# Saxon Calculus Scope and Sequence

<b>Foundations</b>
Real Numbers
dentify the subsets of the real numbers
dentify the order properties of the real numbers
dentify the properties of the real number field
Discuss $0, 1, \pi$ , and $e$
Graph absolute value inequalities
Jse interval notation
Algebra
olve equations and systems of equations
implify expressions
actor
Use factorial notation
Jse summation notation
ranslate verbal descriptions into algebraic equations
Convert between logarithmic and exponential forms
Distinguish between zeros, roots, and <i>x</i> -intercepts
Characterize quadratic equations
Jse the remainder theorem to evaluate polynomials
Jse synthetic division
Jse the rational roots theorem
Derive and use properties of logarithms
decognize conics by their equations
Jse the binomial theorem
olve exponential growth problems without calculus
Inderstand irreducible quadratic factors and their
raphical significance
Geometry
Jse the midpoint and distance formulas
Vrite the equation of a line in various forms
Jse the Pythagorean theorem
Jse similar triangles
ranslate or reflect graphs
Inderstand tangents and slope graphically
∟ogic
dentify the contrapositives, converses, and inverses of a
onditional statement
Inderstand the logical equivalences of conditional
tatements to their contrapositives and of converses to
nverses
Construct biconditional statements using iff (if and only i
<b>Trigonometry</b>
Convert between radian measure and degrees
Define the trigonometric ratios
valuate trigonometric expressions
implify trigonometric expressions

Use the unit circle to evaluate trigonometric functions

Find the centerline, amplitude, phase angle, and period of sinusoids and use them in graphing

Derive or use trigonometric identities

Identify the meaning of *confunctions* 

Identify the inverse trigonometric functions

Solve trigonometric equations

# **Graphing Calculator**

Graph functions

Use zooming features

Use specific window settings

Use tracing features

Change modes

Find intersection points

Find zeros of polynomials

Find zeros of functions

Evaluate functions

Verify domains and ranges of functions

Generate tables of function values

Evaluate exponentials

Evaluate logarithms

Use the absolute value function

Approximate limits

Approximate slopes of curves

Graph conics

Use function variables

Find local extrema

Approximate definite integrals

Graph sequences

Graph parametric equations

Graph polar equations

#### **Basics of Functions**

Represent functions as rules to be applied to specified sets, as tables of values where members in one set are uniquely paired to members of another, and as graphs of such paired values

Evaluate functions

Use function notation

Use the vertical line test

Determine whether mappings are functions

Find the domains and ranges of functions

Add, subtract, multiply, divide, and compose functions

Find and evaluate inverse functions

Understand properties of even and odd functions

# **Functions, Graphs, and Limits**

# **Analysis of Graphs**

Graph functions and equations

Trigonometric functions

Inverse trigonometric functions
Exponential functions
Logarithmic functions
Absolute value functions
Piecewise functions
The greatest integer function
Rational functions
Conic sections
Reciprocal functions
Parametric equations
Polar curves
Vector functions
Using technology
Find points of intersection
Find zeros of functions
Identify the intervals on which a function is increasing (or
decreasing)
Determine local and global extrema
Limits of Functions
Understand limits graphically
Understand limits using epsilon-delta proofs
Calculate limits using algebra
Approximate limits from graphs and data tables
Calculate one-sided limits
Calculate limits that are disguised derivatives
Evaluate $x \xrightarrow{\lim} 0 (1 + x)^{1/x}$
Evaluate $x \xrightarrow{\lim} 0 \frac{\sin x}{x}$
Approximate limits using technology
Find limits of sums, differences, products, and quotients
Use the squeeze theorem
Find limits of compositions
Use change of variables
Evaluate limits using logarithms
Asymptotic and Unbounded Behavior
Understand asymptotes graphically
Understand infinite and undefined limits
Find limits using asymptotes
Find asymptotes of rational polynomial functions
Graph functions with asymptotes
Find asymptotes using limits
Compare relative magnitudes of functions
Continuity as a Property of Functions
Understand continuity graphically
Understand continuity in terms of limits
Use the maximum-minimum value existence theorem
(Extreme Value Theorem)
Use the critical number theorem
Understand point continuity

Understand interval continuity

Use the Intermediate Value Theorem

## Parametric, Polar, and Vector Functions

Understand parametric equations

Convert between parametric and rectangular coordinates

Graph parametric equations

Use parametric equations to describe projectile motion

Understand polar coordinates

Convert between polar and rectangular coordinates

Graph rose curves, limaçons, and lemniscates

Understand vectors

Perform vector addition, subtraction, and scalar multiplication

Find unit and normal vectors

Graph vector functions

# **Derivatives**

### **Concept of the Derivative**

Understand the derivative geometrically

Define *derivative* as the limit of a difference quotient

Understand the derivative as an instantaneous rate of change

Prove the sum and difference rules for derivatives

Prove the product rule for derivatives

Prove the quotient rule for derivatives

Find differentials of functions

Describe the relationship between differentiability and continuity

## **Derivative at a Point**

Calculate slope at a point

Find the line tangent to a curve at a point

Find the line normal to a curve at a point

Approximate slopes using technology

Approximate rate of change from graphs and tables

Find critical numbers

Find instantaneous rate of change

Use the derivative at a point for local linear approximation

#### **Derivative as a Function**

Use various notations for the derivative of a function

Relate the characteristics of the graphs of functions and their derivatives

Relate the increasing and decreasing behavior of functions to the signs of their derivatives

Translate verbal descriptions into equations involving derivatives

Derive the Mean Value Theorem

Understand consequences of the Mean Value Theorem

#### **Second Derivatives**

Find inflection points

Understand the relationships between the graphs of functions, their first derivatives, and their second derivatives

Understand the relationship of the sign of the second derivative to concavity

# **Applications of the Derivative**

Use differentiation to analyze linear motion

Interpret the derivative as a rate of change

Analyze curves in rectangular form

Model rates of change

Solve related-rates problems

Use derivatives in optimization problems

Use L'Hôpital's Rule

Use implicit differentiation to find the derivative of an inverse function

Use Newton's method

Use slope fields

Analyze curves in parametric, polar, and vector forms

Use Euler's method

#### **Computation of Derivatives**

Compute derivatives using the definition

Find derivatives of constant functions

Find derivatives of polynomial functions

Find derivatives of sums, products, differences, and quotients

Find derivatives of exponential functions

Find derivatives of logarithmic functions

Find derivatives of trigonometric functions

Find derivatives of inverse trigonometric functions

Find derivatives of absolute value functions

Compute and evaluate high-order derivatives

Differentiate implicitly

Use substitution

Use the chain rule

Use logarithmic differentiation

Find derivatives of functions defined by definite integrals

Find the derivatives of parametric, polar, and vector functions

# Integrals

#### **Reimann Sums**

Learn the concept of a Reimann sum

Compute Reimann sums using left, right, and midpoint evaluation points

Compute Reimann sums using circumscribed and inscribed (upper and lower) rectangles

# Interpretations and Properties of Definite Integrals

Define *definite integral* as the limit of a Reimann sum

Use geometry to evaluate definite integrals

Interpret the definite integral of the rate of change of a quantity on an interval as the change of the quantity on the interval

Use additive properties of definite integrals

Use linearity of definite integrals

# **Applications of Integrals**

Find the areas of regions determined by rectangular curves Solve mechanical work problems

Solve accumulation problems

Find the volumes of solids of revolution using disks

Find the forces of fluids on sides of tanks

Find the distances traveled by moving particles on lines

Find the volumes of solids of revolution using washers

Find the volumes of solids of revolution using shells

Use the Mean Value Theorem for Integrals

Find the average values of functions

Find the volumes of solids with known cross sections

Define the natural logarithm function using a definite integral

Find the lengths of rectangular curves

Find the lengths of parametric curves

Find the areas of regions determined by polar curves

#### **Fundamental Theorem of Calculus**

Use the Fundamental Theorem to evaluate definite integrals

Use the Fundamental Theorem to represent particular antiderivatives

Analyze functions defined by integrals

Prove the Fundamental Theorem

#### **Techniques of Antidifferentiation**

Use knowledge of derivatives to determine antiderivatives

Find antiderivatives of constants

Find antiderivatives of products of constants and functions

Find antiderivatives of power functions

Antidifferentiate sums

Antidifferentiate  $\frac{1}{x}$ 

Antidifferentiate exponential functions

Antidifferentiate logarithmic functions

Antidifferentiate trigonometric functions

Use substitution of variables

Change limits of definite integrals

Antidifferentiate by parts

Use partial fractions

Use trigonometric substitution

Evaluate improper integrals

Perform piecewise integration

# **Applications of Antidifferentiation**

Use antidifferentiation to analyze linear motion

Find specific antiderivatives using initial conditions

Solve separable differential equations

Model exponential growth by separable differential equations

Model logistic growth by separable differential equations

# Numerical Approximation of Definite Integrals

Use Reimann sums to approximate definite integrals

Use the trapezoidal rule to approximate definite integrals

Use Taylor series to approximate definite integrals

Use technology to approximate definite integrals

# Polynomial Approximations and Series Concept of Series

Define sequence

Define *series* as the limit of a sequence of partial sums

Define *convergence* and *divergence* of series

Use technology to explore convergence and divergence of series

Understand arithmetic of series

#### **Series of Constants**

Represent repeating decimal numbers as series

Determine whether geometric series converge or diverge

Calculate the sums of convergent geometric series

Use geometric series to solve applied problems

Determine whether telescoping series converge or diverge

Calculate the sums of convergent telescoping series

Determine whether p-series converge or diverge

Understand the harmonic series

Use the integral test to determine whether series converge or diverge

Use the integral test to prove the convergence rules for *p*-series

Use the basic comparison test to determine whether series converge or diverge

Use the ratio test to determine whether series converge or diverge

Use the root test to determine whether series converge or diverge

Use the limit comparison test to determine whether series converge or diverge

Determine whether alternating series converge or diverge Calculate error bound of alternating series approximation

#### **Taylor Series**

Find the Maclaurin series for  $e^x$ 

Find the Maclaurin series for sin *x* 

Find the Maclaurin series for  $\cos x$ 

Find the Maclaurin series for  $\frac{1}{1+x}$ 

Compare graphs of functions and their Taylor polynomials

Approximate functions using Taylor polynomials
Express functions as general Taylor series centered at
x = a
Find Lagrange error bound for Taylor polynomials
Determine radius and interval of convergence
Form new Taylor series by differentiating
Form new Taylor series by integrating
Define functions by power series
Form new Taylor series by substituting