In general, the expression $x^2 + bx$ becomes a perfect square when $\left(\frac{b}{2}\right)^2$ is added to it.

Thus, $$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

Solving a quadratic equation using this approach is called the **completing the square method**.

Example 7 Solve the equation $x^2 + 6x - 7 = 0$ by the completing the square method.

Solution

$x^2 + 6x - 7 = 0$

$x^2 + 6x = 7$ — Move the constant term to the RHS.

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 7 + \left(\frac{6}{2}\right)^2$ — Add $\left(\frac{6}{2}\right)^2$ to both sides.

$x^2 + 6x + 3^2 = 7 + 3^2$ — Write the LHS as a perfect square.

$(x + 3)^2 = 16$ — Take the square root of both sides.

$\therefore \quad x + 3 = \pm\sqrt{16}$

$x + 3 = 4$ or $x + 3 = -4$

$\therefore \quad x = 1$ or $x = -7$

Note: The equation $x^2 + 6x - 7 = 0$ can be solved by the factorisation method.

Try It 7! Solve the equation $x^2 + 2x - 15 = 0$ by the completing the square method.

Recall

When $p^2 = q$,

$p = \sqrt{q}$ or $-\sqrt{q}$,

where $q > 0$.

Example 8 Solve the equation $x^2 - x - 3 = 0$ by the completing the square method.

Solution

$x^2 - x - 3 = 0$

$x^2 - x = 3$ — $b = -1$

$x^2 - x + \left(-\frac{1}{2}\right)^2 = 3 + \left(-\frac{1}{2}\right)^2$ — $\left(\frac{b}{2}\right)^2 = \left(-\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = \frac{13}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{13}{4}}$

$\therefore \quad x - \frac{1}{2} = -\frac{\sqrt{13}}{2}$ or $x - \frac{1}{2} = \frac{\sqrt{13}}{2}$

$x = \frac{1}{2} - \frac{\sqrt{13}}{2}$ or $x = \frac{1}{2} + \frac{\sqrt{13}}{2}$

$\therefore \quad x = -1.30$ or $x = 2.30$ (correct to 3 sig. fig.)

Compare this solution with that in Example 4 for the same equation.

MathsBits

A number in the form $a + b\sqrt{n}$, is called a ***number in surd form****, where a and b are integers or fractions, $b \neq 0$, n is a positive integer and $\sqrt{n}$ is not an integer.*

Try It 8! Solve the equation $x^2 - 3x - 5 = 0$ by the completing the square method.