LIFEPAC®

# Consumer Math

Student Book

Unit 10


Alpha Omega Publications®

# CONSUMER MATHEMATICS 10

## CONTENTS




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# CONSUMER MATHEMATICS 10

Now that you have completed the nine units comprising the consumer mathematics series, this LIFEPAC should be beneficial to you as you review the significant principles, procedures, and problems of the preceding LIFEPACs. You may review a specific LIFEPAC if you need more detailed review in any area.

Before reading the objectives, keep in mind that this LIFEPAC is a review. Hence, the objectives listed here are not necessarily those that were defined in the previous LIFEPACs. Instead, these objectives are generally stated to incorporate appropriate and related combinations of specific tasks.

## OBJECTIVES

**Read these objectives.** The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able

1. To apply mathematical principles and operations in solving consumer-related problems;
2. To calculate income, taxes, and interest rates as they affect family finances;
3. To prepare, maintain, and interpret family financial records;
4. To apply and solve mathematical expressions that relate to a variety of occupations;
5. To determine revenue and costs associated with business operations; and
6. To establish, maintain, and interpret financial statements relevant to business operations.

**Survey the LIFEPAC.** Ask yourself some questions about this study. Write your questions here.

# I. MATHEMATICAL OPERATIONS

**OBJECTIVES**

1. To apply mathematical principles and operations in solving consumer-related problems.

To become skilled in handling mathematical relationships is essential in consumer-related activities. To be able to perform operations involving fractions, decimals, percentages, ratios, and proportions is very important also.

Fractions are crucial in occupations that involve finding areas and volumes. Almost all financial transactions require a thorough knowledge of decimals and percentages. Ratios and proportions are used often in working with scaled diagrams.

A basic understanding of fundamental geometry will help you to handle measurement problems that involve plane and solid figures. Some of these measurements are the area, the sum of the interior angles, the circumference, the volume, and the surface area of plane and solid figures.

Many times we are called upon to convert English units of measurement to metric units of measurement. Being able to use scientific notation also makes working with very small or very large numbers much easier.

## FRACTIONS, DECIMALS, AND PERCENTAGES

We shall begin our review by going over the terms and procedures that are pertinent to fractions, decimals, and percentages.

A fraction is a number that is not a whole number. Fractions that are less than one are proper fractions.

Model: $\frac{4}{9}$ is a proper fraction.

Fractions that are greater than one are improper fractions.

Model: $\frac{12}{7}$ is an improper fraction.

An improper fraction can be changed to a mixed number, which contains a whole number and a fraction.

Model: The fraction $\frac{12}{7}$ changed to a mixed number is $1\frac{5}{7}$.

When you work with fractions, try to find a common factor, if one exists, to simplify the operation. A common factor is a number that divides evenly into both the numerator and the denominator of a fraction. A fraction, therefore, is reduced to its lowest terms when the only common factor is 1. Also, the lowest common denominator is the smallest number into which the

denominators of two or more fractions will divide evenly. When you work with mixed numbers, change the number to an improper fraction first. To divide fractions, invert the second fraction and multiply.

Model 1: Add $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{5}{6}$. The lowest common denominator is 6. Multiply each fraction by the appropriate factor of 6 to obtain $\frac{2}{6} + \frac{3}{6} + \frac{5}{6} = \frac{10}{6}$, which is a mixed number that reduces to $1\frac{2}{3}$.

Model 2: Divide $2\frac{2}{5}$ by $\frac{3}{10}$. First, change $2\frac{2}{5}$ to an improper fraction: $\frac{12}{5}$. Then invert the second fraction: $\frac{10}{3}$. Now multiply: $\frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{3}}} = 8$.

**Perform the following activities.**

1.1 Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$. _____

1.2 Subtract $\frac{4}{5}$ from $\frac{19}{20}$. _____

1.3 Reduce $\frac{14}{24}$. _____

1.4 Add $2\frac{1}{4}$, $3\frac{3}{8}$, and $\frac{1}{6}$. _____

1.5 Multiply $\frac{5}{6}$ and $1\frac{1}{2}$. _____

1.6 Multiply $2\frac{1}{3}$ and $\frac{5}{6}$. _____

1.7 Divide $\frac{4}{7}$ by $\frac{2}{7}$. _____

1.8 Divide $3\frac{1}{5}$ by $2\frac{3}{10}$. _____

In our number system, *decimal* means *based on ten*. A decimal fraction, or simply a decimal, is a number less than one, which is represented in place value notation by a decimal point.

Model: 0.62 is a decimal and means $\frac{62}{100}$.

Perform the following operations involving decimals.

1.9 $1.38 + 1.42 + 1.69 =$ _____

1.10 $4.57 - 12.52 =$ _____

1.11 $1.3 \times 1.03 =$ _____

1.12 $2.568 \div 4 =$ _____

Percentages are expressed in terms of per hundred. Percentages need to be converted to decimals to work with them. To convert a percentage to a decimal, move the decimal point to the left two places and remove the per cent symbol.

Model: 40% written as a decimal is 0.40 or 0.4.

Sometimes you can change a per cent to a common fraction to your advantage.

Model 1: The per cent 48% changed to a common fractions is $\frac{48}{100}$ or $\frac{12}{25}$.

Model 2: Increase 25 by 20%. First, change 20% to a decimal, 0.20 and then multiply: 0.20(25) = 5. Now increase 25 by 5 to obtain 30, which is 25 increased by 20%.

Solve the following percentage problems.

1.13 25% of 80 = _____

1.14 15% of 14.8 = _____

1.15 Reduce 38 by 16% = _____

1.16 Increase 50 by 200% = _____

Convert the following expressions to per cents.

1.17 $\frac{1}{8} =$ _____

1.18 $0.375 =$ _____

1.19 $\frac{5}{2} =$ _____

1.20 $0.444 =$ _____

## RATIOS AND PROPORTIONS

A very useful procedure to understand is stating the relationship of one number to another number. This

procedure is known as calculating the ratio.

Model: The ratio 6 to 8, written 6:8, reduces to the ratio 3:4. Both ratios have the same meaning.

An equality between ratios is known as a proportion. The principles of fractions, decimals, and percentages apply to ratios and proportions.

Model: 8:5 = 16:10 is a proportion.

To find a missing number in a proportion, reduce each ratio if possible, then multiply the two outer numbers together and the two inner numbers together. They will be equal; solve for the unknown number.

[redacted] Find the following ratios.

1.21 4:2 = _____

1.22 3 to 9 = _____

1.23 $\frac{68}{4}$ = _____

Model: Solve the proportion problem $15:10 = c:32$. The ratio 15:10 reduces to 3:2 when divided by 5; the proportion is now $3:2 = c:32$. Multiply the outer numbers of the proportion together and the inner numbers together. Set them equal to each other and solve for $c$.

$$3 \times 32 = 2 \times c$$
$$96 = 2c$$
$$c = 48$$

[redacted] Solve the following proportion problems.

1.24 $12:c = 14:7$ $c$ = _____

1.25 $m:5 = 25:15$ $m$ = _____

1.26 $9:27 = c:3$ $c$ = _____

1.27 $\frac{1}{2}:f::\frac{3}{8}:\frac{11}{16}$ $f$ = _____

Since a scale is the size of a plan, map, drawing, or model compared to what it represents, we use the technique for solving proportions to convert scalar quantities to actual quantities.

Model: The scale of a certain map is 1:30,000. What is the map distance between two towns located 3 miles apart? Since the map scale is in inches, convert 3 miles to inches, which is 190,080 inches. Set up a proportion and solve for the distance.

$$1:30{,}000 = d:190{,}080$$
$$30{,}000d = 190{,}080$$
$$d = 6.336 \text{ in.}$$

Perform the following scalar problems.

1.28 A map's scale is 1:25,000. What is the map distance between two cities located 5 miles apart? (Remember to convert 5 miles to inches first.)

1.29 The scale on a shop drawing is $\frac{1}{16}" = 1"$. If the outside diameter of a washer is $4\frac{1}{4}"$, how large will it be in the drawing?

1.30 The blueprint of a house has a scale of $\frac{1}{4}$ in. = 1 ft. If the living room is drawn $4\frac{1}{4}$ in. by $2\frac{3}{4}$ in. on the blueprint, what are its actual dimensions?

## ANGLES, PERPENDICULARS, AND POLYGONS

An angle is either acute, right, or obtuse depending upon whether its magnitude is less than, equal to, or greater than 90°.

A straight line that meets another line so that they form a right angle is called a perpendicular.

A polygon is a closed plane figure of any number of straight sides. The angle formed at a vertex of a polygon is an interior angle. The angle formed by extending one side through a vertex is an exterior angle.

The sum of the interior angles of a polygon is $(n - 2)180°$, where $n$ is the number of sides. Each interior angle is $(1 - \frac{2}{n})180°$.

Model 1: Given a nonagon, calculate the sum of the measures of its interior angles. A nonagon has 9 sides, so the sum of its interior angles
$= (9 - 2)180°$
$= (7)180°$
$= 1,260°$

Model 2: Given a pentagon, calculate the measure of an interior angle. A pentagon has five sides, so each interior angle $= (1 - \frac{2}{5})180°$
$= (\frac{3}{5})180°$
$= 108°$

The areas of regular polygons may be found by the following formulas.

| Name | Number of Sides | Area |
|---|---|---|
| triangle | 3 | $0.433s^2$ |
| square | 4 | $1.000s^2$ |
| pentagon | 5 | $1.7205s^2$ |
| hexagon | 6 | $2.598s^2$ |
| heptagon | 7 | $3.634s^2$ |
| octagon | 8 | $4.828s^2$ |
| nonagon | 9 | $6.182s^2$ |
| decagon | 10 | $7.694s^2$ |

Model: Find the area of a regular octagon with length of each side 4". The area
area $= 4.828(4)^2$
$= 4.828(16)$
$= 77.248$ in.$^2$.

Of course, we do not always work with regular polygons. For most other irregular polygons we have formulas that we can use to find their areas. A few of these formulas are listed.

| Name | Area |
|---|---|
| triangle | $\frac{1}{2}bh$ where $b$ is the base and $h$ is the altitude |
| rectangle | $ab$ where $a$ and $b$ are nonparallel sides |
| trapezoid | $\frac{1}{2}(a + b)h$ where $a$ and $b$ are parallel sides and $h$ is the altitude |

Model: Find the area of a triangle with base of 8 in. and altitude of 12 in.

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(8)(12)$
$A = 4(12)$
$A = 48$ in.$^2$

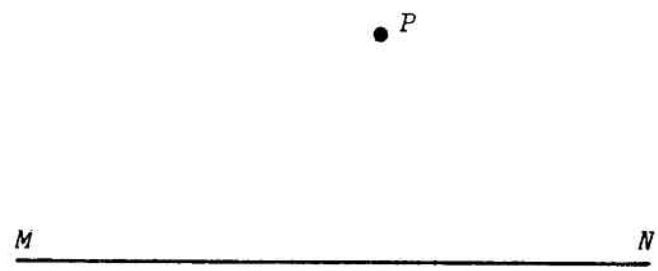
Construct the following angles and perpendicular.

1.31 Construct an acute angle of 30°.

1.32 Construct a right angle.

1.33 Construct any obtuse angle.

1.34 Construct a perpendicular to the line segment shown from the point $P$.



Compute the area of each of the following polygons.

1.35 Type of polygon: regular pentagon
Length of side: 5"
Area = _____

1.36 Type of polygon: triangle
Length of base: 4"
Length of altitude: 6"
Area = _____

1.37 Type of polygon: regular heptagon
Length of side: 3.5"
Area = _____

1.38 Type of polygon: trapezoid
Lengths of parallel sides: 8" and 4.5"
Length of altitude: 5.25"
Area = _____

Perform the following activities.

1.39 Given a decagon, calculate the measure of an interior angle.

_____

1.40 Given a hexagon, calculate the sum of the measures of its interior angles.

_____

## PLANE GEOMETRIC FIGURES WITH CURVED BOUNDARIES

The curved plane figures that are typically found in occupations are the circle and the ellipse.

The area and the circumference are the most critical measurements of these figures.

| Name | Measurement | Formula |
|---|---|---|
| circle | circumference | $C = 2\pi r$ or $\pi d$, where $r$ is the radius and $d$ is the diameter |
| ellipse | circumference | $C = \pi(a + b)\frac{64 - 3R^4}{64 - 16R^2}$, where $R = \frac{a - b}{a + b}$ |
| circle | area | $A = \pi r^2$ or $\pi(\frac{d}{2})^2$, where $d$ is the diameter |
| ellipse | area | $A = \pi ab$, where $a$ is the major semiaxis and $b$ is the minor semiaxis |

Model: Calculate the area of a circle with radius 3 ft. (use $\pi = 3.142$).

$$A = \pi r^2$$
$$A = (3.142)(3)^2$$
$$A = (3.142)(9)$$
$$A = 28.278 \text{ ft.}^2$$

Calculate the following measurements (for $\pi$ use 3.142).

1.41 Find the area of a circle with radius 6". _____

1.42 Find the area of a ellipse with major semiaxis = 5.5" and minor semiaxis = 3.5".

_____

1.43 Find the circumference of a circle with a diameter of 4".

_____

## SOLID GEOMETRIC FIGURES

Most objects are three-dimensional. We need to be able to use measurement formulas to find volumes and surface areas of such figures. The following list includes the formulas for the more common solid figures.

| Name | Measurement Activity | Formula |
|---|---|---|
| cone | surface area | $A = \pi r\sqrt{r^2 + rh^2}$ |
| cone | volume | $V = \frac{1}{3}\pi r^2 h$ |
| cube | surface area | $A = 6s^2$, where $s$ is a side |
| cube | volume | $V = s^3$ |
| cylinder | surface area | $A = 2\pi rh$ |
| cylinder | volume | $V = \pi r^2 h$ |
| pyramid | surface area | $A = \frac{1}{2}$(perimeter of base times slant height) |
| pyramid | volume | $V = \frac{1}{3}$(area of base times altitude) |
| rectangular solid | surface area | $A = 2(ab + ac + bc)$, where $a$, $b$, and $c$ are the lengths of the opposite parallel sides, respectively. |

| | | |
|---|---|---|
| rectangular solid | volume | $V = abc$ |
| sphere | surface area | $A = 4\pi r^2$ |
| sphere | volume | $V = \frac{4}{3}\pi r^3$ |

Model: Given a rectangular pyramid with base of sides 8 ft. and 10 ft. and altitude of 10 ft., find the volume.

$V = \frac{1}{3}$(area of base times altitude)

$V = \frac{1}{3}(8)(10)(10)$

$V = \frac{1}{3}(800)$

$V = \frac{800}{3}$

$V = 266\frac{2}{3}$ cu. ft. or $ft.^3$

Perform the following measurement activities (for $\pi$ use 3.142).

1.44 Rectangular solid
Lengths of sides = 4", 5", and 6".
Find the surface area. _____

1.45 Cone
Radius of base = 3.5"
Altitude = 6"
Find the volume. _____

1.46 Pyramid
Sides of base = rectangle of 18' and 16'
Altitude = 20'
Find the volume. _____

1.47 Cube
Side = 8"
Find the surface area. _____

1.48 Sphere
Radius = 3 cm
Find the surface area. _____

## METRIC CONVERSION

The United States is in the process of converting from the English system of weights and measures to the international metric system of weights and measures.

The basic units of the international metric system are the meter for length, the kilogram for mass, the second for time, the Celsius degree for temperature, the ampere for electrical current, and the candela for luminous intensity.

Consumer Mathematics LIFEPAC 5 includes many of the conversion numbers for various lengths, areas, volumes, weights, and temperatures. A few of these numbers are listed.

| English Unit | Conversion Number | Metric Unit |
|---|---|---|
| mile | 1.609 | kilometer |
| feet | 0.3048 | meter |
| inch | 2.54 | centimeter |
| fluid ounce | 29.574 | cubic centimeter |
| quart (liquid) | 0.9463 | liter |
| gallon | 3.785 | liter |
| ounce | 28.35 | gram |
| pound | 0.4536 | kilogram |
| ton (short) | 907.2 | kilogram |
| degrees Fahrenheit | $\frac{5}{9}(°F - 32)$ | degrees Celsius |
| degrees Kelvin | $273 + °C$ | degrees Celsius |

Model: Convert 5 miles to kilometers.

Multiply 5 by 1.609 to obtain 8.045 kilometers.

Prefixes used in conjunction with the metric system are derived from the Greek language. The most frequently used prefixes are repeated in the following table.

| Prefix | Symbol | Power of Ten |
|---|---|---|
| deca- | *dk* | $10^1$ |
| hecto- | *h* | $10^2$ |
| kilo- | *k* | $10^3$ |
| mega- | *M* | $10^6$ |
| deci- | *d* | $10^{-1}$ |
| centi- | *c* | $10^{-2}$ |
| milli- | *m* | $10^{-3}$ |
| micro- | μ | $10^{-6}$ |
| nano- | *n* | $10^{-9}$ |

An alternative procedure for writing numerals that are either very large or very small is known as scientific notation. A number is expressed in scientific notation when its numeral names a number that is greater than one but less than ten multiplied by some power of ten.

Model 1: Express 0.2495 in scientific notation.

Move the decimal point to the right one place and multiply by $10^{-1}$: $2.495 \times 10^{-1}$.

Model 2: Express 0.0839 in scientific notation.

Move the decimal point to the right two places and multiply by $10^{-2}$: $8.39 \times 10^{-2}$.

Perform the following conversions.

1.49 Convert 5 gallons to liters. _____

1.50 Convert 5,000 miles to kilometers. _____

1.51 Convert 28.5 grams to ounces. (Remember that to convert metric units to English units, you multiply by the reciprocal of the appropriate conversion number.) _____

1.52 Convert 55 miles per hour to kilometers per second. _____

1.53 Express your answer to Problem 1.52 in scientific notation. _____

1.54 Convert 120 tons to kilograms. _____

1.55 Express your answer to Problem 1.54 in the metric equivalent for the next higher prefix. _____

1.56 Convert 4,000°$C$ to °$K$. _____

1.57 Convert 1,800°$F$ to °$C$. _____

1.58 Write 0.005 nanoseconds in scientific notation. _____


**Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.**

## SELF TEST 1

Match these items (each answer, 2 points).

1.01 _____ factor

1.02 _____ fractions that are less than 1

1.03 _____ fractions that are greater than 1

1.04 _____ contains a whole number part and a fraction part

a. a number that divides evenly into another number

b. proper fractions

c. mixed number

d. improper fractions

e. decimal

**Select the letter that correctly completes each statement (each answer, 2 points).**

1.05 A scale is the size of a plan, a map, a drawing, or a model compared to _____.

a. 1:20,000
b. what it represents
c. half-size
d. another scale

1.06 Percentages are expressed _____.

a. in terms of per hundred
b. as reciprocals
c. in tenths
d. in thousandths

1.07 A proportion is _____.

a. a decimal
b. the calculation of ratio
c. an equivalent fraction
d. a statement of equality between ratios

1.08 An obtuse angle is an angle _____.

a. whose magnitude is uncertain
b. that is measured in a clockwise direction
c. that is greater than a right angle
d. that is less than 90°

1.09 An example of a plane geometric figure with curved boundaries is _____.

a. a sphere
b. a rectangular solid
c. an ellipse
d. a cylinder

**Complete these sentences (each answer, 3 points).**

1.010 A method for writing either very large or small numerals is called _____.

1.011 The least common denominator is the smallest number into which the _____.

1.012 To divide one fraction by another, _____.

1.013 In carrying out mathematical operations with mixed numbers, the mixed numbers must first be _____.

1.014 In a polygon the angle formed by extending one side through a vertex is _____.

Perform the following activities (each answer, 3 points).

1.015 Reduce $\frac{38}{84}$. _____

1.016 Form an imporper fraction from the mixed number $3\frac{20}{27}$. _____

1.017 Solve for $c$ in the proportion $7:42 = c:36$. _____

1.018 If the sum of the interior angles of a polygon is 1,260°, what is the name of the polygon? _____

1.019 Given: major semiaxis = 5 cm
minor semiaxis = 3.5 cm
$A = \pi ab$
Find: area of ellipse
(Answer in nearest whole number.) _____

1.020 Given: octagon with side of 4 in.
$A = 4.828s^2$
Find: area expressed in nearest whole square centimeter _____

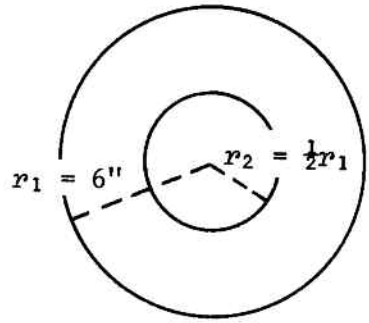
1.021 Given: area of heptagon = 90.85 $cm^2$
$A = 3.634s^2$
Find: length of side in nearest whole centimeter _____

1.022 Express 28,500,000 in scientific notation. _____

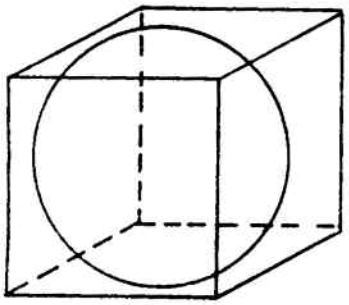
1.023 Convert $27\frac{3}{5}\%$ to a decimal. _____

1.024 Express $4 \times 10^{-9}$ seconds in the appropriate metric unit. _____

1.025 Given: 2 circles with radii as shown
$A = \pi r^2$
Find: area of the smaller circle _____



1.026 Given: Sphere inscribed within a cube with a surface area of 150 in.$^2$.
Surface area = $6s^2$
Volume = $\frac{4}{3}\pi r^3$
Find: volume of the inscribed sphere



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Score _____
Teacher check _____
Initial Date