Lesson 20 Factor Polynomials

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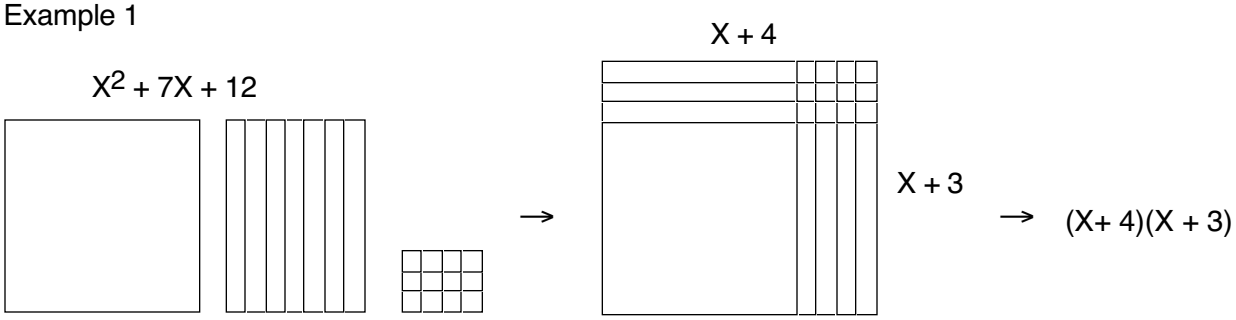
Factoring Trinomials We will be finding the factors of $X^2 + 7X + 12$ using the blocks with the algebra inserts snapped into the back. This is the opposite of multiplying two binomials to find the product, which is a trinomial. In Lesson 19 you were given the factors, and you were to find the product. Now, you are given the product and are asked to find the factors.

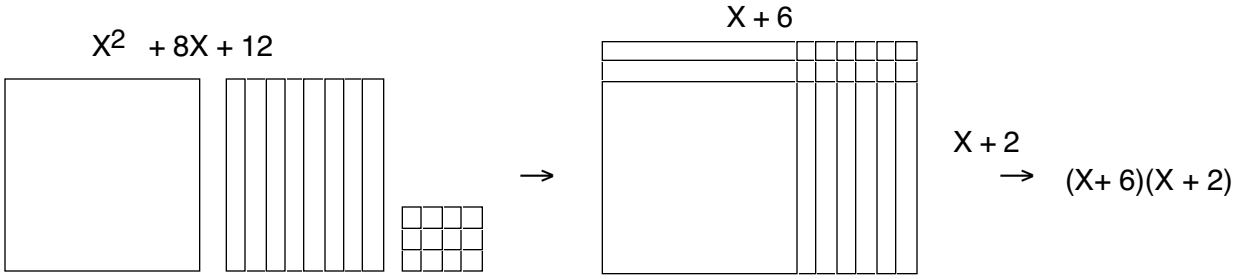
First build $X^2 + 7X + 12$. This is the product, which is given. Now build a rectangle using all the blocks. Then find the factors by reading the length of the over dimension and the up dimension.

Example 1



Example 2

Now find the factors of $X^2 + 8X + 12$. Represent with the manipulatives, build a rectangle, and read the factors.



Notice the relationship between the last term (12), the middle term (7X or 8X), and the factors. This always works when the coefficient of X^2 is 1.

$X^2 + 7X + 12 =$ $(X+ 4)(X + 3)$ The last term is found by multiplying 3 x 4. The middle term by adding 3X + 4X.

$X^2 + 8X + 12 =$ $(X+ 6)(X + 2)$ The last term is found by multiplying 6 x 2. The middle term by adding 6X + 2X.

The factors of the last term are the addends of the middle term.

More on Multiplying Polynomials Polynomials may be multiplied vertically (A), or horizontally (B), using the distributive property.

Example 3

A)
$$\begin{array}{r} 2X + 3 \\ X + 2 \\ \hline 4X + 6 \\ 2X^2 + 3X \\ \hline 2X^2 + 7X + 6 \end{array}$$

B) $(X + 2)(2X + 3) = (X)(2X+ 3) + (2)(2X+ 3) = (2X^2 + 3X) + (4X + 6)$

$2X^2 + 7X + 6$

When multiplying horizontally, there are four partial products just as before, but they are arrived at using a formula called FOIL: F - first, O - outside, I - inside, L - last. Each letter corresponds to a partial product.

F	In $(X + 2)(2X + 3)$,	$X \cdot 2X$ is the First term times the first term	$2X^2$
O	In $(X + 2)(2X + 3)$,	$X \cdot 3$ is the Outside term times the outside term	$3X$
I	In $(X + 2)(2X + 3)$,	$2 \cdot 2X$ is the Inside term times the inside term	$4X$
L	In $(X + 2)(2X + 3)$,	$2 \cdot 3$ is the Last term times the last term	6

$2X^2 + 3X + 4X + 6$

$2X^2 + 7X + 6$

Example 4

A)
$$\begin{array}{r} X + 3 \\ X + 4 \\ \hline 4X + 12 \\ X^2 + 3X \\ \hline X^2 + 7X + 12 \end{array}$$

B) $(X + 4)(X + 3) = (X)(X+ 3) + (4)(X+ 3) = (X^2 + 3X) + (4X + 12)$

$X^2 + 7X + 12$

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Build a rectangle and find the factors. Check by multiplying.

1) $X^2 + 4X + 4$

2) $X^2 + 5X + 6$

3) $X^2 + 11X + 10$

4) $X^2 + 6X + 8$

5) $X^2 + 8X + 7$

6) $X^2 + 8X + 12$

7) $X^2 + 12X + 11$

8) $X^2 + 7X + 6$

9) $X^2 + 9X + 14$

10) $X^2 + 16X + 15$

11) $X^2 + 3X + 2$

12) $X^2 + 4X + 3$

13) $X^2 + 9X + 8$

14) $X^2 + 19X + 18$

15) $X^2 + 9X + 20$

16) $X^2 + 10X + 21$

Find the factors and check by multiplying. (You will not have enough blocks to build some of these)

1) $X^2 + 10X + 16$

2) $X^2 + 11X + 28$

3) $X^2 + 13X + 22$

4) $X^2 + 7X + 12$

5) $X^2 + 8X + 15$

6) $X^2 + 11X + 30$

7) $X^2 + 5X + 4$

8) $X^2 + 6X + 5$

9) $X^2 + 8X + 16$

10) $X^2 + 12X + 20$

11) $X^2 + 11X + 18$

12) $X^2 + 17X + 30$

13) $X^2 + 7X + 10$

14) $X^2 + 2X + 1$

15) $X^2 + 10X + 25$

16) $X^2 + 26X + 25$

Build a rectangle and find the factors.

1) $X^2 + 7X + 12 = (\quad + \quad)(\quad + \quad)$

2) $X^2 + 10X + 16 = (\quad + \quad)(\quad + \quad)$

3) $X^2 + 11X + 24 = (\quad + \quad)(\quad + \quad)$

4) $X^2 + 8X + 12 = (\quad + \quad)(\quad + \quad)$

Build a rectangle and find the area (product)

5) $(X + 4)(X + 2) =$

6) $(X + 5)(X + 3) =$

7) Find the factors: $X^2 + 7X + 6$

8) Check #7 by multiplying the factors to find the product.

9) Find the factors: $X^2 + 2X + 1$

10) Check #9 by multiplying the factors to find the product.

Add.

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11) $2X^2 - 7X - 3$
$+ \; X^2 + 5X + 9$

12) $6X^2 + 2X + 1$
$+ \; X^2 - 4X + 3$

13) Simplify: $(P^{-4})^2 P^3 P^1$

14) Simplify: $(R^{-2}S^3)^{-3}$

15) $15^2 =$

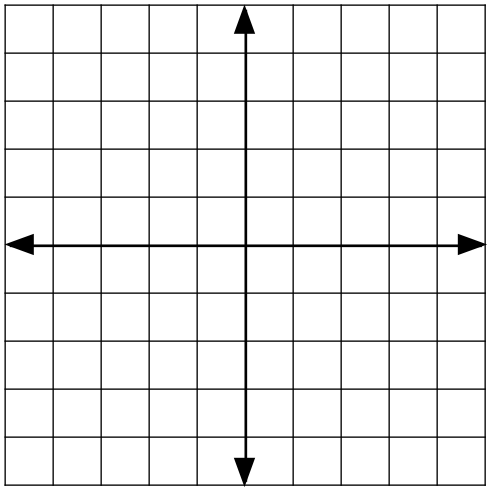
16) $\sqrt{16} =$

17) Find three consecutive odd integers such that eleven times the first, plus two times the second, equals six times the third, plus one.

18) Nine coins, made up of dimes and nickels, have a value of $.60. How many of each?

19) Express in standard form of an equation of a line: $Y = 7X + 3$

20) Graph: $4Y < 3X - 5$



Build a rectangle and find the factors.

1) $X^2 + 11X + 28 = (\quad + \quad)(\quad + \quad)$

2) $X^2 + 4X + 4 = (\quad + \quad)(\quad + \quad)$

3) $X^2 + 6X + 8 = (\quad + \quad)(\quad + \quad)$

4) $X^2 + 8X + 16 = (\quad + \quad)(\quad + \quad)$

Build a rectangle and find the area (product)

5) $(X + 5)(X + 1) =$

6) $(X + 3)(X + 3) =$

7) Find the factors: $X^2 + 12X + 32$

8) Check #7 by multiplying the factors to find the product.

9) Find the factors: $X^2 + 20X + 100$

10) Check #9 by multiplying the factors to find the product.

Add.

11)
$$\begin{array}{r} X^2 + X - 4 \\ +\ X^2 + 3X + 3 \\ \hline \end{array}$$

Sample Student Text Page

12)
$$\begin{array}{r} 2X^2 + 7X + 6 \\ +\ 5X^2 - 4X + 10 \\ \hline \end{array}$$

13) Simplify: $[\,(P^5)^3\,]^{-2}$

14) Simplify: $(S^6\,R^{-3}\,S^2)^0$

15) $11^2 =$

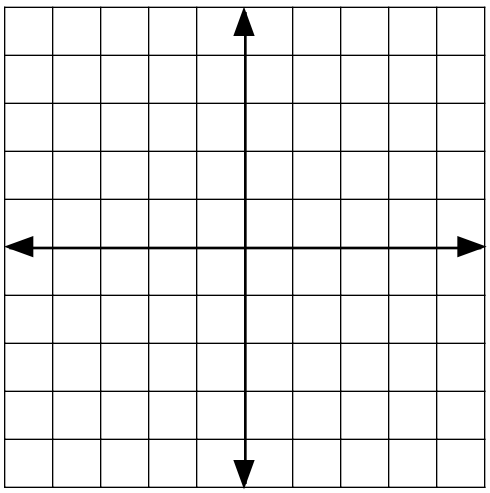
16) $\sqrt{144} =$

17) Find three consecutive odd integers such that fourteen times the second, plus four times the first, equals twelve times the third, minus two.

18) Twenty-seven coins, made up of dimes and nickels, add up to $1.80. How many of each?

19) Graph the line $Y = 3/2\,X - 1$

20) Graph a line perpendicular to #19 through (3, -3).



Build a rectangle and find the factors.

1) $X^2 + 8X + 7 = (\quad + \quad)(\quad + \quad)$

2) $X^2 + 5X + 6 = (\quad + \quad)(\quad + \quad)$

3) $X^2 + 9X + 20 = (\quad + \quad)(\quad + \quad)$

4) $X^2 + 8X + 15 = (\quad + \quad)(\quad + \quad)$

Build a rectangle and find the area (product)

5) $(X + 1)(X + 9) =$

6) $(X + 7)(X + 2) =$

7) Find the factors: $X^2 + 7X + 12$

8) Check #7 by multiplying the factors to find the product.

9) Find the factors: $X^2 + 10X + 21$

10) Check #9 by multiplying the factors to find the product.

Sample Student Text Page

Add.

11)
$$\begin{array}{r} 4X^2 - 4X + 1 \\ +\ X^2 + 2X - 1 \\ \hline \end{array}$$

12)
$$\begin{array}{r} 2X^2 + 3X + 3 \\ +\ X^2 + 7X - 2 \\ \hline \end{array}$$

13) Simplify: $(P^3)^0\, P^4 P^{-1}$

14) Simplify: $(S^2\, R^0\, S^0)^{-2} R^5$

15) $13^2 =$

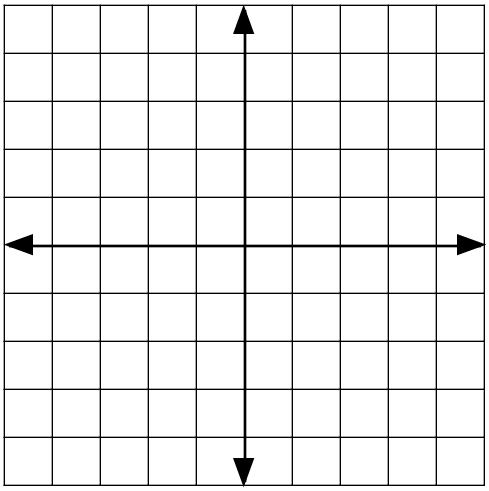
16) $\sqrt{25} =$

17) Find three consecutive integers such that the second, plus seven times the third, equals five times the first.

18) Twenty coins, made up of pennies and nickels, add up to $.76. How many of each?

19) Rewrite in slope-intercept form: $4Y + 3X = 16$

20) Graph: $2Y \geq 3X - 2$



1) If (X + A) is multiplied times (X + B), the final term of the resulting trinomial will be

(A) X^2 (B) (A + B)X (C) BX (D) AX
(E) AB

2) If (X + A) is multiplied times (X + B), the middle term of the resulting trinomial will be

(A) X^2 (B) (A + B)X (C) BX (D) AX
(E) AB

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3) The factors of $X^2 + 3X + 2$ are

(A) (X + 3)(X + 2) (B) (X + 1)(X + 2)
(C) X(X + 2) (D) (X + 5)(X + 2)
(E) (X - 1)(X + 2)

4) The factors of $X^2 + 8X + 15$ are

(A) (X + 2)(X + 4) (B) (X + 1)(X + 8)
(C) (X + 10)(X + 5) (D) (X + 7)(X + 8)
(E) (X + 3)(X + 5)

5) The factors of $X^2 + 12X + 36$ are

(A) (X + 3)(X + 4) (B) (X + 6)(X + 6)
(C) (X + 6)(X + 2) (D) (X + 18)(X + 18)
(E) (X - 6)(X + 6)

6) The factors of $X^2 + 12X + 20$ are

(A) (X + 12)(X - 20) (B) (X + 2)(X + 10)
(C) X(X + 20) (D) (X + 5)(X + 4)
(E) (X + 12)(X + 20)

7) The factors of $X^2 + 11X + 24$ are

(A) (X + 4)(X + 6) (B) (X + 2)(X + 12)
(C) (X + 3)(X + 8) (D) (X + 1)(X + 24)
(E) (X + 5)(X + 6)

8) The factors of $X^2 + 6X + 5$ are

(A) (X + 2)(X + 3) (B) (X + 1)(X + 6)
(C) X(X + 6) (D) (X + 1)(X + 5)
(E) (X + 5)(X + 6)

9) The factors of $X^2 + 14X + 49$ are

(A) (X + 7)(X + 7) (B) (X + 1)(X + 49)
(C) X(X + 7) (D) (X + 2)(X + 7)
(E) (X + 1)(X + 14)

10) The factors of $X^2 + 11X + 10$ are

(A) (X + 2)(X + 5) (B) (X + 1)(X + 10)
(C) X(X + 10) (D) (X + 1)(X + 11)
(E) (X + 5)(X + 5)

11) (A + B)(A + B) is equal to

(A) $A^2 + BA + B^2$ (B) $A^2 + 2BA + AB^2$
(C) $A^2 + 2BA + (AB)^2$ (D) $A^2 + 2BA + B^2$
(E) $A^2 + A + B + B^2$

12) (X + BY)(X + BY) is equal to

(A) $X^2 + 2BYX + BY^2$ (B) $X^2 + BYX + (BY)^2$
(C) $X^2 + 2BY + (BY)^2$ (D) $X^2 + 2BY + BY^2$
(E) $X^2 + 2BYX + (BY)^2$

13) What are the factors of $X^2 + (R + T)X + RT$?

(A) (X + X)(X + T) (B) (R + X)(T + X)
(C) (X + R)(X + T) (D) X(R + T)
(E) (R + T)(R + T)

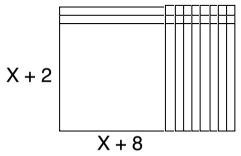
14) What are the factors of $X^2 + 2RX + R^2$?

(A) (X + 2)(X + 2R) (B) (X + R)(X + R)
(C) (X + 2R)(X + 2R) (D) X(RX + R)
(E) (R + X)(R + X)

15) Fill in the blank: The numbers that are added to get the coefficient of the middle term are the _____ of the last term.

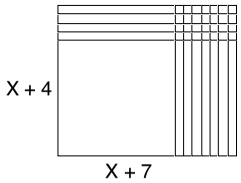
(A) exponents (B) factors (C) inverse
(D) addends (E) products

1)



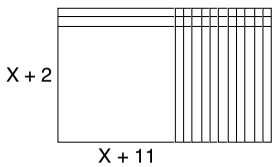
$$\begin{array}{r} X + 8 \\ \times \quad X + 2 \\ \hline 2X + 16 \\ X^2 + 8X \\ \hline X^2 + 10X + 16 \end{array}$$

2)



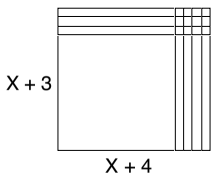
$$\begin{array}{r} X + 7 \\ \times \quad X + 4 \\ \hline 4X + 28 \\ X^2 + 7X \\ \hline X^2 + 11X + 28 \end{array}$$

3)



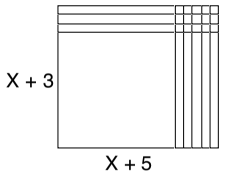
$$\begin{array}{r} X + 11 \\ \times \quad X + 2 \\ \hline 2X + 22 \\ X^2 + 11X \\ \hline X^2 + 13X + 22 \end{array}$$

4)



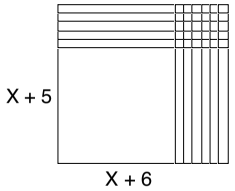
$$\begin{array}{r} X + 4 \\ \times \quad X + 3 \\ \hline 3X + 12 \\ X^2 + 4X \\ \hline X^2 + 7X + 12 \end{array}$$

5)



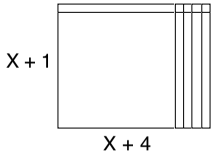
$$\begin{array}{r} X + 5 \\ \times \quad X + 3 \\ \hline 3X + 15 \\ X^2 + 5X \\ \hline X^2 + 8X + 15 \end{array}$$

6)



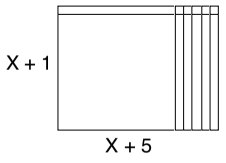
$$\begin{array}{r} X + 6 \\ \times \quad X + 5 \\ \hline 5X + 30 \\ X^2 + 6X \\ \hline X^2 + 11X + 30 \end{array}$$

7)



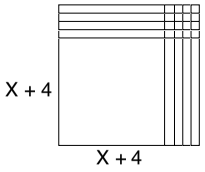
$$\begin{array}{r} X + 4 \\ \times \quad X + 1 \\ \hline X + 4 \\ X^2 + 4X \\ \hline X^2 + 5X + 4 \end{array}$$

8)



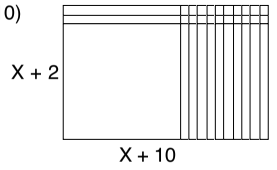
$$\begin{array}{r} X + 5 \\ \times \quad X + 1 \\ \hline X + 5 \\ X^2 + 5X \\ \hline X^2 + 6X + 5 \end{array}$$

9)



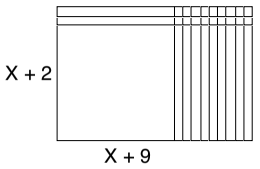
$$\begin{array}{r} X + 4 \\ \times \quad X + 4 \\ \hline 4X + 16 \\ X^2 + 4X \\ \hline X^2 + 8X + 16 \end{array}$$

10)



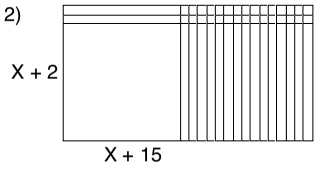
$$\begin{array}{r} X + 10 \\ \times \quad X + 2 \\ \hline 2X + 20 \\ X^2 + 10X \\ \hline X^2 + 12X + 20 \end{array}$$

11)



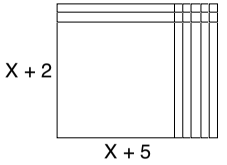
$$\begin{array}{r} X + 9 \\ \times \quad X + 2 \\ \hline 2X + 18 \\ X^2 + 9X \\ \hline X^2 + 11X + 18 \end{array}$$

12)



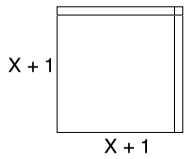
$$\begin{array}{r} X + 15 \\ \times \quad X + 2 \\ \hline 2X + 30 \\ X^2 + 15X \\ \hline X^2 + 17X + 30 \end{array}$$

13)



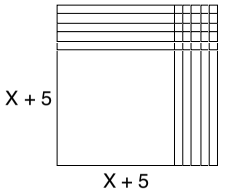
$$\begin{array}{r} X + 5 \\ \times \quad X + 2 \\ \hline 2X + 10 \\ X^2 + 5X \\ \hline X^2 + 7X + 10 \end{array}$$

14)



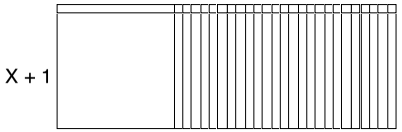
$$\begin{array}{r} X + 1 \\ \times \quad X + 1 \\ \hline X + 1 \\ X^2 + X \\ \hline X^2 + 2X + 1 \end{array}$$

15)



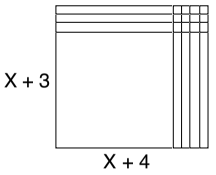
$$\begin{array}{r} X + 5 \\ \times \quad X + 5 \\ \hline 5X + 25 \\ X^2 + 5X \\ \hline X^2 + 10X + 25 \end{array}$$

16)

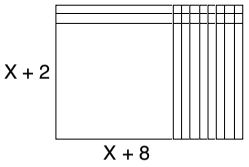


Check using same method as other examples.

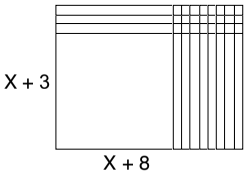
1) $X^2 + 7X + 12 = (X + 4)(X + 3)$



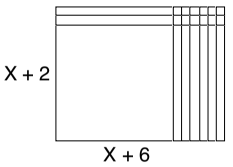
2) $X^2 + 10X + 16 = (X + 8)(X + 2)$



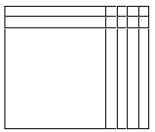
3) $X^2 + 11X + 24 = (X + 8)(X + 3)$



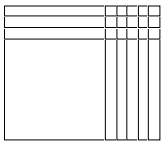
4) $X^2 + 8X + 12 = (X + 6)(X + 2)$



5) $(X + 4)(X + 2) = X^2 + 6X + 8$



6) $(X + 5)(X + 3) = X^2 + 8X + 15$



Sample Teacher Manual Page

7) $X^2 + 7X + 6 = (X + 6)(X + 1)$

8)
$$\begin{array}{r} X + 6 \\ \times \quad X + 1 \\ \hline X + 6 \\ X^2 + 6X \\ \hline X^2 + 7X + 6 \end{array}$$

9) $X^2 + 2X + 1 = (X + 1)(X + 1)$

10)
$$\begin{array}{r} X + 1 \\ \times \quad X + 1 \\ \hline X + 1 \\ X^2 + X \\ \hline X^2 + 2X + 1 \end{array}$$

11)
$$\begin{array}{r} 2X^2 - 7X - 3 \\ X^2 + 5X + 9 \\ \hline 3X^2 - 2X + 6 \end{array}$$

12)
$$\begin{array}{r} 6X^2 + 2X + 1 \\ X^2 - 4X + 3 \\ \hline 7X^2 - 2X + 4 \end{array}$$

13) $P^{-8}P^4 = P^{-4}$

14) $R^{(-2)(-3)}S^{(3)(-3)} = R^6S^{-9}$

15) 225

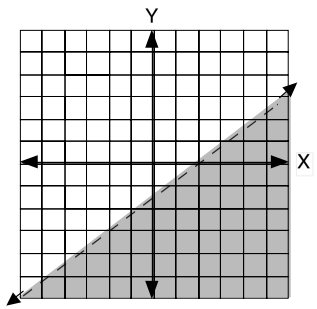
16) ± 4

17) $11N + 2(N + 2) = 6(N + 4) + 1$
$11N + 2N + 4 = 6N + 25$
$7N = 25 - 4$
$N = 3$ 3, 5, 7

18) $.10D + .05N = .60$, $D + N = 9$

$10D + 5N = 60$
$-5D - 5N = -45$
$5D = 15$ $(3) + N = 9$
$D = 3$ $N = 6$

19) $7X - Y = -3$



20) $Y < 3/4\ X - 5/4$