# Lesson 6   Angles of Elevation and Depression

Now we get a chance to apply all of our newly acquired skills in real life applications, otherwise known as word problems. The first section is in elevation and depression problems. I first encountered these in a Boy Scout Handbook many years ago. There was a picture of a tree, a boy, and several lines.

tree

How tall is the tree?

5'

11'   30'
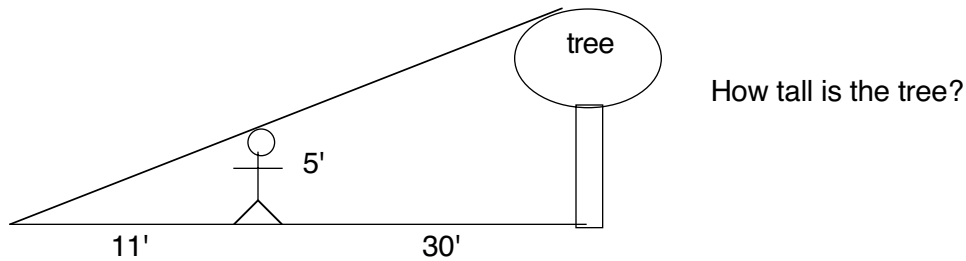
Separating the picture into two triangles helps to clarify our ratios.
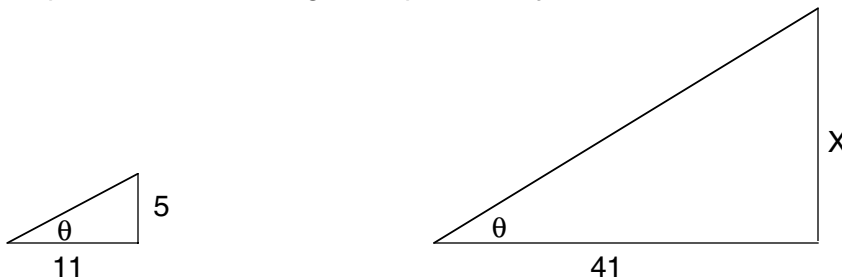
We could solve this with a proportion (two ratios) as

5

θ

11

X

θ

41

$$\frac{5}{11} = \frac{X}{41}$$ and solve for X.

Using our trig. abilities:

in the "boy" triangle $\tan \theta = \dfrac{5}{11} = .4545$

For the large triangle $\tan 24.44° = \dfrac{X}{41}$

$(41)(.4545) = X$

$18.64' = X$

We find that $\theta = 24.44°$, so $\theta = 24.44°$ The tree is 18.64'.

One of the key components in being a good problem solver is drawing a picture using all the data given. It turns a one dimensional group of words into a two dimensional picture.

It is pretty obvious by the words themselves that an angle of elevation measures up, and an angle of depression down. Look at Figure 1.

Figure 1

elevation

depression
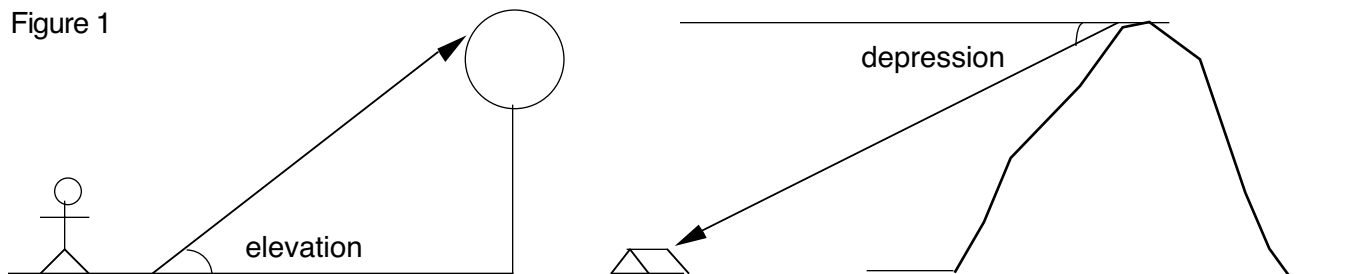
We assume that the line where the angle begins is perfectly flat, or horizontal.

Example 1   A campsite is 9.41 miles from a point directly below the mountain top. If the angle of
             elevation is 12° from the camp to the top of the mountain, how high is the mountain?
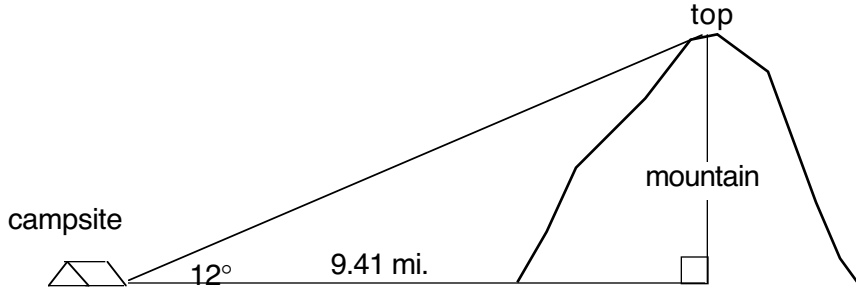
You can now see a right triangle, with the side adjacent to the 12° being 9.41 miles long. To find the height of the mountain, which is the side opposite the 12° angle, the tangent is the best choice.

$$\tan 12° = \frac{\text{height}}{9.41 \text{ mi.}}$$

$$(9.41)(\tan 12°) = \text{height}$$

$$(9.41)(.2126) = \text{height}$$

$$2 \text{ miles} = \text{height}$$
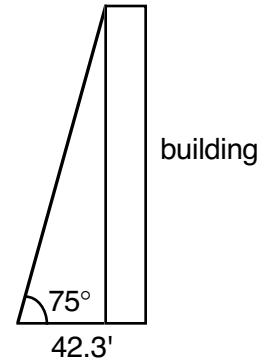
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Example 2   At a point 42.3 feet from the base of a building, the angle of elevation of the top is 75°.
             How tall is the building?

$$\tan 75° = \frac{\text{height}}{42.3'}$$

$$(42.3)(\tan 75°) = \text{height}$$

$$(42.3)(3.7321) = \text{height}$$
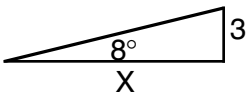
$$157.87' = \text{height of building}$$

building

75°

42.3'

Practice Problems

1) How far from the door must a ramp begin in order to rise 3 feet with an 8° angle of elevation?

2) An A-frame cabin is 26.23 feet high at the center and the angle the floor makes with the base is 53°15'. How wide is the base?
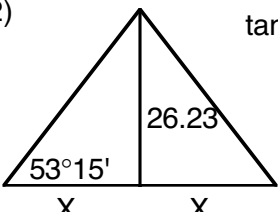
Solutions

1)

8°
X
3

$$\tan 8° = \frac{3}{X}$$

$$X \tan 8° = 3$$

$$X = \frac{3}{\tan 8°}$$

$$X = \frac{3}{.1405}$$

$$X = 21.35 \text{ ft.}$$

2)

53°15'
X   X
26.23

53°15' = 53.25

$$\tan 53.25° = \frac{26.23}{X}$$

$$X = \frac{26.23}{\tan 53.25°}$$

$$X = \frac{26.23}{1.3392}$$

$$X = 19.59 \quad 2X = 39.18 \text{ ft}$$

1) Isaac's camp is 5,280 feet from a point directly beneath Mt. Monadnock. What is the hiking distance along the ridge if the angle of elevation is 25° 16'?

2) How many feet higher is the top of the mountain than his campsite?

Express as a fraction.
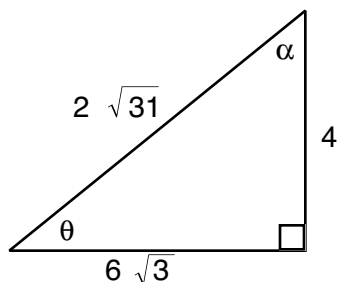
3) csc θ = ——

4) sec θ = ——

5) cot θ = ——

6) csc α = ——

7) sec α = ——

8) cot α = ——

Express as a decimal.

9) sin θ =

10) cos θ =

11) tan θ =

12) sin α =

13) cos α =

14) tan α =
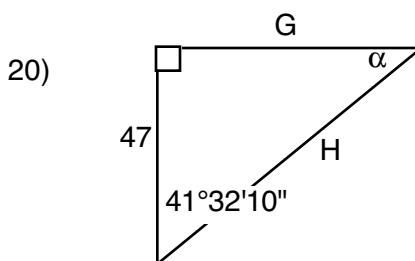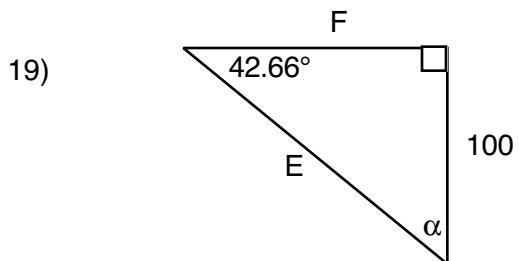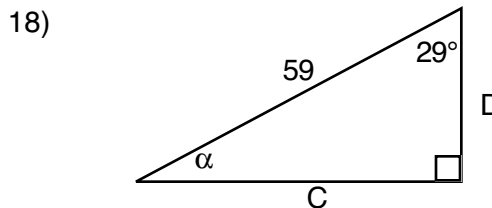
15) Use your answers in #9-11 to find the measure of θ.

16) Use your answers in #12-14 to find the measure of α.

Sample Student Text Page

Solve for the lengths of the sides and the measures of the angles.

17)

18)

19)

20)
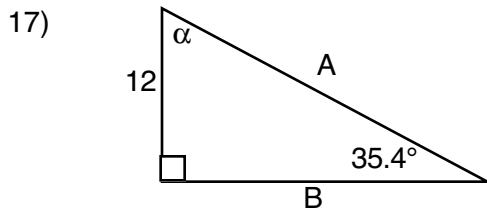
1) The side of a lake has a uniform angle of elevation of 15°30'. How far up the side of the lake does the water rise if, during the flood season, the height of the lake increases by 7.3 feet?

2) A building casts a shadow of 110 feet. If the angle of elevation from that point to the top of the building is 29°3', find the height of the building.

Express as a fraction.
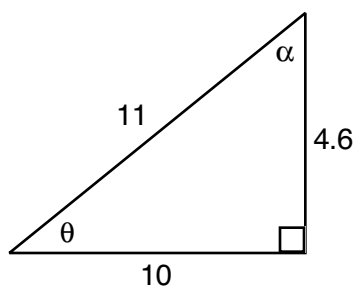
3) csc θ = ⎯⎯⎯

4) sec θ = ⎯⎯

5) cot θ = ⎯⎯

6) csc α = ⎯⎯⎯

7) sec α = ⎯⎯⎯

8) cot α = ⎯⎯⎯

Express as a decimal.

9) sin θ =

10) cos θ =

11) tan θ =

12) sin α =

13) cos α =

14) tan α =

15) Use your answers in #9-11 to find the measure of θ.

16) Use your answers in #12-14 to find the measure of α.

Sample Student Text Page

Solve for the lengths of the sides and the measures of the angles.

17)

18)

19)

20)

1) From a point 120 feet from the base of a church, the angles of elevation of the top of the building and the top of a cross on the building are 38° and 43° respectively. Find the height of the cross. (The ground is flat.)

2) Find the height of the building as well as the height of the cross itself.
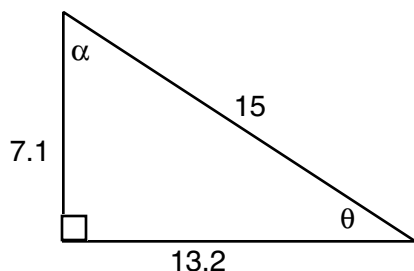
Express as a fraction.

3) csc θ = ——

4) sec θ = ——

5) cot θ = ——

6) csc α = ——

7) sec α = ——

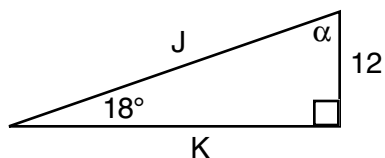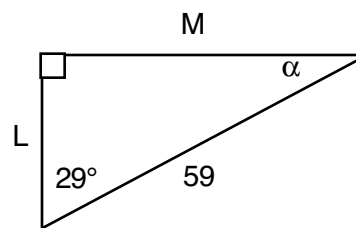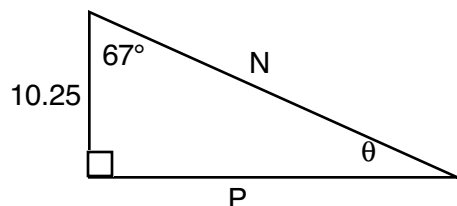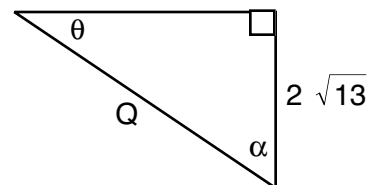8) cot α = ——

Express as a decimal.

9) sin θ =

10) cos θ =

11) tan θ =

12) sin α =

13) cos α =

14) tan α =

15) Use your answers in #9-11 to find the measure of θ. They may vary slightly.

16) Use your answers in #12-14 to find the measure of α. They may vary slightly.

Solve for the lengths of the sides and the measures of the angles.

17)

18)

19)

20)
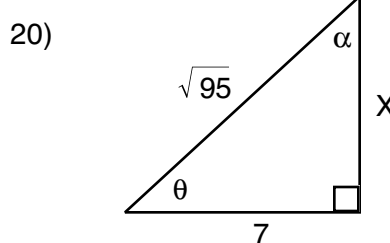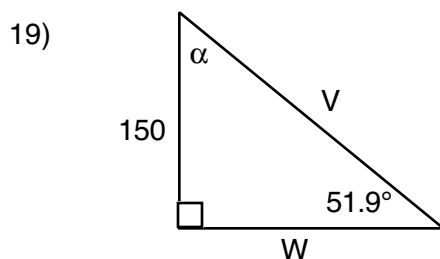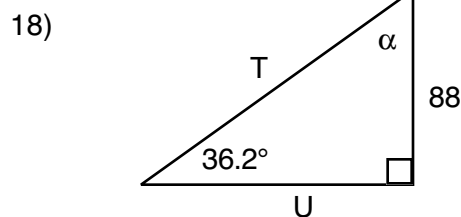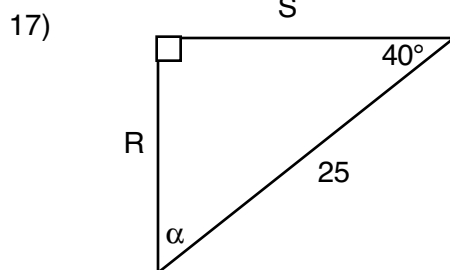
1) A campsite is 12.88 miles from a point directly below Mt. Adams. If the angle of elevation is 15.5° from the camp to the top of the mountain, how high is the mountain?

2) At a point 60.7 feet from the base of a building, the angle of elevation from that point to the top is 64.75°. How tall is the building?

Express as a fraction. First find X.

3) $\csc \theta =$ ———

4) $\sec \theta =$ ——

5) $\cot \theta =$ ——

6) $\csc \alpha =$ ———

7) $\sec \alpha =$ ——

8) $\cot \alpha =$ ——

Express as a decimal.

9) $\sin \theta =$

10) $\cos \theta =$
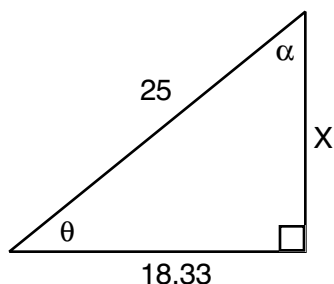
11) $\tan \theta =$

12) $\sin \alpha =$

13) $\cos \alpha =$

14) $\tan \alpha =$

15) Use your answers in #9-11 to find the measure of θ.

16) Use your answers in #12-14 to find the measure of α.

Sample Student Text Page

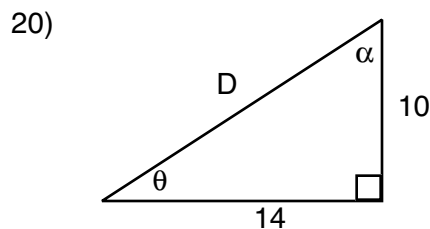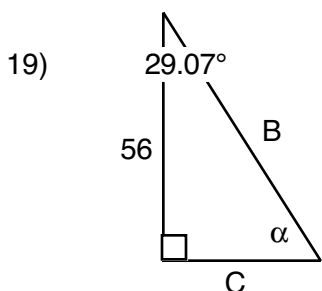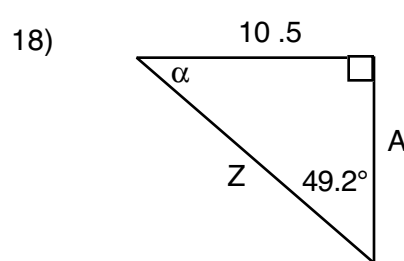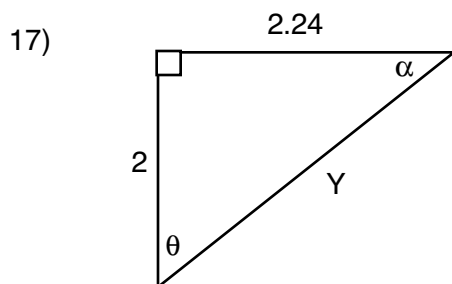*Solve for the lengths of the sides and the measures of the angles.*

17)

18)

19)

20)

Use for questions 1 - 4:   Devan stands 926 meters from a point directly below the peak of a mountain. The angle of elevation between Devan and the top of the mountain is 42°.

1) Which equation can be used to find the height of the mountain (X)?

A) $\sin 42° = \dfrac{X}{926}$

B) $\tan 42° = \dfrac{926}{X}$

C) $\cos 48° = \dfrac{926}{X}$

D) $\tan 42° = \dfrac{X}{926}$

2) What is the height of the mountain?

A) 833.8 m

B) 1028.4 m

C) 619.6 m

D) 1383.9 m

3) A tower 50 meters high is built on top of the mountain. What is the angle of elevation from Devan's position to the top of the tower? (round decimal degrees to tenths)

A) 40°14'44"

B) 43°42'

C) 57°15'

D) 46°20'08"

4) If a bird flew from Devan's position to the top of the mountain, how many meters would it travel?

A) 408.4 m

B) 1246.1 m

C) 1383.9 m

D) 1280.8 m

Use for questions 5 - 8:   From a point 80 meters from the base of a building to the top of the building the angle of elevation is 51°. From the same point to the top of a flag staff on the building the angle of elevation is 54°.

5) What equation can be used to find the combined height (Y) of building and flagpole?

A) Y = 80 tan 51°

B) Y = 80 sin 54°

C) Y = 80 tan 54°

D) $Y = \dfrac{\tan 51°}{80}$

6) What is the height of the building alone?

A) 98.8 m

B) 110.1 m

C) 64.8 m

D) 58.1 m

7) What is the height of the flagpole alone?

A) 15.1 m

B) 45.3 m

C) 4.2 m

D) 11.3 m

8) How long must a cable be in order to stretch from the observation point to the top of the building?

A) 102.9 m

B) 127.1 m

C) 136.1 m

D) 50.3 m

Use for questions 9 & 10:   A car traveled a distance of 100 feet up a ramp to a bridge. The angle of elevation of the ramp was 10°.

9) How high was the bridge above road level?

A) 17.4 ft.

B) 98.5 ft.

C) 10 ft.

D) 100 ft.

10) What is the actual distance from the beginning of the ramp to the base of the bridge?

A) 575 ft.

B) 98.5 ft.

C) 89.4 ft.

D) 17.4 ft.

- -

11) $\dfrac{\sqrt{3}}{3}$ is the ratio for

A) cos 45°

B) cos 30°

C) tan 60°

D) tan 30°

12) Arcsin .8192 =
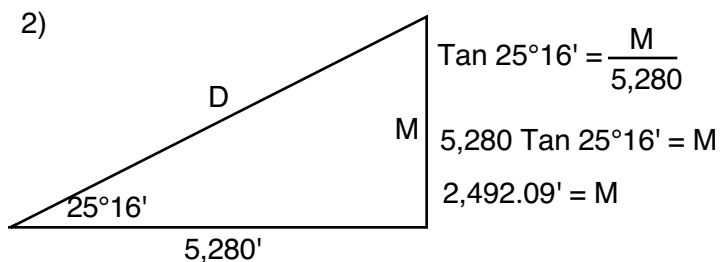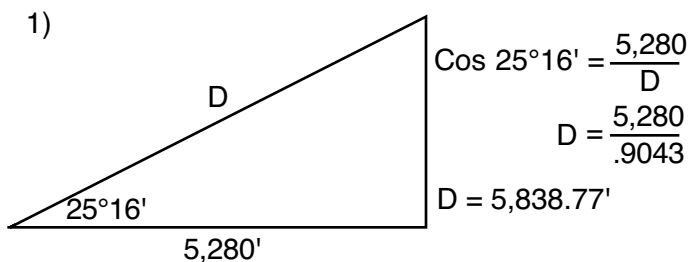
A) 1.22

B) 35°

C) 55°

D) .9999

13) 46°21'02" =

A) 46.21°

B) 46.12°

C) 46.35°

D) 46.4°

14) $\dfrac{\sin \alpha}{\cos \alpha}$ is equal to

A) tan α

B) cot α

C) sec α

D) csc α

15) $\dfrac{1}{\cos \alpha}$ is equal to

A) csc α

B) sec α

C) sin α

D) cos α

1)

$$\text{Cos } 25°16' = \frac{5{,}280}{D}$$

$$D = \frac{5{,}280}{.9043}$$

$$D = 5{,}838.77'$$

D

25°16'

5,280'

2)

$$\text{Tan } 25°16' = \frac{M}{5{,}280}$$

$$5{,}280 \text{ Tan } 25°16' = M$$

$$2{,}492.09' = M$$

D

M

25°16'

5,280'

Express as a fraction.

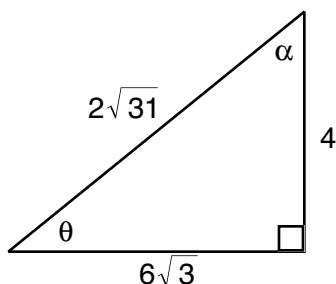3) $\csc \theta = \dfrac{2\sqrt{31}}{4} = \dfrac{\sqrt{31}}{2}$

6) $\csc \alpha = \dfrac{\sqrt{93}}{9}$

4) $\sec \theta = \dfrac{2\sqrt{31}}{6\sqrt{3}} = \dfrac{\sqrt{93}}{9}$

7) $\sec \alpha = \dfrac{\sqrt{31}}{2}$

5) $\cot \theta = \dfrac{6\sqrt{3}}{4} = \dfrac{3\sqrt{3}}{2}$

8) $\cot \alpha = \dfrac{2\sqrt{3}}{9}$

Express as a decimal.

9) $\sin \theta = \dfrac{4}{2\sqrt{31}} = .3592$

12) $\sin \alpha = \dfrac{6\sqrt{3}}{2\sqrt{31}} = .9333$

10) $\cos \theta = \dfrac{6\sqrt{3}}{2\sqrt{31}} = .9333$

13) $\cos \alpha = \dfrac{4}{2\sqrt{31}} = .3592$

11) $\tan \theta = \dfrac{4}{6\sqrt{3}} = .3849$

14) $\tan \alpha = \dfrac{6\sqrt{3}}{4} = 2.598$

α

$2\sqrt{31}$

4

θ

$6\sqrt{3}$

Answers to 15 and 16 may vary slightly.

15) The measure of θ is 21.05°.

16) The measure of θ is 68.95°.

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Solve for the lengths of the sides and the measures of the angles.

17)

54.6°

A

12

35.4°

B

$$\text{Tan } 54.6° = \frac{B}{12}$$

$$12 \text{ Tan } 54.6° = B$$

$$16.89 = B$$

$$\text{Sin } 35.4° = \frac{12}{A}$$

$$A = \frac{12}{\text{Sin } 35.4°}$$

$$A = 20.72$$

18)

59

29°

D

61°

C

$$\text{Sin } 61° = \frac{D}{59}$$

$$(59)(\text{Sin } 61°) = D$$

$$51.6 = D$$

$$\text{Cos } 61° = \frac{C}{59}$$

$$(59)(\text{Cos } 61°) = C$$

$$28.6 = C$$

19)

F

42.66°

100

E

47.34°

$$\text{Tan } 47.34° = \frac{F}{100}$$

$$100 \text{ Tan } 47.34° = F$$

$$108.52 = F$$

$$\text{Sin } 42.66° = \frac{100}{E}$$

$$E = \frac{100}{\text{Sin } 42.66°}$$

$$E = 147.57$$

20)

G

49°27'50"

47

H

41°32'10"

$$41°32'10" = 41.54°$$

$$41.54° = \frac{G}{47}$$

$$47 \text{ Tan } 41.54° = G$$

$$41.64 = G$$

$$\text{Cos } 41.54° = \frac{47}{H}$$

$$H = \frac{47}{\text{Cos } 41.54°}$$

$$H = 62.79$$

1) 15°30' = 15.5°

7.3'   X   D
15°30'

$\sin 15.5 = \dfrac{7.3}{X}$

$X = \dfrac{7.3}{\sin 15.5}$

$X = 27.32'$

2) 29°3' = 29.05°

29°3'   110'   B
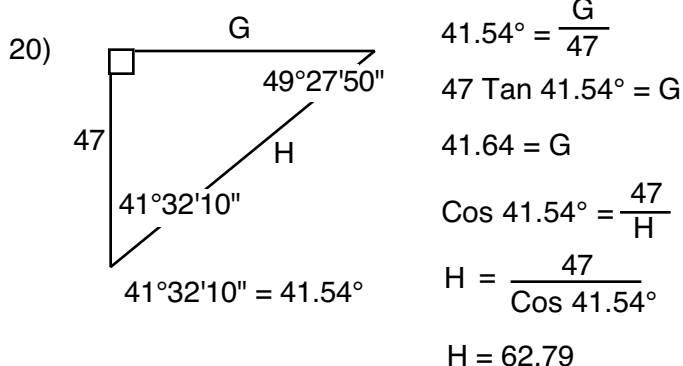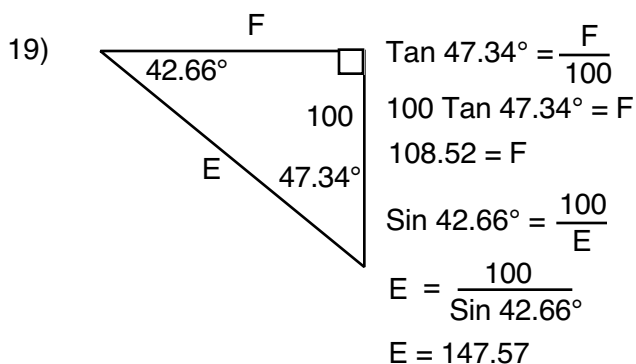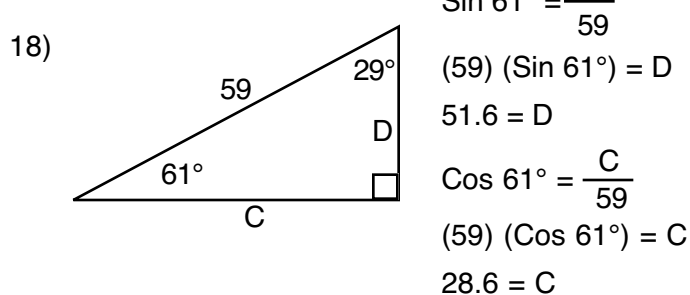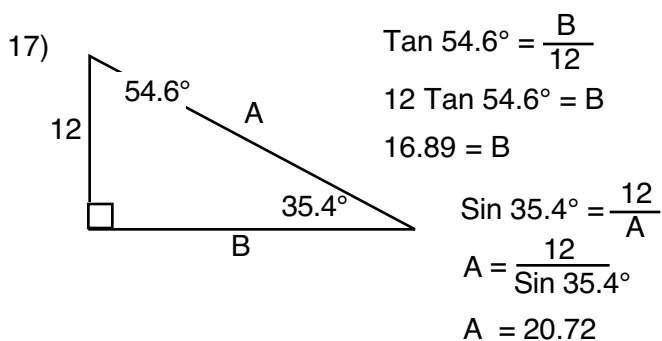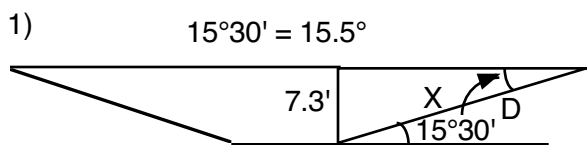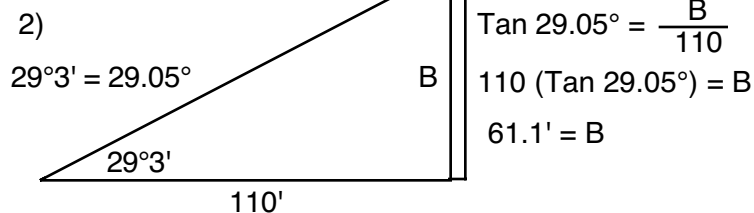
$\text{Tan } 29.05° = \dfrac{B}{110}$

$110 \,(\text{Tan } 29.05°) = B$

$61.1' = B$

Express as a fraction.

3) $\csc \theta = \dfrac{11}{4.6}$

4) $\sec \theta = \dfrac{11}{10}$

5) $\cot \theta = \dfrac{10}{4.6}$

6) $\csc \alpha = \dfrac{11}{10}$

7) $\sec \alpha = \dfrac{11}{4.6}$

8) $\cot \alpha = \dfrac{4.6}{10}$

Express as a decimal.

α
11
4.6
θ
10

9) $\sin \theta = \dfrac{4.6}{11} = .4182$

10) $\cos \theta = \dfrac{10}{11} = .9091$

11) $\tan \theta = \dfrac{4.6}{10} = .46$

12) $\sin \alpha = \dfrac{10}{11} = .9091$

13) $\cos \alpha = \dfrac{4.6}{11} = .4182$

14) $\tan \alpha = \dfrac{10}{4.6} = 2.1739$

15) The measure of θ is 24.7°.

16) The measure of θ is 65.3°.

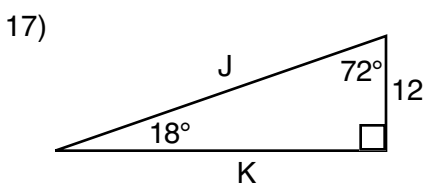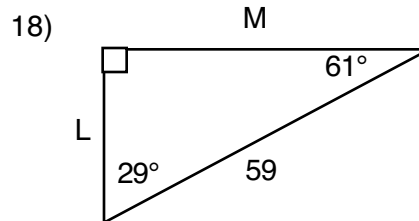*Solve for the lengths of the sides and the measures of the angles.*

17)
J   72°   12
18°
K

$\text{Tan } 72° = \dfrac{K}{12}$

$12 \text{ Tan } 72° = 36.93$

$\text{Sin } 18° = \dfrac{12}{J}$

$J = \dfrac{12}{\text{Sin } 18°} = 38.83$

18)
M
61°
L
29°   59

$\text{Sin } 29° = \dfrac{M}{59}$

$59 \text{ Sin } 29° = M$

$28.6 = M$

$\text{Cos } 29° = \dfrac{L}{59}$

$59 \text{ Cos } 29° = L$

$51.6 = L$

19)
67°   N
10.25
23°
P

$\text{Sin } 23° = \dfrac{10.25}{N}$

$N = \dfrac{10.25}{\text{Sin } 23°} = 26.23$

$\text{Tan } 67° = \dfrac{P}{10.25}$

$10.25 \text{ Tan } 67° = P$

$24.15 = P$

20)
6
50.24°
Q $2\sqrt{13}$
39.76°

$2\sqrt{13} = 7.2111$

$\text{Tan } \theta = \dfrac{7.211}{6}$
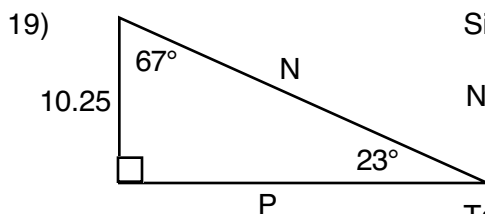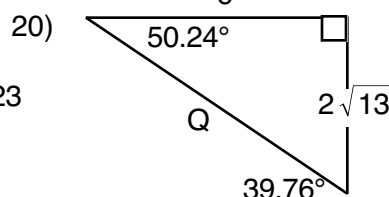
$\text{arctan } \theta = 1.202$

$\theta = 50.24°$

$\alpha = 39.76°$

$(2\sqrt{13})^2 + (6)^2 = Q^2$

$2\sqrt{22} = Q^2$

1)

$$\text{Tan } 43° = \frac{BC}{120}$$

$$BC = 120 \text{ Tan } 43°$$

$$BC = 112'$$

2)

$$\text{Tan } 38° = \frac{B}{120}$$

$$B = 120 \text{ Tan } 38°$$

$$B = 93.75' \text{ or } 93" \ 9"$$

$$112' - 93.75' = 18.25' \text{ or } 18' \ 3"$$
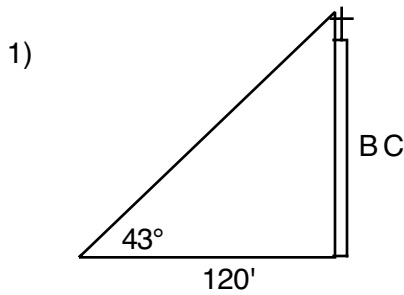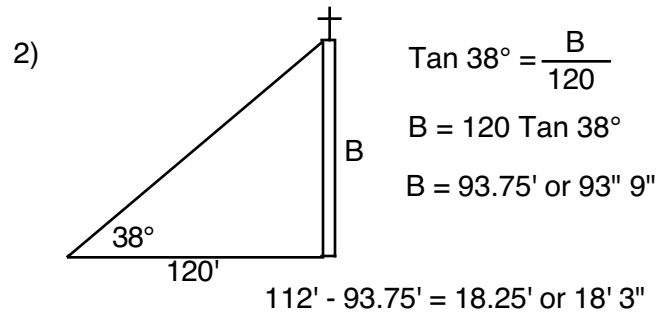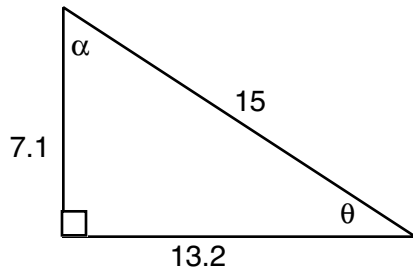
Sample Teacher Manual Page

Express as a fraction.

3) $\csc \theta = \dfrac{15}{7.1}$

4) $\sec \theta = \dfrac{15}{13.2}$

5) $\cot \theta = \dfrac{13.2}{7.1}$

6) $\csc \alpha = \dfrac{15}{13.2}$

7) $\sec \alpha = \dfrac{15}{7.1}$

8) $\cot \alpha = \dfrac{7.1}{13.2}$

Express as a decimal.

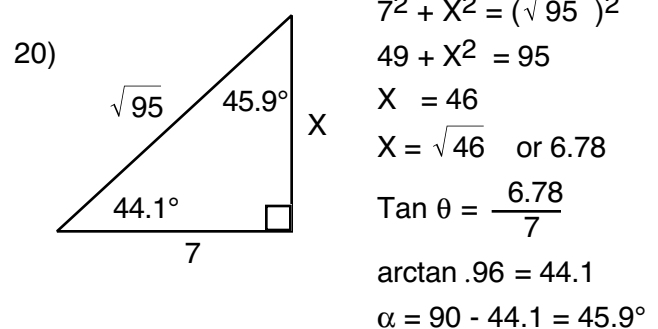**9)** $\sin \theta = \dfrac{7.1}{15} = .4733$

10) $\cos \theta = \dfrac{13.2}{15} = .88$

11) $\tan \theta = \dfrac{7.1}{13.2} = .5379$

12) $\sin \alpha = \dfrac{13.2}{15} = .88$

13) $\cos \alpha = \dfrac{7.1}{15} = .4733$

14) $\tan \alpha = \dfrac{13.2}{7.1} = 1.8592$

15) The measure of θ is 28.25°.

16) The measure of θ is 61.75°.

Solve for the lengths of the sides and the measures of the angles.

17)

$$\text{Sin } 40° = \frac{R}{25}$$

$$25 \text{ Sin } 40° = R$$

$$R = 16.07$$

$$\text{Cos } 40° = \frac{S}{25}$$

$$25 \text{ Cos } 40° = S$$

$$S = 19.15$$

18)

$$\text{Tan } 53.8° = \frac{U}{88}$$

$$88 \text{ Tan } 53.8° = U$$

$$U = 120.24$$

$$\text{Sin } 36.2° = \frac{88}{T}$$

$$T = \frac{88}{\text{Sin } 36.2°}$$

$$T = 149.00$$

19)

$$\text{Tan } 38.1° = \frac{W}{150}$$

$$150 \text{ Tan } 38.1° = W$$

$$W = 117.62$$

$$\text{Sin } 51.9° = \frac{150}{V}$$

$$V = \frac{150}{\text{Sin } 51.9°}$$

$$V = 190.61$$

20)

$$7^2 + X^2 = (\sqrt{95})^2$$

$$49 + X^2 = 95$$

$$X = 46$$

$$X = \sqrt{46} \quad \text{or } 6.78$$

$$\text{Tan } \theta = \frac{6.78}{7}$$

$$\text{arctan } .96 = 44.1$$

$$\alpha = 90 - 44.1 = 45.9°$$

1)

Tan 15.5° = $\dfrac{M}{12.88}$

M = 3.57

15.5°

12.88 mi.

2)

Tan 64.75° = $\dfrac{B}{60.7}$

B = 128.7'

B

64.75°

60.7"
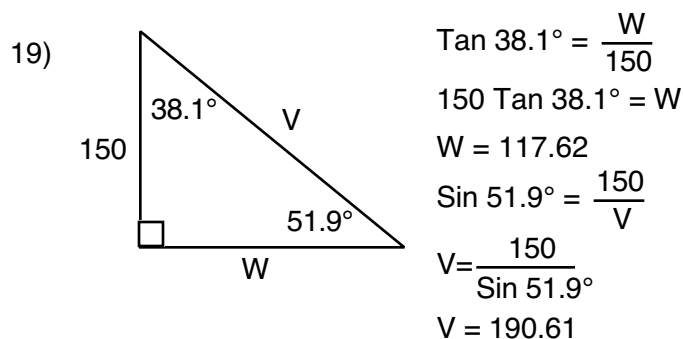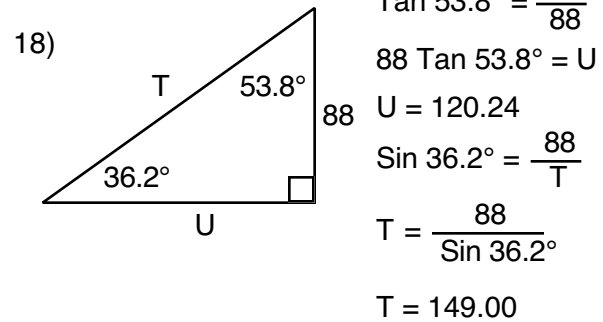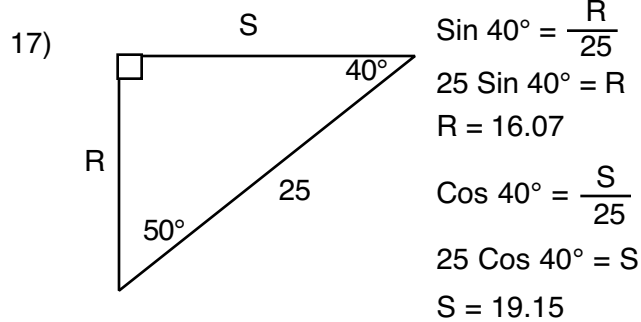
Sample Teacher Manual Page

$(18.33)^2 + X^2 = 25^2$

$336 + X^2 = 625$

$X^2 = 625$

$X = 17$

25

α

X = 17

θ

18.33

Express as a fraction.

3) csc θ = $\dfrac{25}{17}$

4) sec θ = $\dfrac{25}{18.33}$

5) cot θ = $\dfrac{18.33}{17}$

6) csc α = $\dfrac{25}{18.33}$

7) sec α = $\dfrac{25}{17}$

8) cot α = $\dfrac{17}{18.33}$

Express as a decimal.

9) sin θ = $\dfrac{17}{25}$ = .68

10) cos θ = $\dfrac{18.33}{25}$ = .7332

11) tan θ = $\dfrac{17}{18.33}$ = .9274

12) sin α = $\dfrac{18.33}{25}$ = .7332
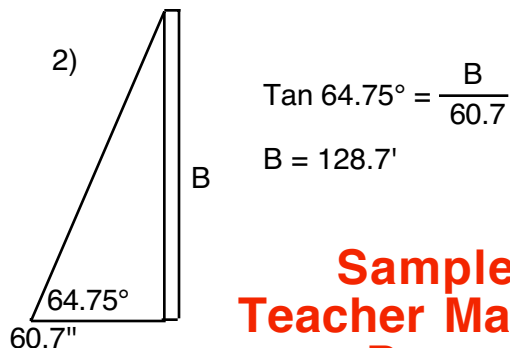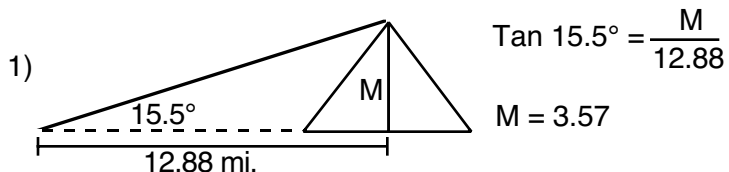
13) cos α = $\dfrac{17}{25}$ = .68

14) tan α = $\dfrac{18.33}{17}$ = 1.0782

15)   The measure of θ is 42.84°.

16)   The measure of θ is 47.16°.

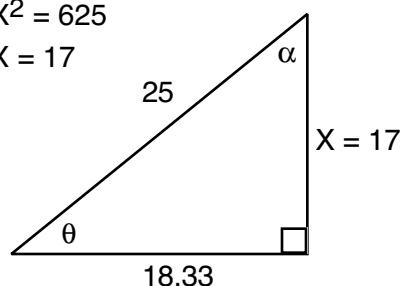*Solve for the lengths of the sides and the measures of the angles.*

17)

2.24

41.81°

2

Y = 3

48.19°

$2^2 + (2.24)^2 = Y^2$

3 = Y

Cos θ = $\dfrac{2}{3}$ = .6667

arccos .6667 = 48.19°

α = 90 - 48.19 = 41.81°

18)

10 .5

40.8°

Z

49.2°

A

Tan 40.08° = $\dfrac{A}{10.5}$

10.5 Tan 40.08° = A

9.06 = A

Sin 49.2° = $\dfrac{10.5}{Z}$

Z = $\dfrac{10.5}{Sin\ 49.2°}$

Z = 13.87

19)

29.07°

56

B

60.93°

C

Tan 29.07° = $\dfrac{C}{56}$

56 Tan 29.07° = C

31.13 = C

$(56)^2 + (31.13)^2 = B^2$

64.07 = B

20)

α

D

θ

10

14

Tan θ = $\dfrac{10}{14}$

arctan .7143 = 35.54°

α = 90 - 35.54 = 54.46°

$10^2 + 14^2 = D^2$

17.2 = D