The polynomial factors may surprise you.

You will need expertise in this section to complete later work with equations and word problems.

## MULTIPLYING WITH MONOMIALS

Certain laws of exponents must be used in operations with polynomials.

### LAWS OF EXPONENTS

A:  $a^m a^n = a^{m+n}$

B:  $(a^m)^n = a^{mn}$

C:  $(ab)^m = a^m b^m$

Models: A:  $x^3 \cdot x^5 = x^8$

B:  $(x^3)^5 = x^{15}$

C:  $(x^2 y^4)^3 = (x^2)^3 (y^4)^3$  or  $x^6 y^{12}$

The distributive property is the basis for multiplying monomials with polynomials.

### DISTRIBUTIVE PROPERTY

$$a(b + c) = ab + ac$$

Model:  $3x(a + y) = 3ax + 3xy$

The associative and commutative properties are used along with the distributive property and the laws of exponents to multiply.

#### ASSOCIATIVE PROPERTY

$$a(bc) = (ab)c$$

#### COMMUTATIVE PROPERTY

$$ab = ba$$

$$\begin{aligned} \text{Model: } 6x^2y(3xy^3 + 14x^2y^2) &= (6x^2y)(3xy^3) + (6x^2y)(14x^2y^2) \\ &= 6 \cdot 3x^2 \cdot xy \cdot y^3 + 6 \cdot 14 \cdot x^2 \cdot x^2 \cdot y \cdot y^2 \\ &= 18x^3y^4 + 84x^4y^3 \end{aligned}$$

**n+1=**

Find the indicated products.

- |      |  |       |
|------|--|-------|
| 1.1  | $x^2 \cdot x^5$  | _____ |
| 1.2  | $a^4 \cdot a^6$  | _____ |
| 1.3  | $(a^4)^5$  | _____ |
| 1.4  | $(x^6)^4$  | _____ |
| 1.5  | $(x^2y)^3$   | _____ |
| 1.6  | $(2a^2b)^4$  | _____ |
| 1.7  | $-2x^2y^3 \cdot 14x^2y^3$                              | _____ |
| 1.8  | $(3x^2y^6)^7$  | _____ |
| 1.9  | $a^2b(3a^2 + 4ab^2)$                                   | _____ |
| 1.10 | $3a^n(a^n + a^{n-1})$                                  | _____ |
| 1.11 | $6xy(\frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{6}y^2)$ | _____ |

### MULTIPLYING POLYNOMIALS BY POLYNOMIALS

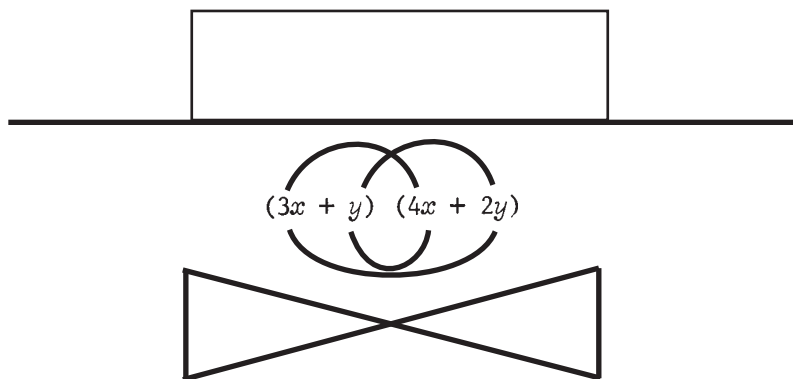
Multiplying two polynomials together involves the distributive, associative, and commutative properties reviewed in the previous section as well as some memory of products.

$$\begin{aligned}
 \text{Model: } (3x + y)(4x + 2y) &= (3x + y)4x + (3x + y)2y \\
 &= 12x^2 + \underbrace{4xy + 6xy} + 2y^2 \\
 &= 12x^2 + 10xy + 2y^2
 \end{aligned}$$

Using either the "foil" method or remembering the unforgettable face of George P. Trinomial can help with finding the product. You should not have to write anything but the answer in finding this product.

The "foil" method involves using the letters in the word to remember that the product consists of the product of the two first terms  $(3x)(4x)$ , plus the product of the outer terms  $(y)(2y)$  combined with the product of the iinner terms  $(y)(4x)$ , and finally the product of the two last terms  $(y)(2y)$ .

George P. Trinomial is the name of the face you see.



George P. Trinomial

Of course the crossed eyebrows, nose, and chin help direct us to the products.

In certain polynomials you may group the terms and treat your selected binomials as monomials. With practice these products can be done mentally.

$$\begin{aligned}
 \text{Model: } (a + b - 3)(a + b + 5) &= [(a + b) - 3][(a + b) + 5] \\
 &\quad \text{let } x = a + b \\
 (x - 3)(x + 5) &= x^2 + 2x - 15 \\
 \text{Now replace } x \text{ by } &= (a + b)^2 + 2(a + b) - 15 \\
 a + b: &= a^2 + 2ab + b^2 + 2a + 2b - 15
 \end{aligned}$$