Saxon *Calculus* Scope and Sequence

<table>
<thead>
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<th>Foundations</th>
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<tbody>
<tr>
<td><strong>Real Numbers</strong></td>
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<tr>
<td>Identify the subsets of the real numbers</td>
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<td>Identify the order properties of the real numbers</td>
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<td>Identify the properties of the real number field</td>
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<tr>
<td>Discuss 0, 1, ( \pi ), and ( e )</td>
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<td>Graph absolute value inequalities</td>
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<td>Use interval notation</td>
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| **Algebra** |
| Solve equations and systems of equations |
| Simplify expressions |
| Factor |
| Use factorial notation |
| Use summation notation |
| Translate verbal descriptions into algebraic equations |
| Convert between logarithmic and exponential forms |
| Distinguish between zeros, roots, and \( x \)-intercepts |
| Characterize quadratic equations |
| Use the remainder theorem to evaluate polynomials |
| Use synthetic division |
| Use the rational roots theorem |
| Derive and use properties of logarithms |
| Recognize conics by their equations |
| Use the binomial theorem |
| Solve exponential growth problems without calculus |
| Understand irreducible quadratic factors and their graphical significance |

| **Geometry** |
| Use the midpoint and distance formulas |
| Write the equation of a line in various forms |
| Use the Pythagorean theorem |
| Use similar triangles |
| Translate or reflect graphs |
| Understand tangents and slope graphically |

| **Logic** |
| Identify the contrapositives, converses, and inverses of a conditional statement |
| Understand the logical equivalences of conditional statements to their contrapositives and of converses to inverses |
| Construct biconditional statements using *iff* (if and only if) |

| **Trigonometry** |
| Convert between radian measure and degrees |
| Define the trigonometric ratios |
| Evaluate trigonometric expressions |
| Simplify trigonometric expressions |
Use the unit circle to evaluate trigonometric functions
Find the centerline, amplitude, phase angle, and period of sinusoids and use them in graphing
Derive or use trigonometric identities
Identify the meaning of *confunctions*
Identify the inverse trigonometric functions
Solve trigonometric equations

**Graphing Calculator**
Graph functions
Use zooming features
Use specific window settings
Use tracing features
Change modes
Find intersection points
Find zeros of polynomials
Find zeros of functions
Evaluate functions
Verify domains and ranges of functions
Generate tables of function values
Evaluate exponentials
Evaluate logarithms
Use the absolute value function
Approximate limits
Approximate slopes of curves
Graph conics
Use function variables
Find local extrema
Approximate definite integrals
Graph sequences
Graph parametric equations
Graph polar equations

**Basics of Functions**
Represent functions as rules to be applied to specified sets, as tables of values where members in one set are uniquely paired to members of another, and as graphs of such paired values
Evaluate functions
Use function notation
Use the vertical line test
Determine whether mappings are functions
Find the domains and ranges of functions
Add, subtract, multiply, divide, and compose functions
Find and evaluate inverse functions
Understand properties of even and odd functions

**Functions, Graphs, and Limits**

**Analysis of Graphs**
Graph functions and equations
  Trigonometric functions
### Inverse trigonometric functions
- Exponential functions
- Logarithmic functions
- Absolute value functions
- Piecewise functions
- The greatest integer function
- Rational functions
- Conic sections
- Reciprocal functions
- Parametric equations
- Polar curves
- Vector functions
- Using technology

### Using technology
- Find points of intersection
- Find zeros of functions
- Identify the intervals on which a function is increasing (or decreasing)
- Determine local and global extrema

### Limits of Functions
- Understand limits graphically
- Understand limits using epsilon-delta proofs
- Calculate limits using algebra
- Approximate limits from graphs and data tables
- Calculate one-sided limits
- Calculate limits that are disguised derivatives
  - Evaluate $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$
  - Evaluate $\lim_{x \to 0} \frac{\sin x}{x}$
- Approximate limits using technology
- Find limits of sums, differences, products, and quotients
- Use the squeeze theorem
- Use change of variables
- Evaluate limits using logarithms

### Asymptotic and Unbounded Behavior
- Understand asymptotes graphically
- Understand infinite and undefined limits
- Find limits using asymptotes
- Find asymptotes of rational polynomial functions
- Graph functions with asymptotes
- Find asymptotes using limits
- Compare relative magnitudes of functions

### Continuity as a Property of Functions
- Understand continuity graphically
- Understand continuity in terms of limits
- Use the maximum-minimum value existence theorem (Extreme Value Theorem)
- Use the critical number theorem
- Understand point continuity
Understand interval continuity
Use the Intermediate Value Theorem

**Parametric, Polar, and Vector Functions**
Understand parametric equations
Convert between parametric and rectangular coordinates
Graph parametric equations
Use parametric equations to describe projectile motion
Understand polar coordinates
Convert between polar and rectangular coordinates
Graph rose curves, limaçons, and lemniscates
Understand vectors
Perform vector addition, subtraction, and scalar multiplication
Find unit and normal vectors
Graph vector functions

**Derivatives**

**Concept of the Derivative**
Understand the derivative geometrically
Define derivative as the limit of a difference quotient
Understand the derivative as an instantaneous rate of change
Prove the sum and difference rules for derivatives
Prove the product rule for derivatives
Prove the quotient rule for derivatives
Find differentials of functions
Describe the relationship between differentiability and continuity

**Derivative at a Point**
Calculate slope at a point
Find the line tangent to a curve at a point
Find the line normal to a curve at a point
Approximate slopes using technology
Approximate rate of change from graphs and tables
Find critical numbers
Find instantaneous rate of change
Use the derivative at a point for local linear approximation

**Derivative as a Function**
Use various notations for the derivative of a function
Relate the characteristics of the graphs of functions and their derivatives
Relate the increasing and decreasing behavior of functions to the signs of their derivatives
Translate verbal descriptions into equations involving derivatives
Derive the Mean Value Theorem
Understand consequences of the Mean Value Theorem

**Second Derivatives**
Find inflection points
Understand the relationships between the graphs of functions, their first derivatives, and their second derivatives
Understand the relationship of the sign of the second derivative to concavity

**Applications of the Derivative**
Use differentiation to analyze linear motion
Interpret the derivative as a rate of change
Analyze curves in rectangular form
Model rates of change
Solve related-rates problems
Use derivatives in optimization problems
Use L’Hôpital’s Rule
Use implicit differentiation to find the derivative of an inverse function
Use Newton’s method
Use slope fields
Analyze curves in parametric, polar, and vector forms
Use Euler’s method

**Computation of Derivatives**
Compute derivatives using the definition
Find derivatives of constant functions
Find derivatives of polynomial functions
Find derivatives of sums, products, differences, and quotients
Find derivatives of exponential functions
Find derivatives of logarithmic functions
Find derivatives of trigonometric functions
Find derivatives of inverse trigonometric functions
Find derivatives of absolute value functions
Compute and evaluate high-order derivatives
Differentiate implicitly
Use substitution
Use the chain rule
Use logarithmic differentiation
Find derivatives of functions defined by definite integrals
Find the derivatives of parametric, polar, and vector functions

**Integrals**

**Reimann Sums**
Learn the concept of a Reimann sum
Compute Reimann sums using left, right, and midpoint evaluation points
Compute Reimann sums using circumscribed and inscribed (upper and lower) rectangles

**Interpretations and Properties of Definite Integrals**
Define *definite integral* as the limit of a Reimann sum
| Use geometry to evaluate definite integrals |
| Interpreting the definite integral of the rate of change of a quantity on an interval as the change of the quantity on the interval |
| Use additive properties of definite integrals |
| Use linearity of definite integrals |

**Applications of Integrals**

- Find the areas of regions determined by rectangular curves
- Solve mechanical work problems
- Solve accumulation problems
- Find the forces of fluids on sides of tanks
- Find the distances traveled by moving particles on lines
- Find the volumes of solids of revolution using washers
- Find the volumes of solids of revolution using shells
- Use the Mean Value Theorem for Integrals
- Find the average values of functions
- Find the volumes of solids with known cross sections
- Define the natural logarithm function using a definite integral
- Find the lengths of rectangular curves
- Find the lengths of parametric curves
- Find the areas of regions determined by polar curves

**Fundamental Theorem of Calculus**

- Use the Fundamental Theorem to evaluate definite integrals
- Use the Fundamental Theorem to represent particular antiderivatives
- Analyze functions defined by integrals
- Prove the Fundamental Theorem

**Techniques of Antidifferentiation**

- Use knowledge of derivatives to determine antiderivatives
- Find antiderivatives of constants
- Find antiderivatives of products of constants and functions
- Find antiderivatives of power functions
- Antidifferentiate sums
- Antidifferentiate \( \frac{1}{x} \)
- Antidifferentiate exponential functions
- Antidifferentiate logarithmic functions
- Antidifferentiate trigonometric functions
- Use substitution of variables
- Change limits of definite integrals
- Antidifferentiate by parts
- Use partial fractions
- Use trigonometric substitution
- Evaluate improper integrals
- Perform piecewise integration

**Applications of Antidifferentiation**
Use antidifferentiation to analyze linear motion
Find specific antiderivatives using initial conditions
Solve separable differential equations
Model exponential growth by separable differential equations
Model logistic growth by separable differential equations

**Numerical Approximation of Definite Integrals**
Use Reimann sums to approximate definite integrals
Use the trapezoidal rule to approximate definite integrals
Use Taylor series to approximate definite integrals
Use technology to approximate definite integrals

**Polynomial Approximations and Series**

**Concept of Series**
Define sequence
Define *series* as the limit of a sequence of partial sums
Define *convergence* and *divergence* of series
Use technology to explore convergence and divergence of series
Understand arithmetic of series

**Series of Constants**
Represent repeating decimal numbers as series
Determine whether geometric series converge or diverge
Calculate the sums of convergent geometric series
Use geometric series to solve applied problems
Determine whether telescoping series converge or diverge
Calculate the sums of convergent telescoping series
Determine whether *p*-series converge or diverge
Understand the harmonic series
Use the integral test to determine whether series converge or diverge
Use the integral test to prove the convergence rules for *p*-series
Use the basic comparison test to determine whether series converge or diverge
Use the ratio test to determine whether series converge or diverge
Use the root test to determine whether series converge or diverge
Use the limit comparison test to determine whether series converge or diverge
Determine whether alternating series converge or diverge
Calculate error bound of alternating series approximation

**Taylor Series**
Find the Maclaurin series for $e^x$
Find the Maclaurin series for $\sin x$
Find the Maclaurin series for $\cos x$
Find the Maclaurin series for \( \frac{1}{1+x} \)
Compare graphs of functions and their Taylor polynomials
Approximate functions using Taylor polynomials
Express functions as general Taylor series centered at $x = a$
Find Lagrange error bound for Taylor polynomials
Determine radius and interval of convergence
Form new Taylor series by differentiating
Form new Taylor series by integrating
Define functions by power series
Form new Taylor series by substituting