

# Contents

<b>1</b>	<b>Exponents and Logarithms</b>	<b>1</b>
1.1	Integer Exponents . . . . .	1
1.2	Fractional Exponents . . . . .	3
1.3	Simplifying Radical Expressions . . . . .	5
1.4	Rationalizing Denominators . . . . .	7
1.5	Logarithms . . . . .	10
<b>2</b>	<b>Complex Numbers</b>	<b>13</b>
2.1	The Square Root of $-1$ . . . . .	13
2.2	Complex Number Operations . . . . .	13
<b>3</b>	<b>Linear Equations</b>	<b>17</b>
3.1	What is a Linear Equation? . . . . .	17
3.2	One Equation, One Variable . . . . .	17
3.3	Two Equations, Two Variables . . . . .	19
3.4	Word Problems . . . . .	22
<b>4</b>	<b>Proportions</b>	<b>28</b>
4.1	Direct and Inverse . . . . .	28
4.2	Manipulating Proportions . . . . .	30
4.3	Conversion Factors . . . . .	31
4.4	Percent . . . . .	33
<b>5</b>	<b>Using the Integers</b>	<b>39</b>
5.1	Divisibility . . . . .	39
5.2	Number Bases . . . . .	39
5.3	The Last Digit . . . . .	42
5.4	Modular Arithmetic . . . . .	42
5.5	Divisibility Tricks . . . . .	45
5.6	Primes . . . . .	47
5.7	Common and Uncommon Factors . . . . .	48
<b>6</b>	<b>Quadratic Equations</b>	<b>52</b>

6.1	What's a Quadratic? . . . . .	52
6.2	Factoring Quadratics . . . . .	52
6.3	The Quadratic Formula . . . . .	56
6.4	Variations on a Theme . . . . .	59
6.4.1	Rearrangements . . . . .	59
6.4.2	Substitutions . . . . .	60
6.5	Square Roots of Irrationals and Imaginaries . . . . .	61
6.6	Beyond Quadratics . . . . .	63
<b>7</b>	<b>Special Factorizations and Clever Manipulations</b>	<b>67</b>
7.1	Factorizations . . . . .	67
7.2	Manipulations . . . . .	70
<b>8</b>	<b>What Numbers Really Are</b>	<b>75</b>
8.1	Integers and Rationals . . . . .	75
8.2	Lowest Terms and Irrationals . . . . .	77
8.3	Complex and Beyond . . . . .	79
<b>9</b>	<b>An Introduction to Circles</b>	<b>81</b>
<b>10</b>	<b>Angles</b>	<b>84</b>
10.1	Lines, Rays, and Segments . . . . .	84
10.2	Classification and Measurement . . . . .	84
10.3	Angles and Parallel Lines . . . . .	86
10.4	Arcs, Segments, Sectors, and Angles . . . . .	87
10.5	Angles Formed By Lines Intersecting a Circle . . . . .	87
10.6	The Burden of Proof . . . . .	90
<b>11</b>	<b>Triangles, a.k.a. Geometry</b>	<b>93</b>
11.1	Classifying Triangles . . . . .	93
11.2	Parts of a Triangle . . . . .	94
11.3	The Triangle Inequality . . . . .	96
11.4	The Pythagorean Theorem . . . . .	97
11.5	Congruent Triangles . . . . .	99
11.6	Similar Triangles . . . . .	102
11.7	Introduction to Trigonometry . . . . .	104
11.8	Area of a Triangle . . . . .	109
11.9	A Handful of Helpful Hints . . . . .	112
<b>12</b>	<b>Quadrilaterals</b>	<b>118</b>
12.1	The Fundamentals . . . . .	118
12.2	Trapezoids . . . . .	118

12.3 Parallelograms . . . . .	120
12.4 Rhombuses (Rhombi?) . . . . .	121
12.5 Rectangles and Squares . . . . .	122
12.6 Hints and Problems . . . . .	123
<b>13 Polygons</b>	<b>127</b>
13.1 Types of Polygons . . . . .	127
13.2 Angles in a Polygon . . . . .	128
13.3 Regular Polygons . . . . .	128
13.4 Regular Hexagons . . . . .	130
<b>14 Angle Chasing</b>	<b>133</b>
<b>15 Area</b>	<b>136</b>
15.1 Similar Figures . . . . .	136
15.2 Same Base/Same Altitude . . . . .	137
15.3 Complicated Figures . . . . .	138
<b>16 The Power of Coordinates</b>	<b>143</b>
16.1 Labelling the Plane . . . . .	143
16.2 What's it Good For? . . . . .	144
16.3 Straight and Narrow . . . . .	145
16.4 Plotting a Line . . . . .	148
16.5 The Distance Formula and Circles . . . . .	149
16.6 Went Down to the Crossroads. . . . .	151
16.7 ... Fell Down on My Knees . . . . .	152
<b>17 Power of a Point</b>	<b>155</b>
17.1 Introduction . . . . .	155
17.2 Power of a Point Proofs . . . . .	157
<b>18 Three Dimensional Geometry</b>	<b>160</b>
18.1 Planes, Surface Area, and Volume . . . . .	160
18.2 Spheres . . . . .	161
18.3 Cubes and Boxes . . . . .	162
18.4 Prisms and Cylinders . . . . .	165
18.5 Pyramids and Cones . . . . .	166
18.6 Polyhedra . . . . .	168
18.7 How to Solve 3D Problems . . . . .	169
<b>19 Shifts, Turns, Flips, Stretches, and Squeezes</b>	<b>173</b>
19.1 Translation . . . . .	173

19.2 Rotation . . . . .	174
19.3 Reflection . . . . .	175
19.4 Distortion . . . . .	176
19.5 Dilation . . . . .	177
19.6 The More Things Change . . . . .	178
19.7 Transformation Proofs . . . . .	178
<b>20 A Potpourri of Geometry</b>	<b>181</b>
<b>21 Functions</b>	<b>187</b>
21.1 Welcome to the Machine . . . . .	187
21.2 Graphing Functions . . . . .	188
21.3 Inputs and Outputs . . . . .	188
21.4 Even and Odd . . . . .	190
21.5 Some Special Functions . . . . .	191
21.5.1 Absolute Values . . . . .	191
21.5.2 Floored . . . . .	192
21.5.3 Split Up . . . . .	193
21.6 Transforming a Function . . . . .	194
<b>22 Inequalities</b>	<b>197</b>
22.1 What They Do . . . . .	197
22.2 Linear Inequalities . . . . .	198
22.3 Quadratic Inequalities . . . . .	199
22.4 Absolute Value Inequalities . . . . .	202
22.5 A Trivial Inequality . . . . .	203
<b>23 Operations and Relations</b>	<b>206</b>
23.1 What is an Operation? . . . . .	206
23.2 Properties of Operations . . . . .	207
23.3 Relations . . . . .	209
<b>24 Sequences and Series</b>	<b>211</b>
24.1 Arithmetic Series . . . . .	211
24.2 Geometric Series . . . . .	212
24.3 Infinite Series . . . . .	213
24.4 $\sum_{i=1}^n$ . . . . .	215
24.5 Sequences . . . . .	216
24.6 Sequences and Means . . . . .	217
<b>25 Learning to Count</b>	<b>221</b>

25.1 What's to Learn? . . . . .	221
25.2 Multiplication . . . . .	221
25.3 Example: The Number of Divisors . . . . .	223
25.4 Restrictions on Multiplication . . . . .	223
25.5 Permutations, Arrangements, and ! . . . . .	225
25.6 Mixing it Up . . . . .	227
25.7 Counting the Wrong Thing, Part I . . . . .	228
25.8 Counting the Wrong Thing, Part II . . . . .	229
25.9 Doing it Another Way . . . . .	232
25.10 The Binomial Theorem . . . . .	232
<b>26 Statistics and Probability</b>	<b>236</b>
26.1 Statistics . . . . .	236
26.2 Probability and Common Sense . . . . .	238
26.3 Multiplying Probabilities . . . . .	240
26.4 Casework . . . . .	241
26.5 Odds . . . . .	242
26.6 What Did You Expect? . . . . .	242
<b>27 Sets</b>	<b>246</b>
27.1 Some Definitions . . . . .	246
27.2 Operating on Sets . . . . .	247
27.3 Venn Diagrams . . . . .	248
27.4 Subsets . . . . .	249
<b>28 Prove It</b>	<b>252</b>
28.1 Words, Words, Words . . . . .	252
28.2 Contradiction . . . . .	254
28.3 Converses Aren't Necessarily True . . . . .	255
28.4 Mathematical Induction . . . . .	256
28.5 Shooting Holes in Pigeons . . . . .	257
28.6 Convincing But Wrong . . . . .	258
<b>29 Parting Shots</b>	<b>262</b>