Life of Fred

Geometry

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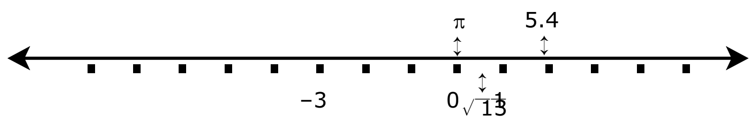
Note to Students

One day in the life of Fred. A Thursday just after his sixth birthday. In those few hours you will experience a full course in geometry. Everything's here: the formulas, the definitions, the theorems, the proofs, and the constructions.

WHAT YOU'LL NEED TO HAVE

You'll need to know a little algebra before you start this Thursday with Fred—not a whole lot. In geometry we will talk about some of the numbers on the real number line.



the real number line

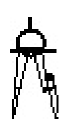
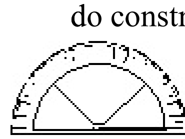
On p. 26 we will go from y + y = z to y = ½ z.

On p. 78 we will have three equations that look something like:

$$a = b$$
$$a + c = 180$$
$$b + d = 180$$

and then we arrive at c = d. If you can figure out on scratch paper how those first three equations can yield c = d, your algebra background is probably just fine.

Besides a bit of algebra, you'll need the usual supplies for geometry:

✓ paper and a pencil
✓ a ruler
✓ a compass  (for use in chapter 11 when we do constructions)
✓ maybe a protractor  (to measure angles)

✓ maybe your old hand-held calculator with +, –, ×, ÷, and √‾ keys on it.

The paper and pencil and ruler you probably already have. A compass may cost you a dollar or two. If you're superrich and you want to blow another buck for a protractor, that's your choice. You won't really need one for this course.

WHAT YOU'LL NEED TO DO

Throughout this book are sections called *Your Turn to Play*, which are opportunities for you to interact with the geometry. Complete solutions are given for all the problems in the *Your Turn to Play* sections. If you really want a solid grounding in geometry, just reading the problems and the solutions without working them out for yourself really won't work.

At the end of each chapter are six sets of questions, each set named after a city in the United States. You may not have heard of some of the cities such as Elmira or Parkdale, but they all exist. Answers are supplied to half of the questions in the Cities at the end of each chapter.

If you would like to learn geometry, the general rule is easy: Personally work out everything for which a solution is supplied. That's all of the *Your Turn to Play* and half of the Cities questions.

WHAT GEOMETRY OFFERS YOU

In the usual sequence of the study of mathematics—

arithmetic
beginning algebra
Geometry
advanced algebra
trig
calculus—

there is one course that is different from all the rest. In five out of these six courses, the emphasis is on calculating, manipulating and computing answers.

In arithmetic, you find what 6% of $1200 is.

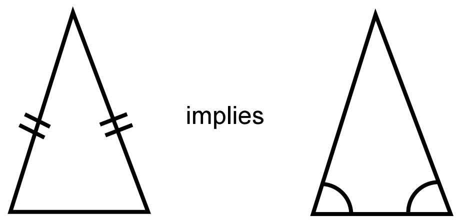
In beginning algebra, you solve $\frac{x^2}{x+2} = \frac{8}{3}$

In advanced algebra, you use logs to find the answer to: *If my waistline grows by 2% each year, how long will it be before my waist is* *one-third larger than it is now?* [Taken from a Cities problem at the end of chapter three in *Life of Fred: Advanced Algebra*.]

In trigonometry, you find the five different answers to $x^5 = 1$.

In calculus, you find the arc length of the curve $y^2 = 4x^3$ from $x = 0$ to $x = 2$. [From a Cities problem at the end of chapter 17 in *LOF: Calculus*. The answer, in case you're wondering, is $(2/27)(19^{3/2} - 1)$.]

In contrast, in geometry there are proofs to be created. It is much more like solving puzzles than grinding out numerical answers. For example, if you start out with a triangle that has two sides of equal length, you are asked to show that it has two angles that have the same size.



There are at least four different ways to prove that this is true. The proof that you create may be different than someone else's. Things in geometry are much more creative than in the other five courses. It was because of the fun I experienced in geometry that I decided to become a mathematician.

The surprising (and delightful) truth is that geometry is much more representative of mathematics than are arithmetic, beginning algebra, advanced algebra, trig, or calculus. Once you get beyond all the "number stuff" of those five courses, math becomes a playground like geometry. On p. 209 I describe eight (of the many) math courses which follow calculus. All of them have the can-you-show-that spirit that geometry has.

I wish you the best of luck in your adventures in geometry.

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Chapter One

Points and Lines

Fred's two little eyes popped open. It was several minutes before dawn on an early spring morning. He awoke with a smile. There were so many things for which to be grateful. He ticked them off in his mind one by one.

✔ He was home, safe and sound.

✔ It was Thursday—one of his seven favorite days of the week.

✔ He had received a wonderful pet llama at his sixth birthday party last night.

✔ His math teaching position at KITTENS University.

What a wonderful life! was his morning prayer. He stood up and stretched to his full height of thirty-six inches. Fred's home for the last five years or so was his office on the third floor of the math building at the university. By most standards he had been quite young when he first came to KITTENS.

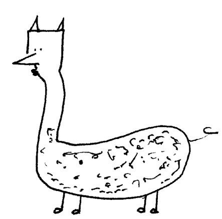
Years ago, when he had arrived at the school, they had assigned him his office. He had never had a room of his own before. He was so tuckered out from all the newness in his life (a new job, a new state to live in, a new home) that he had just closed the door to his office and found a nice cozy spot (under his desk) and had taken a nap.

And every night since then, that is where he slept.

He had the world's shortest commute to work. And no expenses for an apartment or a car.

"Good morning Lambda!" he said to his llama. He had named her Lambda in honor of the Greek letter lambda (λ). He enjoyed the alliteration of "Lambda the llama."

She was busy chewing on the wooden fence that he had erected in his office.

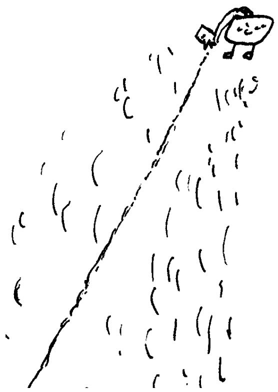


λ

"I just thought you'd like to get out and get some exercise with me," he continued. Fred hopped into his jogging clothes. The pair headed down the two flights of stairs and out into an icy Kansas morning.

Fred was worried that his new pet wouldn't be able to keep up with him as he jogged. He had been jogging for years and a ten-mile run was nothing for him. Fred's fears, however, were unfounded. His six-foot-tall llama had no trouble matching the pace of Fred's little legs.

In fact, after a few minutes, Lambda spotted the new bocci ball lawn and raced ahead to enjoy some breakfast. By the time Fred caught up to her, she had mowed a straight line right across the lawn. Fred's little snacks that he had given her last night had left her hungry.

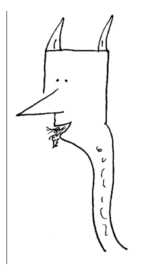


Oh no! Fred thought. Larry is gonna be mad. He put a lot of effort into that lawn. The international bocci ball tournament is scheduled to be here next week.

"Lambda, please come here. You're not supposed to be on Mr. Wistrom's lawn."

She, being a good llama, obeyed. On her way back to Fred she munched a second line in the grass. Fred loved his pet and didn't know that he was supposed to discipline her. Besides, he thought to himself, those are such nice parallel lines.

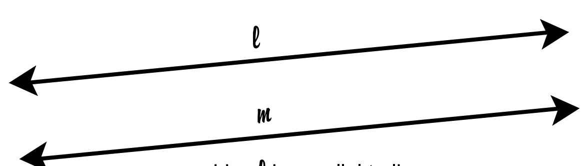
Parallel lines



Who could discipline such a lovely creature?

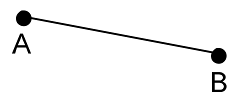
Actually, to be perfectly accurate, those are line segments, he corrected himself. A line segment is just part of a line. It has two endpoints. A line is infinitely long in both directions. Anybody who's studied geometry knows that. Fred enjoyed the precision that mathematical language afforded him.

When Fred drew lines on the blackboard in his geometry class, he'd put arrows on ends to indicate that the lines went on forever. He labeled lines with lower case letters.



Line l is parallel to line m

To draw a line segment was easy. You didn't need any arrows.

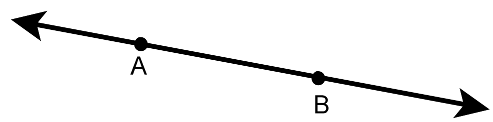


Line segment $\overline{AB}$ with endpoints A and B

Points like A and B are written with capital letters, and lines like ℓ and m are written with lower case letters.

$\overline{AB}$ is the notation for the line segment with endpoints A and B.

$\overleftrightarrow{AB}$ is the notation for the line which contains A and B.



Line $\overleftrightarrow{AB}$ which passes through A and B

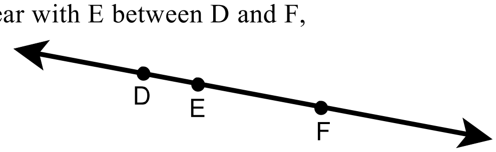
And just to make things complete: AB is the *distance* between points A and B. That makes AB a number (like six feet) whereas $\overline{AB}$ and $\overleftrightarrow{AB}$ are geometrical objects (a segment and a line).

Your Turn to Play

1. Line segments (sometimes called segments for short) can come in lots of different lengths. Name a number that *couldn't* be a length of a line segment. (Please try to figure out the answer before you look at the solution furnished below.)

2. If AB = 4 and AC = 4, does that mean that A, B and C all lie on the same line?

3. If points D, E and F are collinear with E between D and F, it would *not* be right to say that $\overline{DE} + \overline{EF} = \overline{DF}$. Why not?



4. (A tougher question) If GH + HI = GI, must it be true that:

a) points G, H and I are collinear?

b) H is between G and I?

The way to figure out the answer to this question is to get out a piece of paper and draw three points so that the distance from G to H plus the distance from H to I is equal to the distance from G to I. Be convinced in your own mind what the answers to questions a) and b) are before you look at the solutions below. You will learn very little by just reading the questions and then glancing at the answers.

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