Lesson 12 Quadratic Formula

A Quadratic is an equation that has an unknown or variable, raised to the second power, as in Y^2 or A^2. In factoring and completing the square, we have been dealing exclusively with quadratic equations. So far we can find the solution to a quadratic equation by factoring it, or if this fails, by completing the square. In this lesson we are going to discover a formula to solve all quadratics, by completing the square with variables. If you've mastered the previous lesson, try solving the following equation by completing the square, then compare your solution with mine.

$$AX^2 + BX + C = 0$$

Divide by the coefficient of X^2.

$$\frac{AX^2}{A} + \frac{BX}{A} + \frac{C}{A} = 0$$

$$X^2 + \frac{BX}{A} + \frac{C}{A} = 0$$

Add the inverse of the third term to both sides.

$$X^2 + \frac{BX}{A} = -\frac{C}{A}$$

Take 1/2 of the coefficient of the middle term, square it, and add it to both sides.

$$X^2 + \frac{BX}{A} + \left(\frac{B}{2A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{2A}\right)^2$$

Factor the left side.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

Combine terms on the right.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{4AC}{4A^2} + \frac{B^2}{4A^2}$$

Take the square root of both sides.

$$\sqrt{\left(X + \frac{B}{2A}\right)^2} = \pm\sqrt{-\frac{4AC}{4A^2} + \frac{B^2}{4A^2}}$$

$$X + \frac{B}{2A} = \pm\sqrt{\frac{-4AC + B^2}{4A^2}} = \frac{\pm\sqrt{-4AC + B^2}}{2A}$$

Subtract B/2A from both sides, and combine.

$$X = -\frac{B}{2A} \pm \frac{\sqrt{B^2 - 4AC}}{2A}$$

The quadratic formula!

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 1

Let's try one that we know the answer to by factoring, and "plug in" the values for A, B, & C. Remember, to find A, B, & C, you have to be in the form $AX^2 + BX + C = 0$.

$$X^2 + 5X + 6 = 0$$

$A = 1$, $B = 5$, & $C = 6$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$X = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$X = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

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We can also solve $X^2 + 5X + 6 = 0$ by factoring.

$$X^2 + 5X + 6 = 0$$
$$(X+2)(X+3) = 0$$
$$X+2 = 0 \qquad X+3 = 0$$
$$X = -2 \qquad X = -3$$

For this problem it would have much easier to solve by factoring. Try factoring first and if it doesn't work, then use the quadratic formula. Here is another problem to try.

Example 2 Find the factors of $2X^2 = -7X - 4$

To find A, B, & C, you have to be in the form $AX^2 + BX + C = 0$.

$$2X^2 + 7X + 4 = 0$$

$A = 2$, $B = 7$, & $C = 4$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$X = \frac{-7 \pm \sqrt{49 - 32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$$

$$X = \frac{-7 \pm \sqrt{17}}{4}$$

$$X = \frac{-7 + \sqrt{17}}{4} \text{ or } X = \frac{-7 - \sqrt{17}}{4}$$

Practice Problems *Solve for X. First try factoring, then use the quadratic formula if necessary.*

1) $X^2 - 25 = 0$

2) $X^2 + 5 = -3X$

3) $2X^2 + 7X + 6 = 0$

4) $3X^2 + X - 4 = 0$

5) $4A^2 - 36 = 0$

6) $X^2 - 18X = -81$

7) $7X^2 = -2X + 1$

8) $2X^2 + 2X - 5 = 0$

9) $\frac{5}{X+3} + \frac{2}{X-3} = 5 \quad (X \neq \pm 3)$

10) $4X^2 = 9$

11) $3Q^2 = -4Q - 2$

12) $4X^2 + 20X = -25$

Solutions

1) $(X+5)(X-5) = 0$
$X+5=0 \quad X-5=0$
$X = 5, -5$

2) $X = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$
$X = \frac{-3 \pm \sqrt{-11}}{2}$
$X = \frac{-3 + \sqrt{-11}}{2}$ or $X = \frac{-3 - \sqrt{-11}}{2}$

3) $(2X+3)(X+2) = 0$
$2X+3=0 \quad X+2=0$
$X = -3/2, -2$

4) $(3X+4)(X-1) = 0$
$3X+4=0 \quad X-1=0$
$X = -4/3, 1$

5) $(2A-6)(2A+6) = 0$
$2A-6=0 \quad 2A+6=0$
$X = 3, -3$

6) $(X-9)(X-9) = 0$
$X-9=0 \quad X-9=0$
$X = 9$

7) $X = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot -1}}{2 \cdot 7}$
$X = \frac{-2 \pm 4\sqrt{2}}{14}$
$X = \frac{-1 + 2\sqrt{2}}{7}$ or $X = \frac{-1 - 2\sqrt{2}}{7}$

8) $X = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2}$
$X = \frac{-2 \pm 2\sqrt{11}}{4}$
$X = \frac{-1 + \sqrt{11}}{2}$ or $X = \frac{-1 - \sqrt{11}}{2}$

9) $(X^2 - 9)\left(\frac{5}{X+3} + \frac{2}{X-3}\right) = 5$
$5(X-3) + 2(X+3) = 5(X^2 - 9)$
$7X - 9 = 5X^2 - 45$
$5X^2 - 7X - 36 = 0$
$X = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 5 \cdot -36}}{2 \cdot 5}$
$X = \frac{7 \pm \sqrt{769}}{10}$
$X = \frac{7 + \sqrt{769}}{10}$ or $X = \frac{7 - \sqrt{769}}{10}$

10) $(2X-3)(2X+3) = 0$
$2X-3=0 \quad 2X+3=0$
$X = 3/2, -3/2$

11) $X = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$
$X = \frac{-2 + i\sqrt{2}}{3}$ or $X = \frac{-2 - i\sqrt{2}}{3}$

12) $(2X+5)(2X+5) = 0$
$2X+5=0 \quad 2X+5=0$
$X = -5/2$

Find the roots, using the quadratic equation when necessary.

1) $X^2 - 5X + 6 = 0$

2) $X^2 + 4X + 2 = 0$

3) $X^2 - 3X + 1 = -6X$

4) $X^2 + 4X - 12 = 0$

5) $2X^2 + 2X + 5 = 0$

6) $X^2 + 8X = -16$

Complete the square.

7) $X^2 - 26X +$ _____

8) $2X^2 + 9X +$ _____

9) $X^2 +$ ____ $+ 400$

10) $X^2 -$ ____ $+ 14$

Solve for X by completing the square, if necessary.

11) $X^2 + 1/3X - 4/3 = 0$

12) Check the answer to #11 by placing it in the original equation.

13) Expand $(X - A)^6$.

14) What is the second term of $(1/2X - 3A)^4$?

15) Expand $(5 - 2A)^3$.

16) Find the cube root of $X^3 - 6X^2Y + 12XY^2 - 8Y^3$

Put in standard form.

17) $\dfrac{6 + 5i}{3i - 2}$

18) $\dfrac{2 + \sqrt{-49}}{2 - \sqrt{-49}}$

Simplify, and combine like terms when possible.

19) $\dfrac{2}{3 - \sqrt{7}}$

20) $\dfrac{2 + \sqrt{5}}{2\sqrt{5} - 4}$

Find the roots, using the quadratic equation when necessary.

1) $2X^2 - 9X - 7 = 0$

2) $X^2 + 5X - 2 = 0$

3) $3X^2 + 7X + 4 = 0$

4) $X^2 - 6X + 12 = 0$

5) $5X^2 - 3X - 2 = 0$

6) $4X^2 + 1 = 4X$

Complete the square.

7) $X^2 + 5X +$ _____

8) $X^2 - 1/2X +$ _____

9) $25X^2 +$ ____ $+ 1$

10) $49X^2 -$ ____ $+ 4$

Solve for X by completing the square, if necessary.

11) $X^2 - 12X + 20 = 0$

12) Check the answer to #11 by placing it in the original equation.

13) Expand $(X + 1)^4$.

14) What is the fifth term of $(1/2X - 3A)^4$?

15) Expand $(10 - 1/X)^3$.

16) Find the cube root of $X^3 + 6X^2 + 12X + 8$

Put in standard form.

17) $\dfrac{4 - 3i}{2i}$

18) $\dfrac{10 + \sqrt{\square A}}{10 \square \sqrt{\square A}}$

Simplify, and combine like terms when possible.

19) $\dfrac{9}{7 + \sqrt{10}}$

20) $\dfrac{4 \square \sqrt{6}}{3\sqrt{7} + 5}$

Find the roots, using the quadratic equation when necessary.

1) $X^2 + 2X - 8 = 0$

2) $X^2 - 6X = -8$

3) $2X^2 - 15X + 7 = 0$

4) $3X^2 + 4X = 7$

5) $2 = 5X + X^2$

6) $X^2 + 2X - 15 = 0$

Complete the square.

7) $4X^2 + 28X +$ _____

8) $9X^2 - 36X +$ _____

9) $36X^2 +$ ____ $+ 25$

10) $81X^2 -$ ____ $+ 121$

Solve for X by completing the square, if necessary.

11) $X^2 + 5X - 14 = 0$

12) Check the answer to #11 by placing it in the original equation.

13) Expand $(2X + 1)^5$.

14) What is the third term of $(1/3X + 2)^5$?

15) Expand $(X - 3/5)^3$.

16) Find the cube root of $8X^3 + 12X^2 + 6X + 1$

Put in standard form.

17) $\dfrac{10 + i}{5i}$

18) $\dfrac{10}{5 \square \sqrt{8}}$

Simplify, and combine like terms when possible.

19) $\dfrac{2 + 3\sqrt{6}}{1 \square \sqrt{6}}$

20) $\dfrac{6 \square \sqrt{2}}{10\sqrt{3} \square 8}$

Find the roots, using the quadratic equation when necessary.

1) $9X^2 + 4 = 12X$

2) $2X^2 + 7X + 6 = 0$

3) $16X^2 + 9 = 24X$

4) $X^2 - 6X = -1$

5) $4X^2 + 20X = -25$

6) $2X^2 - 3X = 5$

Complete the square.

7) $25X^2 - 80X +$ _____

8) $36X^2 + 60X +$ _____

9) $X^2Y^2 -$ ____ $+ 1/4$

10) $4X^2 -$ ____ $+ 1/25$

Solve for X by completing the square, if necessary.

11) $3X^2 - 4X - 2 = 0$

12) Check the answer to #11 by placing it in the original equation.

13) Expand $(X - 4)^5$.

14) What is the fourth term of $(1/3X + 2)^5$?

15) Expand $(2X + 9)^3$.

16) Find the cube root of $X^3 - 3/5X^2 + 3/25X - 1/125$

Put in standard form.

17) $\dfrac{6 - 2i}{9i}$

18) $\dfrac{5 + \sqrt{7}}{\sqrt{7}}$

Simplify, and combine like terms when possible.

19) $\dfrac{X}{\sqrt{X} \square \sqrt{4}}$

20) $\dfrac{\sqrt{XA}}{2\sqrt{A} + 3\sqrt{X}}$

Sample Test Booklet Page

1) Which of the following can not be solved using the quadratic equation?

A) $X^2 - 64 = 0$ B) $X^3 + 3Y + 1 = 0$

C) $4A^2 + 8A = 16$ D) $Y^2 = 2Y + 4$

2) The part of the quadratic formula written under the radical is

A) $B^2 + 4AC$ B) $B^2 - 4AC$

C) $-B^2 \pm 4AC$ D) $A^2 + 4BC$

3) All quadratic equations can be solved by

A) factoring B) both factoring and the quadratic formula

C) the quadratic formula D) none of the above

4) In order to find values of A, B, and C in the quadratic formula, an equation should be in the form

A) $AX^2 = BX + C$ B) $X^2 + AX = B - C$

C) $AX^2 + BX + C = 0$ D) $AX^2 + BX = -C$

5) The solution to $7X^2 + 2X - 1 = 0$ can be written as

A) $X = \frac{-2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$

B) $X = \frac{2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$

C) $X = \frac{-2 \pm \sqrt{2^2 + (4)(7)(-1)}}{2(7)}$

D) $X = \frac{-2 \pm \sqrt{(-2)^2 - (4)(7)(1)}}{2}$

For 6-10, solve using the best method.

6) $X^2 - 36 = 0$

A) X = 6, -6 B) X = 4, 9

C) X = 0, 6 D) $X = \pm 9$

7) $X^2 + 3 = -3X$

A) $X = \frac{-3 \pm \sqrt{3}}{2}$ B) $X = \frac{-3 \pm i\sqrt{3}}{6}$

C) $X = \frac{3 \pm i\sqrt{3}}{2}$ D) $X = \frac{-3 \pm i\sqrt{3}}{2}$

8) $5X^2 = -2X + 1$

A) $X = \frac{-1 \pm \sqrt{5}}{5}$ B) $X = \frac{-1 \pm \sqrt{6}}{5}$

C) $X = \frac{-1 \pm 2\sqrt{6}}{5}$ D) $X = \frac{1 \pm \sqrt{5}}{5}$

9) $4X^2 + 20X = -25$

A) $X = \pm 5/2$ B) X = 4, 5

C) X = 5/2 D) X = -5/2

10) $4X^2 + 4X - 10 = 0$

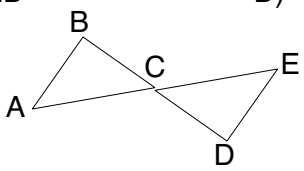
A) $X = \frac{-1 \pm i\sqrt{11}}{2}$ B) X = i, -2i

C) $X = \frac{-1 \pm \sqrt{11}}{2}$ D) $X = \frac{-1 \pm 3i}{2}$

11) Δ ABC is congruent to Δ EDC. $\overline{AB}$ corresponds to

A) $\overline{BA}$ B) $\overline{AC}$

C) $\overline{ED}$ D) $\overline{BC}$



12) A quadrilateral with only one pair of parallel sides is a

A) rhombus B) trapezoid

C) parallelogram D) regular polygon

13) Two sides of triangle A are congruent to the corresponding sides of triangle B. The angle formed by the corresponding sides is 25° in both triangles. What postulate may be used to prove triangles A and B congruent?

A) SSS B) SSA

C) SAS D) cannot be proved congruent

14) Each angle of triangle ABC is congruent to the corresponding angle of triangle DEF. What postulate may be used to prove Δ ABC and Δ DEF congruent?

A) SSS B) AAA

C) SAS D) cannot be proved congruent

15) Five yards are a little less than

A) 5 meters B) 10 meters

C) 2 meters D) 6 meters

1) $(X - 3)(X - 2) = 0$
$X = 3 \qquad X = 2$

2) $\frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$

3) $X^2 + 3X + 1 = 0$

$\frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-3 \pm \sqrt{5}}{2}$

4) $(X + 6)(X - 2) = 0$
$X = -6 \qquad X = 2$

5) $\frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2} = \frac{-2 \pm \sqrt{-36}}{4} =$
$\frac{-2 \pm 6i}{4} = \frac{-1 \pm 3i}{2}$

6) $X^2 + 8X + 16 = 0$
$(X + 4)(X + 4) = 0 \qquad X = -4$

7) 169

8) $\frac{2X^2}{2} + \frac{9X}{2} + \_\_\_\_$

$X^2 + \frac{9X}{2} + \frac{81}{16}$

9) $40X$

10) $2\sqrt{14}\ X$

11) $X^2 + 1/3\ X - 4/3 = 0$
$X^2 + 1/3\ X + 1/36 = 4/3 + 1/36$
$(X + 1/6)^2 = 48/36 + 1/36$
$\sqrt{(X + 1/6)^2} = \sqrt{49/36}$
$X + 1/6 = \pm 7/6$
$X = -1/6 + 7/6 \qquad X = -1/6 - 7/6$
$X = 6/6 = 1 \qquad X = -8/6 = -4/3$

12) $1^2 + 1/3(1) - 4/3 = 0$
$4/3 - 4/3 = 0$
$(-4/3)^2 + 1/3(-4/3) - 4/3 = 0$
$16/9 - 4/9 - 12/9 = 0$

13) $X^6 + 6X^5(-A) + 15X^4(-A)^2 + 20X^3(-A)^3 +$
$15X^2(-A)^4 + 6X(-A)^5 + (-A)^6 =$
$X^6 - 6X^5A + 15X^4A^2 - 20X^3A^3 +$
$15X^2A^4 - 6XA^5 + A^6$

14) $4/1(1/2X)^3(-3A)^1 = 4(1/8X)^3(-3A) =$
$-3/2\ X^3A$

15) $5^3 + 3(5)^2(-2A) + 3(5)(-2A)^2 + (-2A)^3 =$
$125 - 150A + 60A^2 - 8A^3$

16) $(X - 2Y)^3$

17) $\frac{(6 + 5i)(3i + 2)}{(3i - 2)(3i + 2)} =$

$\frac{18i + 15i^2 + 12 + 10i}{9i^2 - 4} = \frac{28i - 3}{-13}$

18) $\frac{(2 + \sqrt{-49})(2 + \sqrt{-49})}{(2 - \sqrt{-49})(2 + \sqrt{-49})}$

$\frac{4 + 4\sqrt{49} - 49}{4 - (-49)} = \frac{-45 + 4 \cdot 7i}{53} =$

$\frac{-45 + 28i}{53}$

19) $\frac{2(3 + \sqrt{7})}{(3 - \sqrt{7})(3 + \sqrt{7})} = \frac{6 + 2\sqrt{7}}{9 - 7} =$

$\frac{6 + 2\sqrt{7}}{2} = 3 + \sqrt{7}$

20) $\frac{(2 + \sqrt{5})(2\sqrt{5} + 4)}{(2\sqrt{5} - 4)(2\sqrt{5} + 4)} =$

$\frac{4\sqrt{5} + 8 + 2 \cdot 5 + 4\sqrt{5}}{4 \cdot 5 - 16} =$

$\frac{8\sqrt{5} + 18}{4} = \frac{4\sqrt{5} + 9}{2}$

1) $\frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-7)}}{2(2)} = \frac{9 \pm \sqrt{137}}{4}$

2) $\frac{-5 \pm \sqrt{5^2 - 4(1)(-2)}}{2} = \frac{-5 \pm \sqrt{33}}{2}$

3) $(3X + 4)(X + 1) = 0$
$X = -4/3,\ -1$

4) $\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)} = \frac{6 \pm \sqrt{-12}}{2} =$

$\frac{6 \pm 2i\sqrt{3}}{2} = 3 \pm i\sqrt{3}$

5) $(5X + 2)(X - 1) = 0$
$X = -2/5,\ 1$

6) $4X^2 - 4X + 1 = 0$
$(2X - 1)(2X - 1) = 0 \qquad X = 1/2$

7) 25/4

8) 1/16

9) $10X$

10) $28X$

11) $(X - 10)(X - 2) = 0$
$X = 10,\ 2$

12) $(10)^2 - 12(10) + 20 = 0$
$100 - 120 + 20 = 0$
$(2)^2 - 12(2) + 20 = 0$
$4 - 24 + 20 = 0$

13) $X^4 + 4X^3 + 6X^2 + 4X + 1$

14) $\frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4}\ (1/2\ X)^0(3A)^4 = 81A^4$

15) $10^3 + 3(10)^2(-1/X) + 3(10)(-1/X)^2 + (-1/X)^3$
$1000 - 300/X + 30/X^2 - 1/X^3$

16) $(X + 2)^3$

17) $\frac{(4 - 3i)(i)}{(2i)(i)} = \frac{4i - 3i^2}{2i^2} =$

$\frac{4i + 3}{-2}$

18) $\frac{(10 + \sqrt{-A})(10 + \sqrt{-A})}{(10 - \sqrt{-A})(10 + \sqrt{-A})}$

$\frac{100 + 20i\sqrt{A} - A}{100 - (-A)} =$

$\frac{100 + 20i\sqrt{A} - A}{100 + A}$

19) $\frac{(9)(7 - \sqrt{10})}{(7 + \sqrt{10})(7 - \sqrt{10})} = \frac{63 - 9\sqrt{10}}{49 - \sqrt{10}} =$

$\frac{63 - 9\sqrt{10}}{39} = \frac{21 - 3\sqrt{10}}{13}$

20) $\frac{(4 - \sqrt{6})(3\sqrt{7} - 5)}{(3\sqrt{7} + 5)(3\sqrt{7} - 5)} =$

$\frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{9(7) - 25} =$

$\frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{38}$