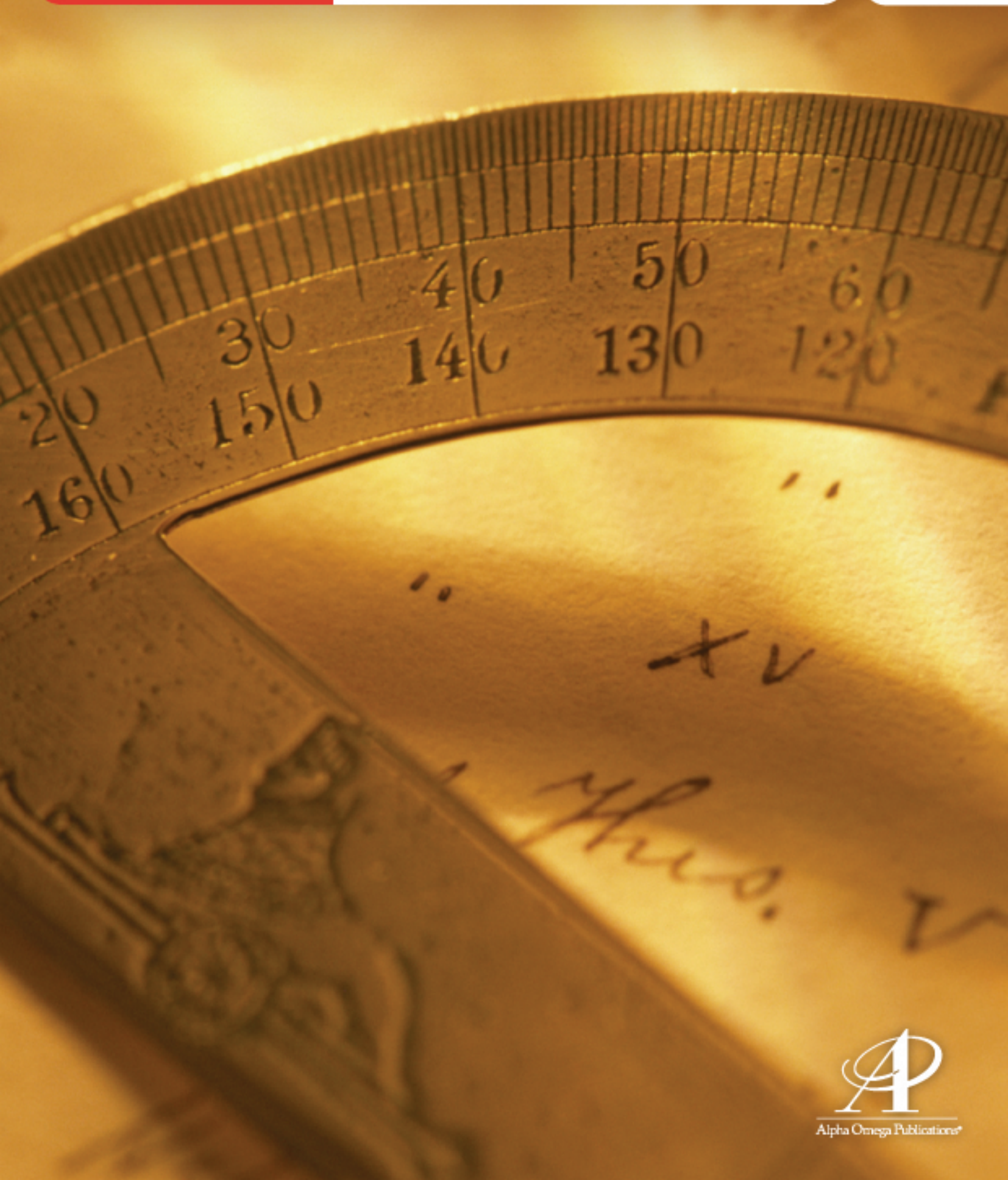
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1102
NUMBERS, SENTENCES, AND PROBLEMS

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NUMBERS, SENTENCES, AND PROBLEMS

Someone has said, "Numbers make expressions, expressions make sentences, and sentences solve problems." Our ultimate goal in the study of mathematics, and particularly of algebra, is to *symbolize* a problem and solve it. Many skills at all levels are needed to accomplish this task. This LIFEPAC® consists of a review of some basic skills relating to numbers, methods of solving equations and inequalities of one variable, and the application of the sentence to problem solving.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. Evaluate expressions involving absolute values and integers.
2. Solve linear equations of one variable.
3. Solve and graph linear inequalities.
4. Solve and graph compound sentences.
5. Solve application problems.

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

I. NUMBERS

SECTION OBJECTIVE

1. Evaluate expressions involving absolute values and integers.

Numbers referred to here are called the set of rational numbers. They include positive and negative integers and fractions. In this section, you will study properties of order, absolute values of numbers and their graphs, and rules for adding and multiplying signed numbers.

ORDER AND ABSOLUTE VALUE

This part of Section I will deal with solving and graphing equations involving absolute values. Because absolute value terms yield positive and negative numbers, properties of order will first be considered.

DEFINITION

Trichotomy: Given any two numbers, a and b, exactly one of the following is true:

$$a < b,\ a = b,\ a > b$$

$a < b$ means that the number a is to the left of b on the number line.

$a = b$ means that the number a is at the same place as b on the number line.

$a > b$ means that the number a is to the right of b on the number line.

DEFINITION

Transitive Property:

A. If $a < b$ and $b < c$, then $a < c$.

B. If $a > b$ and $b > c$, then $a > c$.

On the number line, b is between a and c. In the first case the numbers are in order a, b, c left to right. In the second case the order is c, b, a.

$|a| = K$ means that $a = -K$ or $a = K$.

Model 1: $|a| = 5$ means $a = 5$ or $a = -5$

Model 2: $|a + 2| = 6$ means $a + 2 = 6$ or $a + 2 = -6$

$a = 4$ or $a = -8$

Check: replace a with 4,

$|4 + 2| = 6$ True

replace a with (-8),

$|-8 + 2| = |-6| = 6$ True

Model 3: Solve for x: $|x| = 10$

In set notation, we write

$x = 10$ or $x = -10$

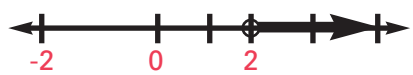
$x = \{-10, 10\}$

Model 4: Graph $|x| = 2$.

The solution is $x = 2$ or $x = -2$. The graph is shown.

Model 5: Graph $x > 2$.

The solution is any number to the right of 2 not including 2. The graph is shown. (The circle on 2 means that the graph does *not* include 2.)



Model 6: If $a = 6$ and $b = 7$, find a number c such that $a < c < b$.

c can be any number between 6 and 7. Let $c = 6\frac{1}{2}$. Then $6 < 6\frac{1}{2} < 7$ is a true statement.

Evaluate each of the following expressions.

1.1 $|-3|$ _____ 1.3 $|-7| + 3|4|$ _____

1.2 $|6| + |-4|$ _____ 1.4 $|8 + 3|$ _____

1.5 $2|-1|$ _____

1.6 $-|0|$ _____

1.7 $3|-4|$ _____

1.8 $4|-2| - 3|-3|$ _____

1.9 $-|-8 - 2|$ _____

1.10 $4 - |-4|$ _____

n+1= Write the solution set for each of the following expressions.

1.11 $|a| = 4$ _____

1.12 $|a + 1| = 4$ _____

1.13 $|x - 1| = 3$ _____

1.14 $|K| = -5$ _____

1.15 $|x| = x$ _____

1.16 $|x| + |7| = 10$ _____

1.17 $|p| = \frac{2}{3}$ _____

1.18 $2|x| = 20$ _____

1.19 $|R - 3| = 10$ _____

1.20 $|2x + 6| = 12$ _____

n+1= Replace the question mark with one of the following symbols that will make the statement true. $<$, $=$, $>$, $\leq$, $\geq$. The variables represent real numbers.

1.21 $6 \; ? \; 8$ _____

1.22 $\frac{1}{2} \; ? \; \frac{1}{3}$ _____

1.23 $|-a| \; ? \; |a|$ _____

1.24 $|x| + 1 \; ? \; |x|$ _____

1.25 $-|y| \; ? \; |-y|$ _____

1.26 $|-x| \; ? \; 0$ _____

1.27 $3 \; ? \; 4 \; ? \; 5$ _____

1.28 $|-4| \; ? \; -|-4|$ _____

n+1= Graph each of the following expressions on a number line.

1.29 $|x| = 2$

1.30 $|x + 1| = 2$

1.31 $|x - 1| = 3$

1.32 $|2x| = 6$

1.33 $|2x - 1| = 7$

1.34 $|x| = 0$

1.35 $|2 - x| = 4$