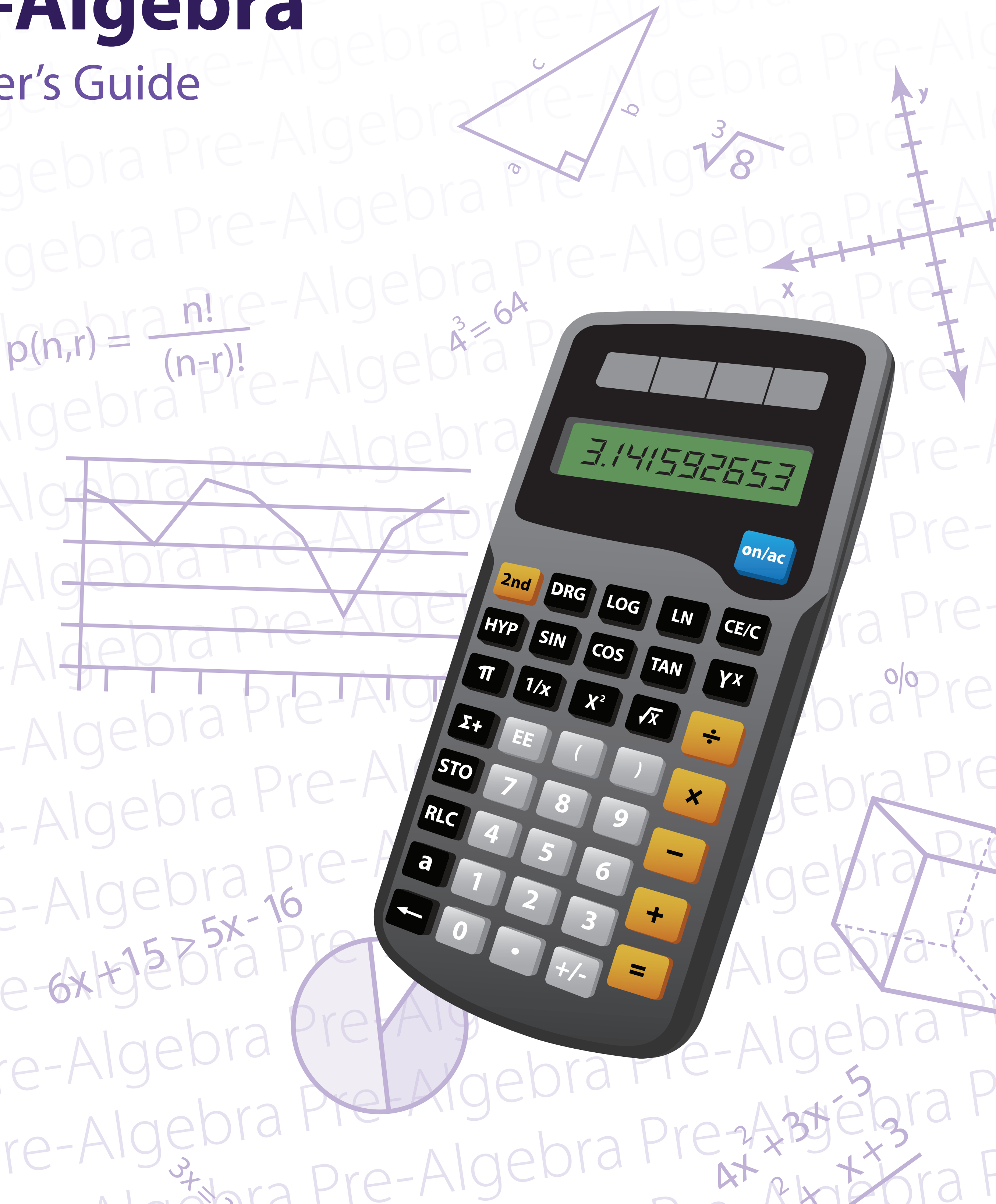
Horizons

Pre-Algebra

Teacher's Guide



Lesson 31

Concepts

- Metric system: Centi-
- Metric system: Milli-
- Metric system conversions
- Order of operations
- Greatest common factor
- Powers of 10
- Math in the real world

Learning Objectives

The student will be able to:

- Define *centi-*
- Define *milli-*
- Apply powers of 10 to convert values within the metric system

Materials Needed

- Student Book, Lesson 31

Teaching Tips

- Review the rules for multiplying and dividing by powers of 10. (See Lessons 28-29)
- Teach the metric prefixes *centi-* and *milli-*. Make sure students correctly identify *centi-* as equal to one hundred*th* and *milli-* as equal to one thousand*th*.
- Explain the additional information on the metric system in the right column. As students progress through their high school science courses, they will see metric units referred to as SI units. Emphasize that these are the same thing.
- Emphasize the capitalization rules for metric unit names and symbols.

Lesson 31

Metric System: Centi- and Milli-

The metric prefix ***centi-*** (abbreviated c) means *one hundredth*, and the metric prefix ***milli-*** (abbreviated m) means one *thousandth*. One gram is equal to 100 centigrams or 1000 milligrams. 4 = 4 centigram = cg; millimeter = mm; To determine the number of base units (grams, meters, liters, etc.) in a given number of centi-units, divide the number of centi-units by 100. For example, 400 cg = 400 ÷ 100 = 4 grams 437 cm = 437 ÷ 100 = 4.37 meters Note that this is the same as dividing by 10^2. To determine the number of base units (grams, meters, liters, etc.) in a given number of milli-units, divide the number of milli-units by 1000. 4000 mg = 4000 ÷ 1000 = 4 grams 4370 mg = 4370 ÷ 1000 = 4.37 grams Note that this is the same as dividing by 10^3. Rule of thumb when converting measurements: When going from a larger unit to a smaller unit (such as from meters to millimeters), multiply. When going from a smaller unit to a larger unit (such as from millimeters to meters), divide.	① **Classwork** 1 g = ____100____ cg 1 g = ____1000____ mg 3 m = ____300____ cm 4.2 L = ____420____ cL 4.2 L = ____4200____ mL 6.17 W = ____6,170____ mW 2000 cg = ____20____ g 2000 mg = ____2____ g 5976 cL = ____59.76____ L 728 cm = ____7.28____ m 728 mm = ____0.728____ m

Activities

② Multiply or divide the appropriate powers of 10 to complete the metric conversions.

7 g = ____700____ cg	8000 cg = ____80____ g
.9 m = ____900____ mm	7348 mL = ____7.348____ L
6.4 L = ____640____ cL	186 cm = ____1.86____ m
13.57 W = ____13,570____ mW	39.5 mw = ____0.0395____ W

③ Simplify.

$(4^2 + 5) \div (7 - 4) = (16 + 5) \div 3 = 21 \div 3 = 7$

$4 - (7 - 5^2) - (5 - 8) = 4 - (7 - 25) - (-3) = 4 - (-18) - (-3) = 4 + 18 + 3 = 22 + 3 = 25$

$(5^2 - (1^2 - 6^2)) \div (2 - 7) =$

$(25 - (1 - 36)) \div (-5) = (25 - (-35)) \div (-5) = (25 + 35) \div (-5) = 60 \div (-5) = -12$

$3^3 \times (9 - 3) \div (8 + 1) - 5 = 27 \times 6 \div 9 - 5 = 162 \div 9 - 5 = 18 - 5 = 13$

Additional Information on the Metric System

- The metric system is now referred to as the SI system, which stands for the *Système International d'Unités*, or the International System of Units.
- All units are spelled with all lowercase letters. The only exceptions are units that are the first word in a sentence, and Celsius, which is always capitalized.
- Prefix symbols representing values less than one million are written in lowercase. (m for milli- or one thousandth)
- Prefix symbol representing values of one million or more are capitalized. (M for mega- or one million)

It's College Test Prep Time!

1. The aerial bucket ride at an amusement park allows a maximum of 8 park guests to exit or board at each stop. The chart below shows how many guests boarded and exited the bucket ride in each of the first 5 stops. If there were 38 guests on the ride at the start, how many were on the ride after the 5^{th} stop?

Stop	A	B	C	D	E
Boarded	6	4	7	8	8
Exited	2	8	5	4	3

A. 10 — 6 + 4 + 7 + 8 + 8 = 33 guests boarded.
B. 16 — 2 + 8 + 5 + 4 + 3 = 22 guests exited.
C. 34 — 33 – 22 = 11 additional guests were on the ride.
D. 38 — 38 + 11 = 49
E. 49

2. Given $x+3=7$ and $y+12=20$, what is the value of $x+y$?

A. 4 — $x = 7 - 3$ $x = 4$
B. 8 — $y = 20 - 12$ $y = 8$
C. 12 — $x + y = 4 + 8 = 12$
D. 32
E. 42

3. In a football game, a touchdown with an extra point is worth a total of 7 points. A field goal is worth 3 points. If a team has 23 points, how many field goals have they scored? (Assume all extra points were made and no safeties or 2-point conversions were scored.)

A. 1
B. 2 — Use trial and error to solve.
C. 3 — 3 touchdowns = 21 points. Only 2 points remain.
D. 4 — 2 touchdowns = 14 points, leaving 9 points.
E. 5 — 9 ÷ 3 = 3 field goals.

4. Given *x* is the square of an integer and a multiple of 9 and 18, find the value of *x*.

A. 3 — Since 18 is a multiple of 9, look for multiples of 18
B. 6 — that are perfect squares. 18, 36, 54, . . .
C. 9 — You should recognize 36 as a perfect square. This
D. 18 — is the value of *x*. Notice that *x* is the square, not
E. 36 — the integer.

A Math Minute with . . .

Steve G. – Race Car Driver

What is your hobby? Auto Racing

Where do you do this? I race on $\frac{3}{8}$- to $\frac{1}{2}$-mile asphalt tracks around the state of Georgia.

Did you attend college? If so, what was your major? I studied Automotive Management and Service at Western Michigan University.

What parts of your hobby require the use of math? Before each race I use math to set up the race car suspension, weight balance, and tire dimensions.

What is the biggest "problem" you have faced that required the use of math to solve? A race car has four footprints, or tire prints. Each tire must carry a certain percentage of the total vehicle weight in order to handle well. I place the car on electronic scales to determine front to rear and side to side bias, or balance. Those numbers help me to determine the cross weight, which is the right front tire and left rear tire, as a percentage of the total weight, or *wedge*.

In addition to calculating weight percentages, the angles on the suspension parts are critical in keeping the tires as flat as possible on the track when cornering. These angles help determine the center of gravity, roll centers, and moment of inertia.

The circumference of the tires, measured around the center, is critical to the car's handling. This is called *tire stagger*.

Mathematical formulas help calculate all these different car measurements. I use a race car computer program to set up the calculations to make the car race faster.

Are there any other interesting math uses you have experienced? The biggest challenge in preparing to race is to set up the car so the front and rear suspensions are balanced. This ensures they work together instead of against each other. While this takes a great deal of measuring and calculating, it hopefully pays off during the race. Sometimes determining what will help a car's performance is simply trial and error.

Assignments

- Complete *It's College Test Prep Time!*
- Read *A Math Minute with …* Steve G. – Race Car Driver

Lesson 83

Concepts

- Perimeter and area of rectangles
- Perimeter and area of squares
- Simple interest
- Probability and odds
- Math in the real world

Learning Objectives

The student will be able to:

- Define *rectangle* and *square*
- Calculate the perimeter and area of rectangle
- Calculate the perimeter and area of a square

Materials Needed

- Student Book, Lesson 83
- Worksheet 42

Teaching Tips

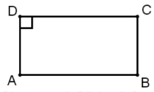
- ➢ Review parallelograms and rhombuses. (See Lesson 82.)
- ➢ Teach rectangles and squares from the teaching box. Ask students if rectangles are parallelograms. (Yes.) Why? (The opposite sides are parallel.) Ask students if squares are parallelograms. (Yes.) Why? (The opposite sides are parallel.)
- ➢ Ask students if rectangles are rhombuses. (Sometimes.) Why or why not? (The opposite sides are not always equal.) Ask students if squares are rhombuses. (Yes.) Why? (They are parallelograms and the opposite sides are equal.)

Rectangles and Squares

A **rectangle** is a parallelogram with four congruent angles. Because a rectangle is a parallelogram, the formulas for perimeter and area remain the same.

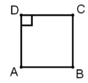
A **square** is a rectangle with four congruent sides. Because a square has four congruent angles and four congruent sides, the formulas for perimeter and area can be simplified as follows:
$P = 4s$, where s is the length of a side
$A = s^2$, where s is the length of a side

List everything you know to be true about the diagram below. Find the perimeter and area.



Given: ▱$DCBA$; $DC = 7$; $CB = 4$
What you know:
It is a parallelogram. It is rectangle.
$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$
Each of the angles is equal to 360° ÷ 4 = 90°.
Perimeter is 2(7) + 2(4) = 14 + 8 = 22 units.
The area is 7(4) = 28 square units.

① **Classwork**
List everything you know to be true about the diagram below. Include the perimeter and area.

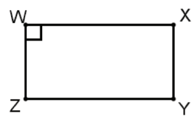


Given: ▱$DCBA$; $DC = 5$; $CB = 5$

The figure is a parallelogram.
The figure is a rhombus.
The figure is a rectangle.
The figure is a square.
The opposite sides are parallel.
The opposite sides are congruent.
Each of the four angles is equal to 360° ÷ 4 = 90°.
Each of the four sides is equal to 5.
The perimeter is 4(5) = 20 units.
The area is 5(5) = 25 square units.

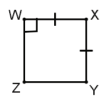
Activities

② List everything you know to be true about the diagrams below. Include the perimeter and area.



Given: ▱$WXYZ$; $WX = 4\frac{1}{2}$; $XY = 2\frac{1}{4}$

WXYZ is a parallelogram and a rectangle.
$\overline{WX} \parallel \overline{ZY}$, $\overline{WZ} \parallel \overline{XY}$, $\overline{WZ} \cong \overline{XY}$, $\overline{WX} \cong \overline{ZY}$
Each of the angles is 360° ÷ 4 = 90°.
The perimeter is
$2\left(4\frac{1}{2}\right) + 2\left(2\frac{1}{4}\right) = 9 + 4\frac{1}{2} = 13\frac{1}{2}$ units.
The area is $\left(4\frac{1}{2}\right)\left(2\frac{1}{4}\right) = \frac{9}{2} \times \frac{9}{4} = \frac{81}{8} = 10\frac{1}{8}$ square units.



Given: ▱$WXYZ$; $WX = 4\frac{1}{2}$

The figure is a parallelogram, a rhombus, a rectangle, and a square.
$\overline{WX} \parallel \overline{ZY}$, $\overline{WZ} \parallel \overline{XY}$, $\overline{WZ} \cong \overline{XY} \cong \overline{WX} \cong \overline{ZY}$
Each angle is 90°; each side is $4\frac{1}{2}$.
The perimeter is $4\left(4\frac{1}{2}\right) = \cancel{4}^{2}\left(\frac{9}{\cancel{2}}\right) = 18$ units.
The area is $\left(4\frac{1}{2}\right)\left(4\frac{1}{2}\right) = \left(\frac{9}{2}\right)\left(\frac{9}{2}\right) = \frac{81}{4} = 20\frac{1}{4}$ square units.

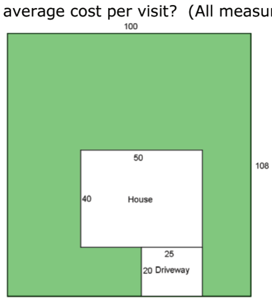
③ Complete the chart to show simple interest.

Principal	Interest Rate	Time, in Years	Amount of Interest
\$1500	6.9%	3	i = \$1500(0.069)(3) = \$310.50
\$3900	4.5%	5	i = \$3900(0.045)(5) = \$877.50
\$5200	9.9%	8	i = \$5200(0.099)(8) = \$4118.40

④ Complete the chart to show probability and odds.

Scenario	Number of Possible Outcomes	Desired Event (E)	P(E)	Odds (in favor of E)	Odds (Against E)
Choosing an integer from 1 to 50	50	Multiple of 7 (7, 14, 21, 28, 35, 42, 49)	P(E) = $\frac{7}{50}$	7:43	43:7
Choosing an integer from 1 to 100	100	Multiple of 11 (11,22,33,44,55, 66,77,88,99)	P(E) = $\frac{9}{100}$	9:91	91:9

⑤ Solve.
Mike offers his customers an annual lawn care plan. He charges \$0.01 per square foot of grass per month for 12 months to cover mowing, edging, and blowing. What should Mike charge each month for the rectangular yard pictured below? What is the total cost of an annual contract for this lawn? If Mike's contract includes 20 lawn care visits, what is the average cost per visit? (All measurements are given in feet.)



Area of lot: 100 × 108 = 10,800 sq. ft.
Area of house: 40 × 50 = 2000 sq. ft.
Area of driveway: 20 × 25 = 500 sq. ft.
Area of grass: 10,800 – (2000 + 500) = 10,800 – 2500 = 8300 sq. ft.

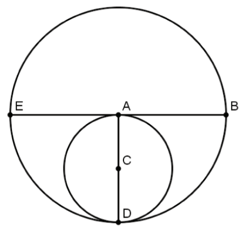
Cost per month of an annual contract is 8300 × \$0.01 = \$83

Cost per year for an annual contract is \$83 × 12 = \$996

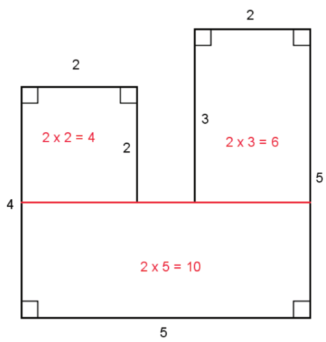
Average cost for each of 20 visits is \$996 ÷ 20 = \$49.80

What percent of the above lot is covered with grass? Round your answer to the nearest tenth of a percent.
8300 ÷ 10,800 = 0.7685 = 76.9%

It's College Test Prep Time!



1. In the figure above, *A* is the center of the large circle and *C* is the center of the small circle. If *CD* = 3, what is the length of $\overline{EB}$?
   - A. 6
   - B. 9
   - C. <u>12</u> $\overline{CD}$ is the radius of circle *C*, so *AD* = 6, which is the radius of circle *A*. $\overline{EB}$ is the diameter of circle *A*, so it is twice the radius, or 12.
   - D. 15
   - E. 18



2. What is the area of the figure above?
   - A. <u>20</u> Divide the figure into smaller shapes: 4 + 6 + 10 = 20.
   - B. 22
   - C. 23
   - D. 24
   - E. 25

A Math Minute with . . .

Scott M. – Crane Operator

What is your occupation? I work in crane safety operations.

Where do you work? I work at a railroad intermodal terminal.

Did you attend college? If so, what was your major? No.

What parts of your job require the use of math? One place where I use math is when lining up trains. I also use math to determine what types of cars and engines are needed for a train. Each train has a maximum total length. The individual train cars have different lengths, so the lengths of each car must be considered when designing and building a train. I am also responsible for ordering the safety equipment for the terminal. Because I must stay within a budget, I calculate the cost of different equipment in order to find the least expensive way to get the equipment we need to meet safety standards.

What is the biggest "problem" you have faced that required the use of math to solve? A train that was 7000 feet long came into the terminal. I needed to add more cars to the train. In order to do this, I had to consider the different types of cars available and the length and capacity of each car. I rearranged the train to get the maximum cargo load without exceeding the maximum train length allowed.

Are there any other interesting math uses you have experienced? The process of designing trains is interesting. We input all the available cars and train data into a computer. We use the mouse to move the cars around on the screen to design the train exactly the way we want. We then send the information to the yard to begin assembling the actual train.

Teaching Tips, Cont.

- When all students are finished taking the test, introduce *It's College Test Prep Time* from the student book. This page may be completed in class or assigned as homework.
- Have students read the Math Minute interview for lessons 91-100.

Assignments

- Complete *It's College Test Prep Time!*
- Read *A Math Minute with…* Scott M. – Crane Operator

Lesson 112

Concepts

- Functions
- Graphs of functions
- Graphing two-variable equations

Learning Objectives

The student will be able to:

- Define *function*
- Define *domain*
- Define *range*
- Use the vertical line test to determine if a graph is a function
- Identify the domain and range of a function
- Draw the graph of a function

Materials Needed

- Student Book, Lesson 112
- Worksheet 56
- Graph paper
- Straightedge

Teaching Tips

- Have students complete Worksheet 56 in class. This may be for added practice of earlier topics or graded as a quiz, if desired.
- Teach the definition of *function* from the teaching box. Explain that the domain is the same as the values of the independent variable in a two-variable equation and the range is the same as the values of the dependent variable in a two-variable equation.
- Write these equations on the board.

$y = x^2$

$x = y^2$

Functions and Graphs

A **function** is an equation in which each value of the independent variable has exactly one corresponding value of the dependent variable.

The values assigned to the independent variable are called the **domain**.

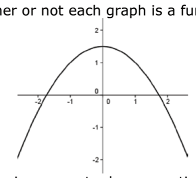
The corresponding values of the dependent variable are called the **range**.

A function is written in the format $f(x)$ and is read, "the function *f* of *x*," or, "the *f* of *x*."

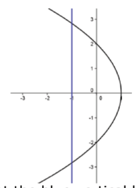
When graphing a function, the $f(x)$ side of the equation corresponds to the *y* portion of an equation. Plot points as usual and graph.

To look at a graph and instantly determine whether or not the graph is a function, use the **vertical line test**. If you can draw a vertical line on the graph and cross the graph in two or more points, the graph is not a function. Otherwise, the graph is a function.

Tell whether or not each graph is a function.

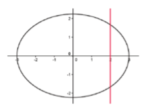


Yes. There is no way to draw a vertical line that intersects the graph in more than one point.

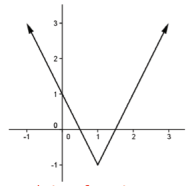


No. Notice that the blue vertical line intersects the graph in two places.

① **Classwork**
Tell whether or not each graph is a function.

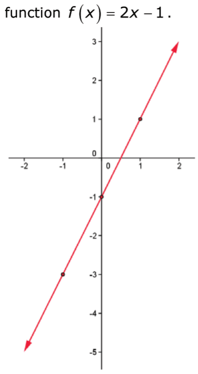


No. A vertical line intersects the graph in two places.



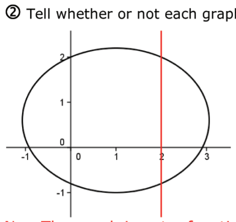
Yes. The graph is a function.

Graph the function $f(x) = 2x - 1$.

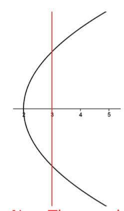


Activities

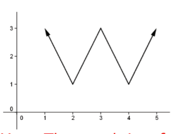
② Tell whether or not each graph is a function.



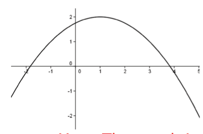
No. The graph is not a function.



No. The graph is not a function.



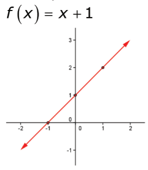
Yes. The graph is a function.



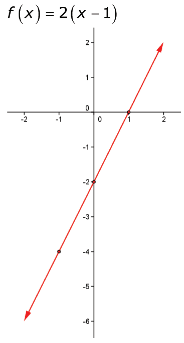
Yes. The graph is a function.

③ Graph the following functions on your own graph paper.

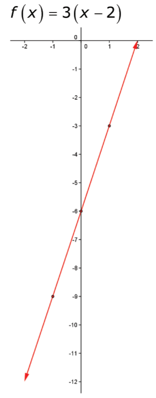
$f(x) = x + 1$



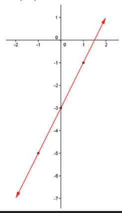
$f(x) = 2(x - 1)$



$f(x) = 3(x - 2)$



$f(x) = 2x - 3$



Lesson 129

Concepts
- Algebra tiles
- Multiplying binomials by monomials
- Math in the real world

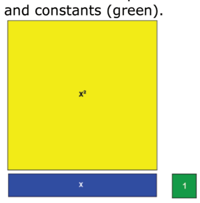
Learning Objectives
The student will be able to:
- Use algebra tiles to represent algebraic expressions
- Show the solution of an algebraic problem using algebra tiles

Materials Needed
- Student Book, Lesson 129
- Algebra tiles from the *Tests and Resources* book
- Scissors
- Zip-top sandwich bags
- Tape

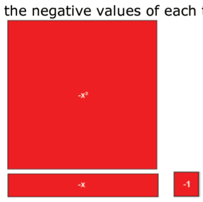
Teaching Tips
- Review signed numbers. (See Lesson 3)
- Have the students cut out the algebra tiles from the Tests and Resources book if they have not already done so. Tiles should be stored in a zip-top bag taped inside the cover of the book for use in future lessons.
- Introduce algebra tiles as a method of representing expressions with variables. Tell the students that their set has four colors: yellow, blue, green, and red. The yellow tiles represent x^2 terms; the blue tiles represent x terms; the green tiles represent constant terms; the corresponding red tiles represent negative terms.

Using Algebra Tiles

Algebra tiles can help you see what is happening in an algebra problem. There are three different sizes of tiles to represent x^2 (yellow), x (blue), and constants (green).



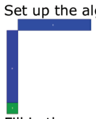
There is also a second set in red to represent the negative values of each term.



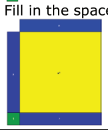
To show the multiplication of terms, place the multiplicand and the multiplier perpendicular to each other. Fill in the rectangular area with the appropriate pieces to get the answer.

Solve $x(x+1)$ using algebra tiles.

Set up the algebra tiles to look like this.

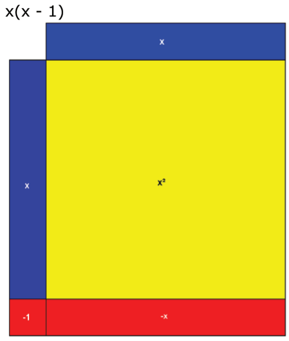


Fill in the space to show $x^2 + x$ as the answer.



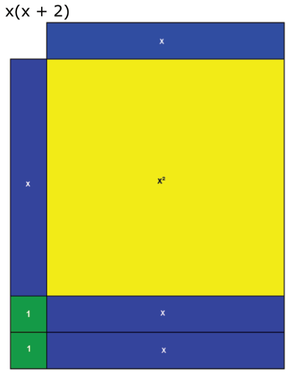
① **Classwork.**
Solve using algebra tiles.

x(x - 1)



$x^2 - x$

x(x + 2)



$x^2 + 2x$

Lesson 156

Concepts

- Area of plane figures
- Volume of prisms
- Volume of cones
- Surface area of cones
- Parts of a circle
- Types of angles
- Parallel lines

Learning Objectives

The student will be able to:

- Calculate the area of plane figures
- Calculate the volume of prisms having different bases
- Calculate the volume and surface area of cones
- Identify parts of a circle
- Identify angles as acute, right, obtuse, or straight
- Calculate the measure of complementary and supplementary angles
- Apply properties of parallel lines cut by a transversal to find the measure of angles

Materials Needed

- Student book, Lesson 156
- Formula strip, Lesson 156

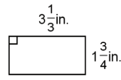
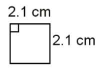
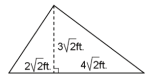
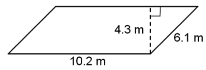
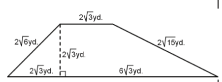
Teaching Tips

- ➢ Review Lessons 91-107.
- ➢ Review finding the volume of solid figures. (See Lessons 91-96.)
- ➢ Review parts of a circle. (See Lesson 103.)

Review

Activities

① Find the area of each base, and the volume of a prism having the indicated height.

Base of Prism	Area of Base	Prism Height	Volume of Prism
	$A = bh$ $A = (3\frac{1}{3}\text{ in.})(1\frac{3}{4}\text{ in.})$ $A = (\frac{10}{3}\text{ in.})(\frac{7}{4}\text{ in.})$ $A = \frac{35}{6} = 5\frac{5}{6}$ sq. in.	$3\frac{3}{5}$ in.	$V = Bh$ $V = (5\frac{5}{6}\text{ sq. in.})(3\frac{3}{5}\text{ in.})$ $V = \left(\frac{\overset{7}{\cancel{35}}}{\cancel{6}}\text{ sq. in.}\right)\left(\frac{\overset{3}{\cancel{18}}}{\cancel{5}}\text{ in.}\right)$ $V = 21$ cubic in.
	$A = s^2$ $A = (2.1\text{ cm})^2$ $A = 4.41\text{ cm}^2$	2.1 cm	$V = s^3$ $V = (2.1\text{ cm})^3$ $V = 9.261\text{ cm}^3$
	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(2\sqrt{2}\text{ ft.} + 4\sqrt{2}\text{ ft.})(3\sqrt{2}\text{ ft.})$ $A = \frac{1}{2}(6\sqrt{2}\text{ ft.})(3\sqrt{2}\text{ ft.})$ $A = (3\sqrt{2}\text{ ft.})(3\sqrt{2}\text{ ft.})$ $A = 18$ sq. ft.	$4\sqrt{3}$ ft.	$V = Bh$ $V = (18\text{ sq. ft.})(4\sqrt{3}\text{ ft.})$ $V = 72\sqrt{3}$ cubic ft.
	$A = bh$ $A = (10.2\text{ m})(4.3\text{ m})$ $A = 43.86\text{ m}^2$	7.03 m	$V = Bh$ $V = (43.86\text{ m}^2)(7.03\text{ m})$ $V = 308.3358\text{ m}^3$
	$A = \frac{1}{2}(b_1 + b_2)h$ $A = \frac{1}{2}(2\sqrt{3}\text{ yd.} + 6\sqrt{3}\text{ yd.} + 2\sqrt{3}\text{ yd.})(2\sqrt{3}\text{ yd.})$ $A = \frac{1}{2}(10\sqrt{3}\text{ yd.})(2\sqrt{3}\text{ yd.})$ $A = (5\sqrt{3}\text{ yd.})(2\sqrt{3}\text{ yd.})$ $A = 30$ sq. yd.	$5\sqrt{2}$ yd.	$V = Bh$ $V = (30\text{ sq. yd.})(5\sqrt{2}\text{ yd.})$ $V = 150\sqrt{2}$ cubic yd.

② Complete the chart for cones.

Radius	Height	Slant Height	Volume	Lateral Area	Surface Area
5.2 in.	1.8 in.	1.8 in.	$V = \frac{1}{3}\pi(5.2)^2(1.8)$ $V = \frac{1}{3}\pi(27.04)(1.8)$ $V = 16.224\pi$ cu. in.	$LA = \pi(5.2)(1.8)$ $LA = \pi(9.36)$ $LA = 9.36\pi$ sq. ft.	$SA = 9.36\pi + \pi(5.2)^2$ $SA = 9.36\pi + \pi(27.04)$ $SA = 9.36\pi + 27.04\pi = 36.4\pi$ sq. in.
6 m	8 m	10 m	$V = \frac{1}{3}\pi(6)^2(8)$ $V = \frac{1}{\cancel{3}}\pi(\overset{12}{\cancel{36}})(8)$ $V = 96\pi\text{ m}^3$	$LA = \pi(6)(10)$ $LA = \pi(60)$ $LA = 60\pi\text{ m}^2$	$SA = 60\pi + \pi(6)^2$ $SA = 60\pi + \pi(36)$ $SA = 60\pi + 36\pi = 96\pi\text{ m}^2$