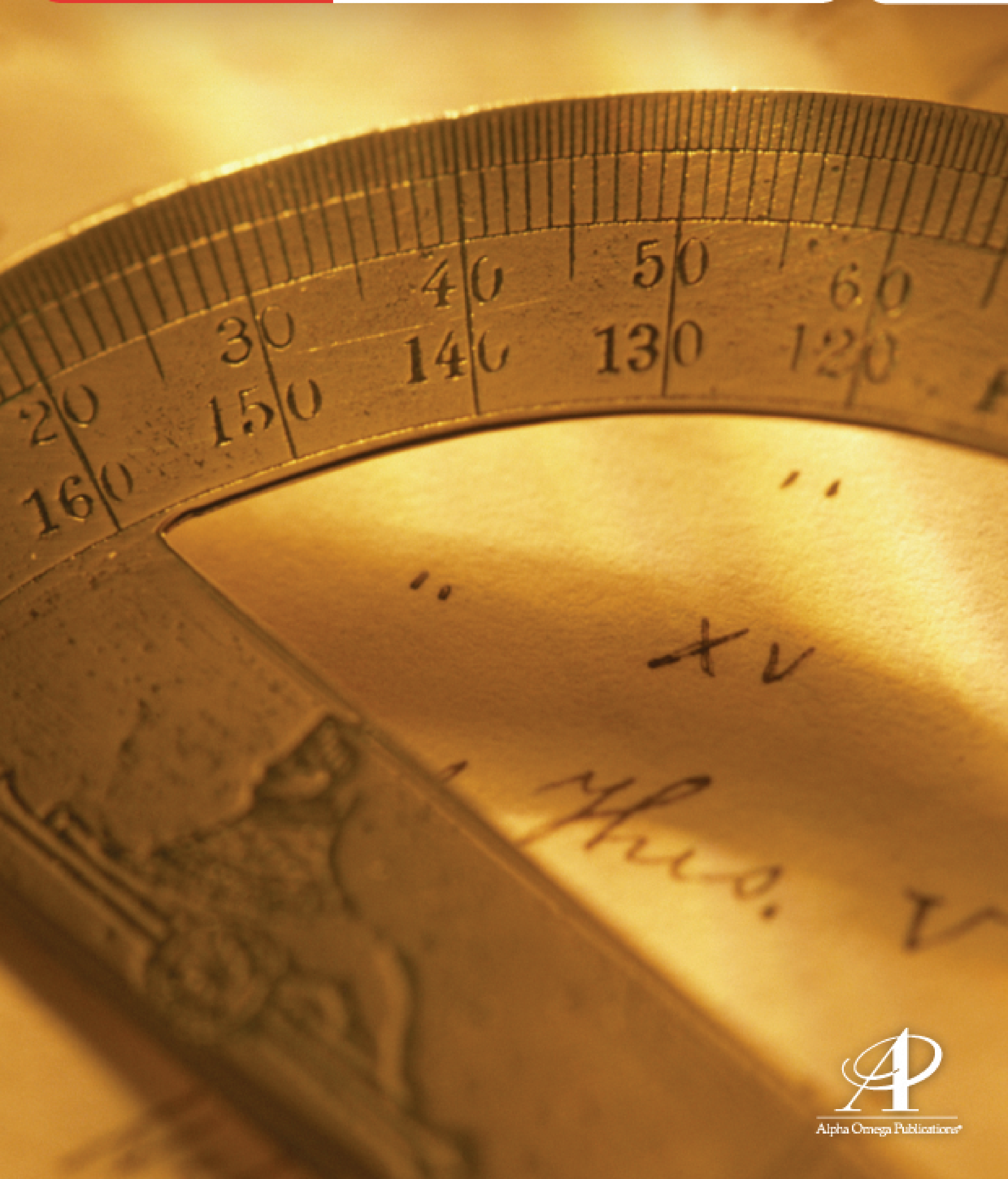
LIFEPAC® **Math**



Alpha Omega Publications®

MATHEMATICS 1210
CALCULUS AND REVIEW

CONTENTS

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CALCULUS AND REVIEW

This LIFEPAC® contains four sections. The first section, Mathematical Induction, is a form of proof to be used on expressions or formulas that have consecutive integers in their results. The second section is an introduction to limits and an application of limits. The limit is the foundation of the derivative and the integral that make up the differential calculus and the integral calculus. The third section is an introduction to slope of functions. The fourth section is a review that consists of major concepts of all ten LIFEPACs.

OBJECTIVES

Read these objectives. The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC.

When you have completed this LIFEPAC, you should be able to:

1. Evaluate summation statements.
2. Prove propositions by mathematical induction.
3. Evaluate functions.
4. Evaluate difference quotients.
5. Evaluate limits of functions.
6. Find the slope of a function by the limit process.
7. Find the angle of intersection between two lines.
8. Solve problems representative of precalculus.

Survey the LIFEPAC. Ask yourself questions about this study. Write your questions here.

I. MATHEMATICAL INDUCTION

OBJECTIVES

1. Evaluate summation statements.
2. Prove propositions by mathematical induction.

Many expressions and formulas appear in mathematics that are a function of integers. An example is the rule of exponents, $A^m \cdot A^n = A^{m+n}$, where m and n are integers. We need to prove that this role is true for all integers. Mathematical induction is the method of proving a proposition for all integers. Many of the propositions that are proved by mathematical induction are in the form of a mathematical series. Summation notation is used to express the indicated sums of these series.

SUMMATION

Many formulas can be expressed in summation notation. For example, the indicated sum of the first hundred integers would be a very long expression: $1 + 2 + 3 + \ldots + 100$. Summation notation allows us to write the indicated sum as

$$1 + 2 + 3 + \ldots + 100 = \sum_{i=1}^{100} i.$$

The summation notation has three parts.

a. The symbol used for summation is $\sum$, the Greek letter sigma.

b. A function of an integer is usually represented by i or j.

c. The lower limit is the beginning integer and the upper limit is the ending integer

The sum of the first thirty odd integers can be symbolized as

$$\sum_{i=1}^{30} 2i - 1.$$

To evaluate the preceding summation, we substitute the lower limit 1 for i in the function for the first term. We substitute 2 for i for the second term. We substitute 3 for i for the third term, and so on for all thirty terms.

Examples are

$$\sum_{i=1}^{30} f(i) = f(1) + f(2) + f(3) + \ldots + f(30)$$

and

$$\sum_{i=1}^{30} 2i - 1 = 1 + 3 + 5 + 7 + \ldots + 59.$$

Model 1: Evaluate $\sum_{i=1}^{5} i^2$.

$$\begin{aligned}\sum_{i=1}^{5} i^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2\\ &= 1 + 4 + 9 + 16 + 25\\ &= 55\end{aligned}$$

Model 2: Find $\sum_{i=2}^{4} 5i - 1$.

$$\begin{aligned}\sum_{i=2}^{4} 5i - 1 &= [5(2) - 1) + (5(3) - 1]\\ &\quad + (5(4) - 1)\\ &= 9 + 14 + 19\\ &= 42\end{aligned}$$

Model 3: Write the terms of $\sum_{i=1}^{3} \sum_{j=1}^{4} a(i, j)$.

$a(1, 1) + a(1, 2) + a(1, 3) + a(1, 4) +$
$a(2, 1) + a(2, 2) + a(2, 3) + a(2, 4) +$
$a(3, 1) + a(3, 2) + a(3, 3) + a(3, 4)$.

Evaluate the following summations.

1.1 $$\sum_{i=1}^{5} 2i + 1$$

1.2 $$\sum_{i=1}^{7} i - 1$$

1.3 $$\sum_{i=1}^{3} i^3$$

1.4 $$\sum_{i=1}^{4} i^{-1}$$

1.5 $$\sum_{i=1}^{5} 2^i$$

1.6 $$\sum_{i=1}^{3} 2^{-i}$$

1.7 $$\sum_{i=3}^{6} (i - 1)^2$$