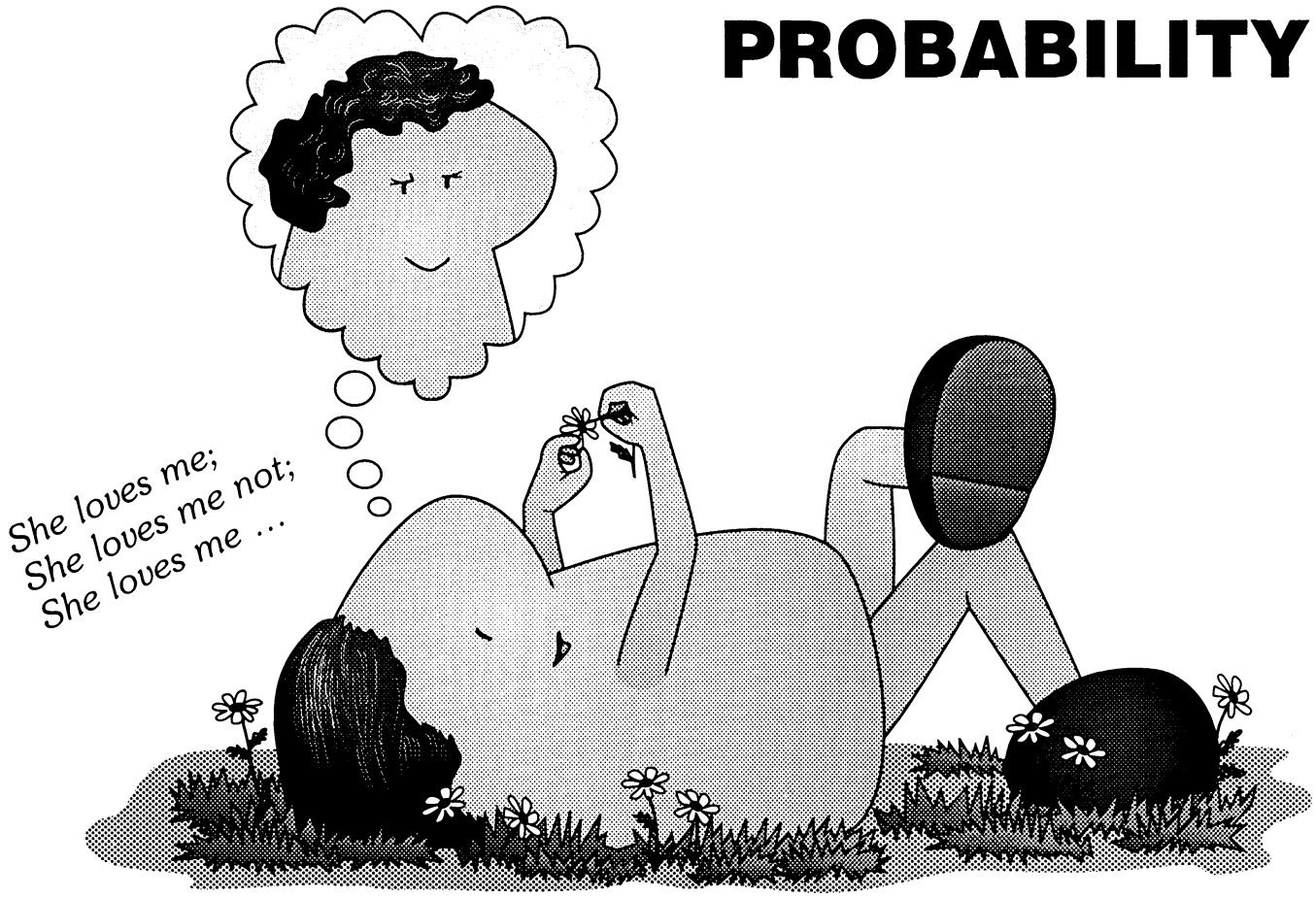
PROBABILITY



TASK CARD SERIES

Conceived and written by

RON MARSON

Illustrated by

PEG MARSON

TOPS LEARNING SYSTEMS

10970 South Mulino Road
Canby, Oregon 97013

WHAT CAN YOU COPY?

Dear Educator,

Please honor our copyright restrictions. We offer liberal options and guidelines below with the intention of balancing your needs with ours. When you buy these labs and use them for your own teaching, you sustain our work. If you "loan" or circulate copies to others without compensating TOPS, you squeeze us financially, and make it harder for our small non-profit to survive. Our well-being rests in your hands. Please help us keep our low-cost, creative lessons available to students everywhere. Thank you!

PURCHASE, ROYALTY and LICENSE OPTIONS

TEACHERS, HOMESCHOOLERS, LIBRARIES:

We do all we can to keep our prices low. Like any business, we have ongoing expenses to meet. We trust our users to observe the terms of our copyright restrictions. While we prefer that all users purchase their own TOPS labs, we accept that real-life situations sometimes call for flexibility.

Reselling, trading, or loaning our materials is prohibited unless one or both parties contribute an Honor System Royalty as fair compensation for value received. We suggest the following amounts – let your conscience be your guide.

HONOR SYSTEM ROYALTIES: If making copies from a library, or sharing copies with colleagues, please calculate their value at 50 cents per lesson, or 25 cents for homeschoolers. This contribution may be made at our website or by mail (addresses at the bottom of this page). Any additional tax-deductible contributions to make our ongoing work possible will be accepted gratefully and used well.

Please follow through promptly on your good intentions. Stay legal, and do the right thing.

SCHOOLS, DISTRICTS, and HOMESCHOOL CO-OPS:

PURCHASE Option: Order a book in quantities equal to the number of target classrooms or homes, and receive quantity discounts. If you order 5 books or downloads, for example, then you have unrestricted use of this curriculum for any 5 classrooms or families per year for the life of your institution or co-op.

2-9 copies of any title: 90% of current catalog price + shipping.

10+ copies of any title: 80% of current catalog price + shipping.

ROYALTY/LICENSE Option: Purchase just one book or download *plus* photocopy or printing rights for a designated number of classrooms or families. If you pay for 5 additional Licenses, for example, then you have purchased reproduction rights for an entire book or download edition for any **6** classrooms or families per year for the life of your institution or co-op.

1-9 Licenses: 70% of current catalog price per designated classroom or home.

10+ Licenses: 60% of current catalog price per designated classroom or home.

WORKSHOPS and TEACHER TRAINING PROGRAMS:

We are grateful to all of you who spread the word about TOPS. Please limit copies to only those lessons you will be using, and collect all copyrighted materials afterward. No take-home copies, please. Copies of copies are strictly prohibited.

Ask us for a ***free shipment*** of as many of our ***TOPS IDEAS Catalogs*** as you need to support your efforts. Every catalog is a rich, attractive resource magazine packed with free sample teaching ideas.

For licensing, honor system royalty payments, or catalog requests, contact: **www.TOPScience.org**; or

TOPS Learning Systems, 10970 S Mulino Rd, Canby OR 97013; or inquire at **tops@canby.com**

ISBN 978-0-941008-78-5

CONTENTS

INTRODUCTION

TEACHING NOTES

REPRODUCIBLE MATERIALS

Gathering Materials

Listed below is everything you'll need to teach this module. You already have many of these items. The rest are available from your supermarket, drugstore and hardware store. Laboratory supplies may be ordered through a science supply catalog.

Keep this classification key in mind as you review what's needed:

special in-a-box materials:	general on-the-shelf materials:
Italic type suggests that these materials are unusual. Keep these specialty items in a separate box. After you finish teaching this module, label the box for storage and put it away, ready to use again the next time you teach this module.	Normal type suggests that these materials are common. Keep these basics on shelves or in drawers that are readily accessible to your students. The next TOPS module you teach will likely utilize many of these same materials.
(substituted materials):	*optional materials:
Parentheses enclosing any item suggests a ready substitute. These alternatives may work just as well as the original, perhaps better. Don't be afraid to improvise, to make do with what you have.	An asterisk sets these items apart. They are nice to have, but you can easily live without them. They are probably not worth an extra trip to the store, unless you are gathering other materials as well.

Everything is listed in order of first use. Start gathering at the top of this list and work down. Ask students to bring recycled items from home. The teaching notes may occasionally suggest additional student activity under the heading "Extensions." Materials for these optional experiments are listed neither here nor in the teaching notes. Read the extension itself to find out what new materials, if any, are required.

Needed quantities depend on how many students you have, how you organize them into activity groups, and how you teach. Decide which of these 3 estimates best applies to you, then adjust quantities up or down as necessary:

$Q_1 / Q_2 / Q_3$

- **Single Student:** Enough for 1 student to do all the experiments.
- **Individualized Approach:** Enough for 30 students, working in pairs, all self-paced.
- **Traditional Approach:** Enough for 30 students, working in pairs, all doing the same lesson.

KEY: *special in-a-box materials* (substituted materials) general on-the-shelf materials *optional materials

$Q_1 / Q_2 / Q_3$:

1/15/15	**towels (sweaters or other fabric to muffle sound) — see notes 1*
7/105/150	pennies
1/15/15	hand calculators
1/8/8	rolls masking tape
1/15/15	*bottle caps, unbent, all same brand*
1/15/15	empty cereal box, 32 ounce Grape-Nuts boxes recommended
2/30/30	pieces corrugated cardboard, about 25 x 30 cm
1/4/8	*heavy scissors — see notes 7
1/15/15	*quarts package filler — see notes 7*
1/4/4	rolls clear tape that you can write on
1/15/15	index cards, 4 x 6 inch
10/150/150	paper clips
1/10/15	size D dry cells, dead or alive
1/10/15	thick rubber bands
50/750/750	straight pins, about 1 inch long
1/5/15	metric rulers
1/15/15	plastic drinking straws
1/15/15	scissors
1/4/4	wire cutters
1./1/1	cups modeling clay
1/1/1	pkg beans, pinto or other
1/15/15	plastic sandwich bags or equivalent
4/60/60	jar lids or other shallow containers
2/10/30	*dice*
1/5/15	styrofoam cups — see notes 19
10/50/150	*flat thumb tacks, all the same brand*
1/1/5	old urban phone book with white pages
1/1/1	roll black tape, cloth or plastic
1/2/8	desk calculators with tape
10/150/150	flat wooden toothpicks
1/1/1	trigonometry textbook with sine tables

Sequencing Task Cards

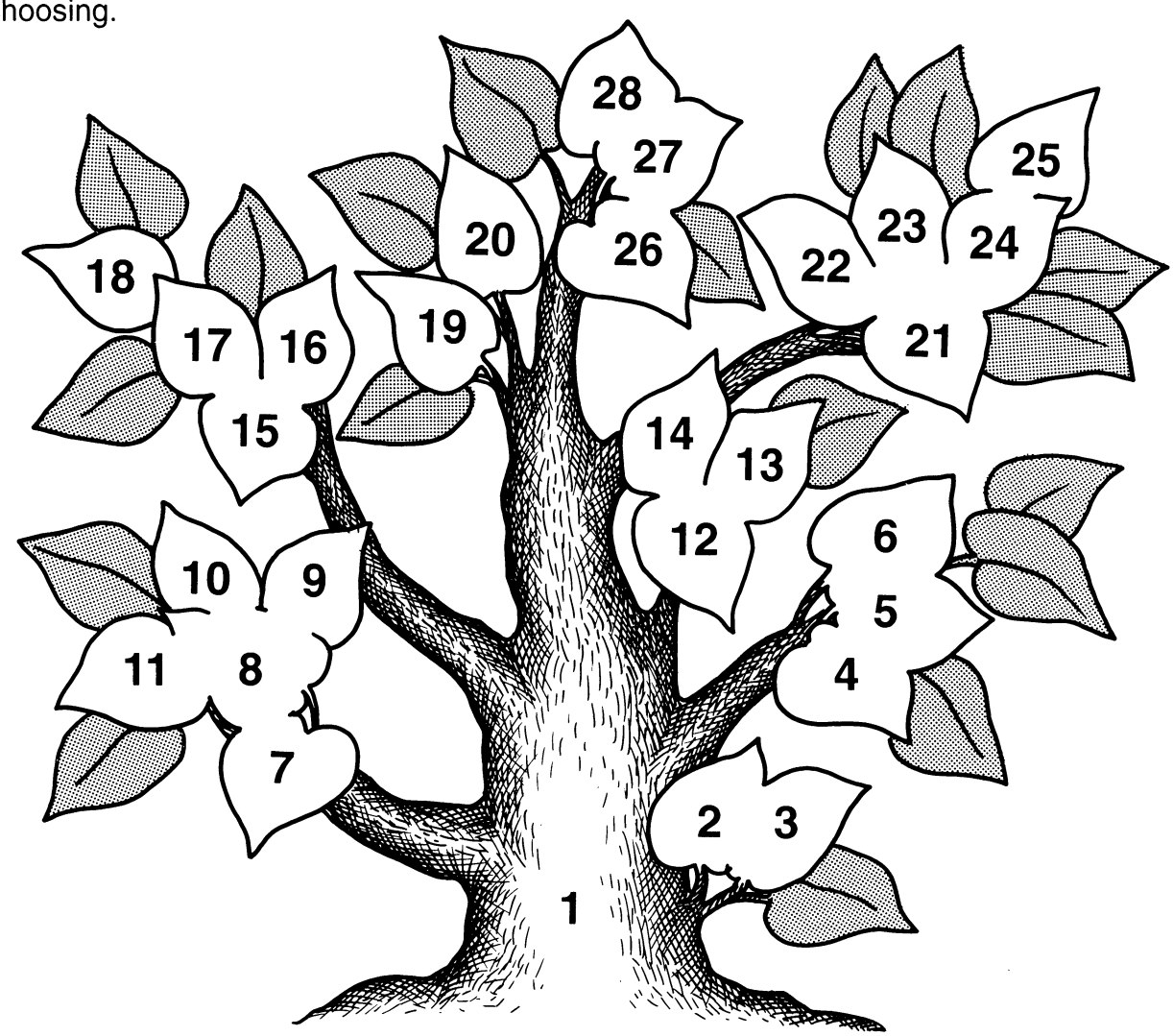
This logic tree shows how all the task cards in this module tie together. In general, students begin at the bottom of the tree and work up through the related branches. As the diagram suggests, upper level activities build on lower level activities.

At the teacher's discretion, certain activities can be omitted, or sequences changed, to meet specific class needs. The only activities that must be completed in sequence are indicated by leaves that open *vertically* into the ones above them. In these cases the lower activity is a prerequisite to the upper.

When possible, students should complete the task cards in the same sequence as numbered. If time is short, however, or certain students need to catch up, you can use the logic tree to identify concept-related *horizontal* activities. Some of these might be omitted, since they serve only to reinforce learned concepts, rather than introduce new ones.

On the other hand, if students complete all the activities at a certain horizontal concept level, then experience difficulty at the next higher level, you might move back down the logic tree to have students repeat specific key activities for greater reinforcement.

For whatever reason, when you wish to make sequence changes, you'll find this logic tree a valuable reference. Parentheses in the upper right corner of each task card allow you total flexibility; they are left blank so you can pencil in sequence numbers of your own choosing.



PROBABILITY 08

Review / Test Questions

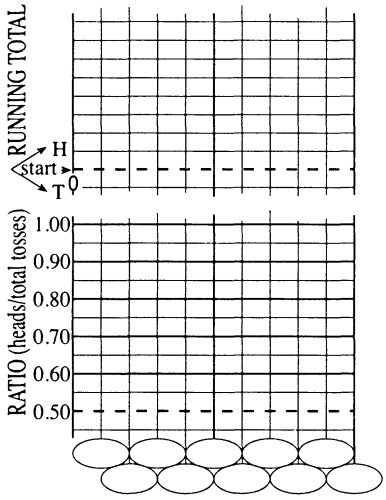
tasks 1-3, 19

Use a hand calculator:

A fair coin is tossed 10 times with these results: HHTHHHTTHT

a. Plot these outcomes in the graphs below as a running total; as a ratio.

b. Where will each graph be relative to its dashed line after 1,000 tosses? Make reasoned predictions.



tasks 1-3, 23 A

Coins A and B are tossed many times with these outcome ratios for heads:

A: 6/10 55/100 511/1,000 5,036/10,000

B: 4/10 52/100 589/1,000 5,918/10,000

Convert each outcome to a decimal. Are these coins fair? Evaluate.

tasks 1-3, 23 B

Pete and Repeat have played a game of checkers together, every day, for 17 years. Pete's win record has remained close to Repeat's over these years, but never seems to quite catch up. Is Repeat the better player? Explain.

tasks 4-6

A coin is tossed 3 times. Jane records the outcomes as THH. Joe records these same outcomes as 2H, 1T. What kind of information did each record? What does it mean?

tasks 4-11

Each letter is a pin position on Pascal's triangle. ABDHM is one permutation leading to M. List a total of 6 leading to this letter:

A
B C
D E F
G H I J
K L **M** N O

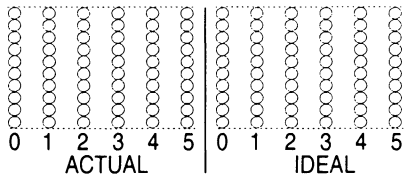
tasks 5-11, 19

Five pennies are tossed 32 times. You record these numbers of heads after each throw:

2, 4, 5, 1, 4, 4, 2, 2, 1, 2, 3,
2, 3, 2, 1, 2, 4, 4, 4, 3, 4, 3,
5, 3, 5, 3, 3, 2, 0, 2, 3, 1.

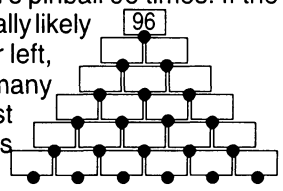
a. Plot the ACTUAL frequency distribution in the graph below. Plot an IDEAL bell curve next to it.

b. Find the mode, median and mean outcomes for the ACTUAL numbers.



tasks 7-11

Imagine dropping a penny onto the top pin of Pascal's pinball 96 times. If the penny is equally likely to fall right or left, record how many times the rest of these pins are hit.



tasks 10, 21-22

Name an activity that...

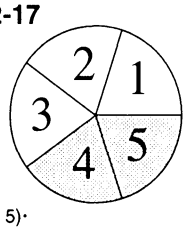
a. ...has even binomial probability.

b. ...has uneven binomial probability.

c. ...does not have binomial probability.

tasks 12-17

A circle is divided into 5 equal parts:



a. Find these 1-spin probabilities:

$P_{(even)}$, $P_{(odd)}$, $P_{(not\ 5)}$.

b. Find these 2-spin probabilities:

$P_{(2\ and\ 2)}$, $P_{(odd\ and\ odd)}$, $P_{(doubles)}$.

c. Does $P_{(grey\ and\ 4)} = P_{(grey)} \times P_{(4)}$? Why?

tasks 13-14, 18

Consult Pascal's triangle:

You answer 4 questions on a true/false test by guessing. What are your chances of getting them all right? Half right?

tasks 16-17

You meet a stranger and say, "Today is your birthday!" What are your chances of being right? Of being wrong?

tasks 17

Nine white marbles and 1 black are placed in a paper bag. Express the probability, of blindly drawing 9 white marbles out of this bag (one by one) without getting the black one.

tasks 18

Compare your chances of rolling 7 on two dice with 6 on one die.

tasks 20

Of the billions and billions of snowflakes that fall from the sky, this particular flake landed on the tip of Tommy's tongue. Do you agree that this is a rare event?

tasks 20-25

What is the probability that 2 adjacent numbers in a phone book will end in the same digit?

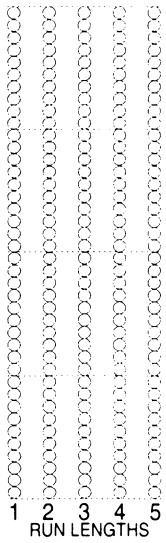
tasks 21-25

A coin is tossed 120 times with these results (read across):

H H T H H T T T T T H H H H T H T H
T T T H T H H H H T H H T T H H
H T H T H H T H H T H T H T T H T
H H T T H H T H H T
T T H T H T T T H T
T T H T T H T T H T
H H T H T H T T H H
H T T H H H T H T T
T T H T T H H H T H
H T H T H H T T

a. Graph the run frequency of this string. Show that all these runs contain exactly 120 outcomes.

b. Find the mode, median and mean run length.



tasks 21-25

A string of heads and tails contains one longest run of 7. What is the most likely length of this string? Show your math.

tasks 26-28

Darts are thrown randomly at a red and black checker board. What is the probability that the dart will land inside a red square? Explain your reasoning.

tasks 26-28

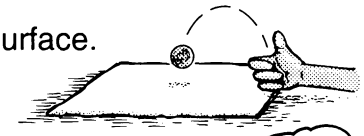
An iron needle is tossed on a grid of parallel lines that are spaced 1 needle width apart. It is constrained by a magnetic field to always land at a 45° angle with these lines. How likely is it that the needle will hit a line?

sin 0° = .0000
sin 45° = .7071
sin 90° = 1.0000.)

Task Objective (TO) toss a coin and record the outcomes as an up-and-down graph line. To appreciate that win-loss records that appear to favor one team or the other may really be a matter of chance.

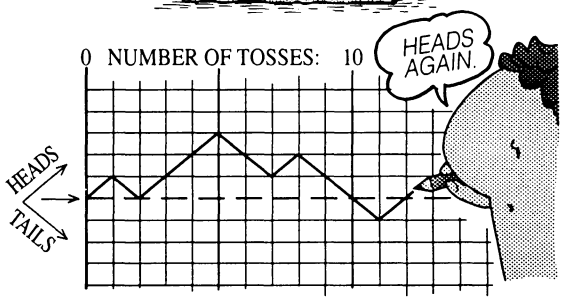
HEADS OR TAILS? ○ Probability ( )

1. Flip a penny onto a towel or other cushioned surface.



2. Record your result on a Probability Grid: if you get *heads*, draw a diagonal line *upward* from the starting arrow; if you get *tails*, draw it *downward*.

3. Fairly flip the penny a total of 50 times onto the towel. Record each *outcome* (result) as an up and down graph line along the upper dashed line on your grid.

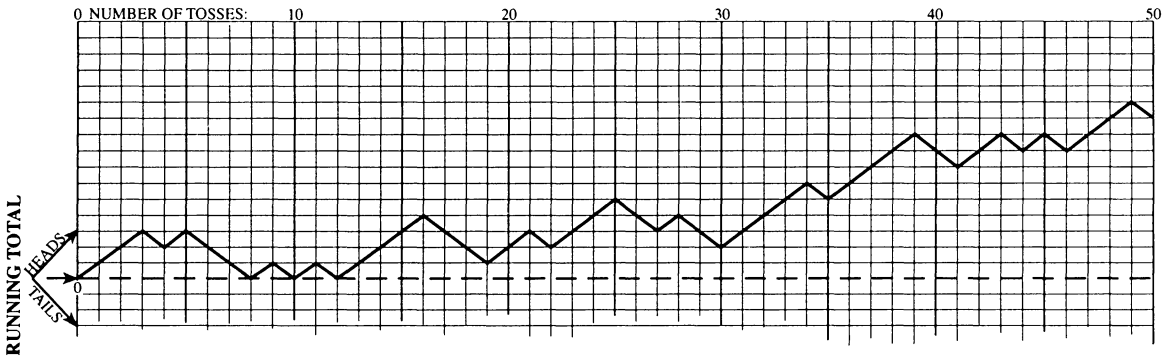


4. Let your results represent a marathon 50-game basketball tournament between the "Heads" and the "Tails".
   a. Who won or was usually ahead?
   b. Is the team with the best results really the better team? Explain. (Hint: Is the coin fair?)
   c. Does a coin "know" when one side or the other is ahead, and try to even the score?

Answers / Notes

1. *This cushioned surface is optional. It will reduce excessive noise in your classroom and minimize stray pennies rolling across the floor.*

2-3. *A huge number of different graph shapes (2^{50}) are possible. Answers here and elsewhere can only model the structure of randomness. We have chosen to represent an outcome that seems to favor Heads, but is nevertheless random. In more extreme cases, the graph line might even stray off the paper. If this happens, simply tape on an extra piece of grid to accommodate.*



4a. In our graph, Heads won, finishing a full 10 games ahead of Tails. *(In general, either side is equally likely to be ahead at any given time. But whoever is ahead after 50 tosses will probably have been ahead almost the whole tournament.)*

4b. No. In our case, Heads is not a more skilled team, just luckier. Their win-loss record was determined by the flip of a *fair* coin. Other outcomes are just as likely.

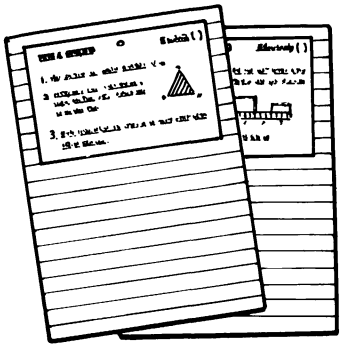
4c. A fair coin "knows" nothing at all. With every fair flip it is equally likely to land heads or tails no matter what its previous history.

Materials

☐ A towel or other fabric to form a cushioned surface (optional).
☐ A penny.
☐ A Probability Grid. Photocopy this from the supplementary pages at the back of this book. Duplicate a double quantity to use here and in activity 3.

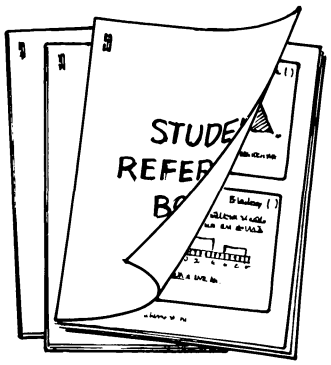
Task Cards Options

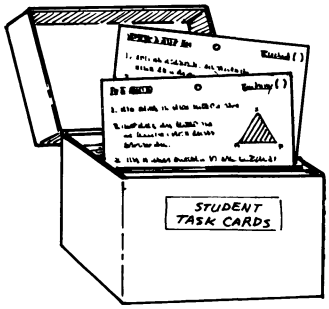
Here are 3 management options to consider before you photocopy:



1. Consumable Worksheets: Copy 1 complete set of task card pages. Cut out each card and fix it to a separate sheet of boldly lined paper. Duplicate a class set of each worksheet master you have made, 1 per student. Direct students to follow the task card instructions at the top of each page, then respond to questions in the lined space underneath.

2. Nonconsumable Reference Booklets: Copy and collate the 2-up task card pages in sequence. Make perhaps half as many sets as the students who will use them. Staple each set in the upper left corner, both front and back to prevent the outside pages from working loose. Tell students that these task card booklets are for reference only. They should use them as they would any textbook, responding to questions on their own papers, returning them unmarked and in good shape at the end of the module.



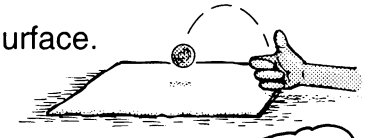


3. Nonconsumable Task Cards: Copy several sets of task card pages. Laminate them, if you wish, for extra durability, then cut out each card to display in your room. You might pin cards to bulletin boards; or punch out the holes and hang them from wall hooks (you can fashion hooks from paper clips and tape these to the wall); or fix cards to cereal boxes with paper fasteners, 4 to a box; or keep cards on designated reference tables. The important thing is to provide enough task card reference points about your classroom to avoid a jam of too many students at any one location. Two or 3 task card sets should accommodate everyone, since different students will use different cards at different times.

HEADS OR TAILS?

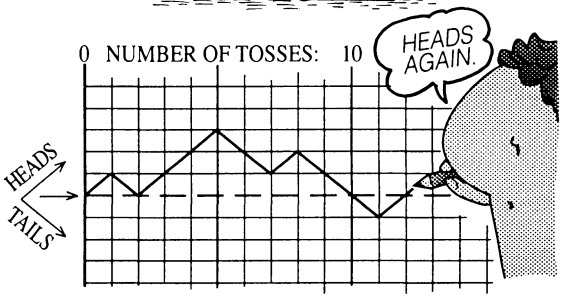
Probability ( )

1. Flip a penny onto a towel or other cushioned surface.



2. Record your result on a Probability Grid: if you get *heads*, draw a diagonal line *upward* from the starting arrow; if you get *tails*, draw it *downward*.

3. Fairly flip the penny a total of 50 times onto the towel. Record each *outcome* (result) as an up and down graph line along the upper dashed line on your grid.

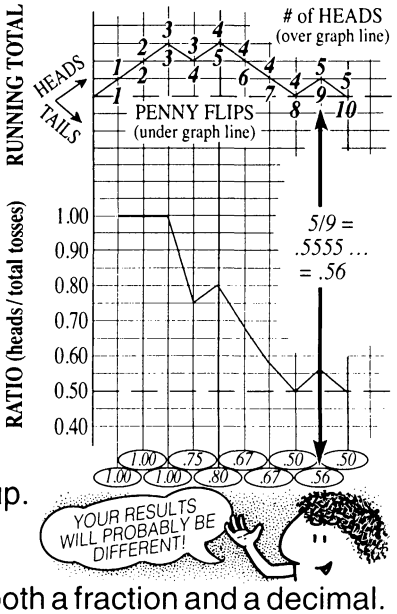


4. Let your results represent a marathon 50-game basketball tournament between the "Heads" and the "Tails".
   a. Who won or was usually ahead?
   b. Is the team with the best results really the better team? Explain. (Hint: Is the coin fair?)
   c. Does a coin "know" when one side or the other is ahead, and try to even the score?

EVEN ODDS

Probability ( )

1. Get your Probability Grid from before.
   a. Write a running total of penny flips *under* your graph line.
   b. Write a running total of heads outcomes *over* your graph line.
2. Each set of numbers above and below this graph line reads like a ratio.
   a. Convert each ratio to a 2-place decimal. Record each result in its oval space below the grid.
   b. Graph these ratios along the *lower* dashed line on your grid.
   c. Compare and contrast the running total graph (above) with the ratio graph (below).



3. A coin is equally likely to land Heads or Tails up. The Probability of Heads is: $P_{(H)} = 1/2 = .50$.
   a. Write the Probability of flipping Tails.
   b. Write your actual outcome after 50 tosses as both a fraction and a decimal.
4. Write your name on your grid and turn it in.