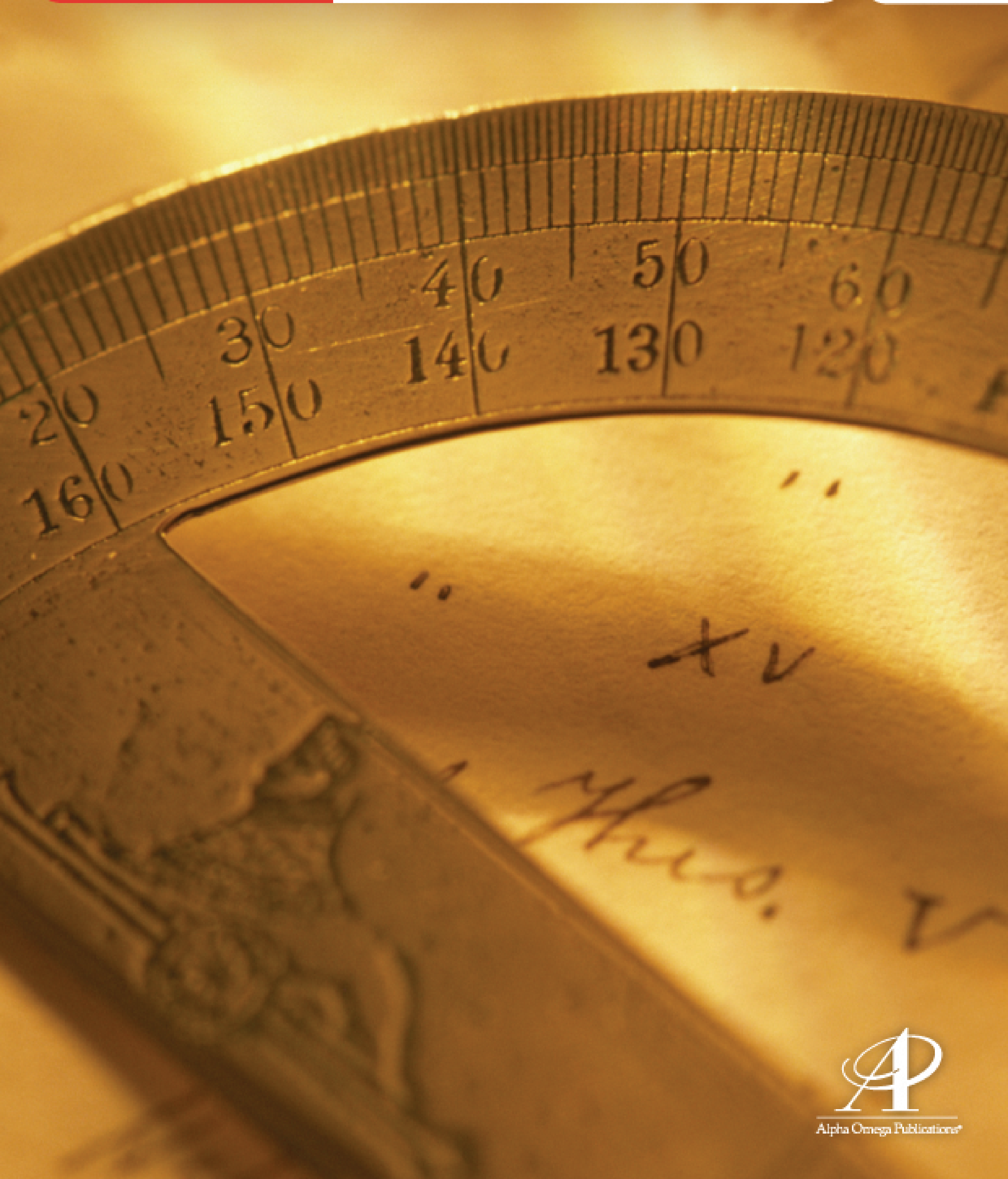
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1208
QUADRATIC EQUATIONS

CONTENTS

Authors:	**Larry L. Welch, M.A.**
	Robert L. Zenor, M.A., M.S.
Editor-in-Chief:	Richard W. Wheeler, M.A.Ed.
Editor:	Stephany L. Sykes
Consulting Editor:	Rudolph Moore, Ph.D.
Illustrator:	Thomas R. Rush

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804 N. 2nd Ave. E., Rock Rapids, IA 51246-1759

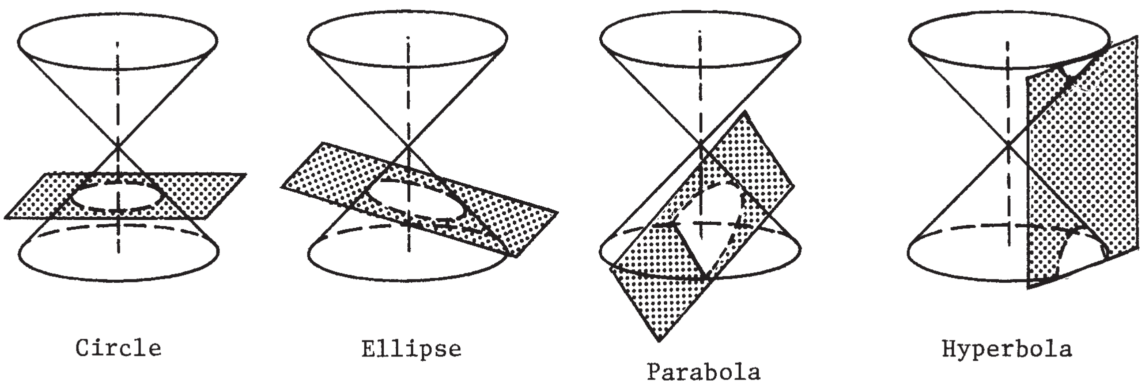
QUADRATIC EQUATIONS

The general form for all *quadratic equations* in x and y is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where the coefficients, A, B, C, D, E, and F *are real numbers and at least one coefficient,* A, B, or C, is a number other than zero.

The second-degree equations are derived from a plane intersecting a right circular cone.



The circle, the ellipse, the parabola, and the hyperbola are called the *conic sections*. When the intersecting plane is perpendicular to the axis of the cone, the section formed is a *circle*. When the intersecting plane is rotated so that it is not perpendicular to the axis of the cone but intersects both edges of the cone, the section formed is an *ellipse*. When the intersecting plane is parallel to one edge of the cone, the section formed is a *parabola*. When the intersecting plane is parallel to the axis of the cone, the section formed is a *hyperbola*.

The graphs of the conic sections can assume any position on the coordinate axis. Transformation of the equation simplifies the equation and puts its graph in standard form.

OBJECTIVES

Read these objectives. The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC®.

When you have completed this LIFEPAC, you should be able to:

1. Identify the conic sections, find specific information relating to each of them, and graph related equations.
2. Rotate and translate an equation into a simpler equation and sketch its curve.

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

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I. CONIC SECTIONS: CIRCLE AND ELLIPSE

OBJECTIVE

1. Identify the conic sections, find specific information relating to each of them, and graph related equations.

The equation for each of the conic sections will be derived from a geometric definition of *locus of points*.

DEFINITION

Locus of points is the set of all the points, and only those points, that satisfy a given condition.

You should become familiar with a formal definition of *conic* sections, sometimes simply called *conics*.

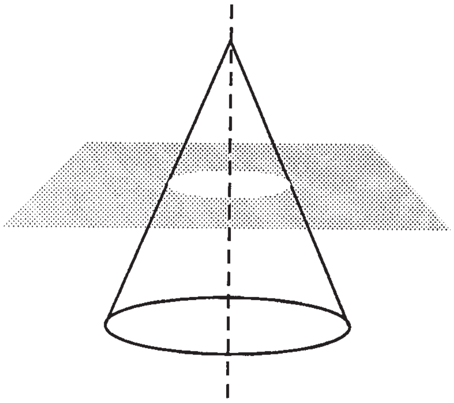
DEFINITION

Conic section is the locus of a point that moves so that the ratio of its distance from a fixed point to its distance from a fixed line is constant.

The ratio of a curve is called its *eccentricity* and is always denoted by e. When $e = 1$, the conic is a parabola. When $e < 1$, the conic is an *ellipse*. When $e > 1$, the conic is a hyperbola. As e approaches zero, the corresponding ellipses become more and more circular, approaching the circle as a limit. The fixed line is called the *directrix*.

THE CIRCLE

The circle is probably the most widely used geometric configuration in our world today. The circle appears equally as often in nature as it does in man's inventions. The study of the circle is responsible for the discovery of the number π by early mathematicians and also the study of irrational numbers.

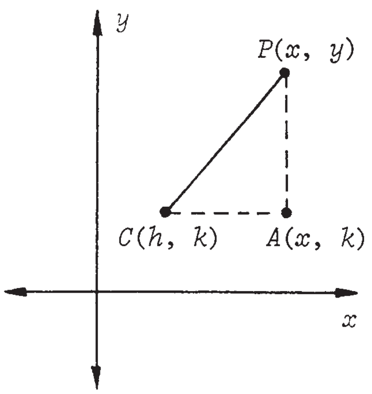


A circle is the section of a cone formed when the cutting plane intersects the edges of a cone and is perpendicular to the axis of the cone.

DEFINITION

A *circle* is the locus of points that are at a constant distance from a fixed point.

The fixed point (h, k) is the *center* of the circle and the constant is the *radius*, r, of the circle. The point (h, k) and the constant r are the elements of the set of real numbers.



STANDARD FORM

On the coordinate axis, locate a point C with center (h, k) as the fixed point and point $P(x, y)$ as the radius, or the point that moves. Draw $\overline{PA} \perp \overline{CA}$. ΔCAP is a right triangle and $\overline{CP} = r$.

Using the distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

$$\overline{CA} = \sqrt{(x - h)^2 + (k - k)^2} = \sqrt{(x - h)^2} = |x - h|$$

and $\overline{AP} = \sqrt{(x - x)^2 + (y - k)^2} = \sqrt{(y - k)^2} = |y - k|$.

By the Pythagoreon Theorem, $(x - h)^2 + (y - k)^2 = r^2$, which is the standard form of the equation with (h, k) as the center and $|r|$ as the radius. If $(h, k) = (0, 0)$, which means that the center is at the origin, the equation becomes $x^2 + y^2 = r^2$.

STANDARD EQUATIONS OF THE CIRCLE

Center at origin: $x^2 + y^2 = r^2$

Center at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

Model 1: Find the standard equation of a circle with center at (5, -1) and radius of 6.

Given: $(h, k) = (5, -1)$ and $r = 6$.

Using the distance formula square, $(x - h)^2 + (y - k)^2 = r^2$,

we obtain $(x - 5)^2 + (y + 1)^2 = 36$.

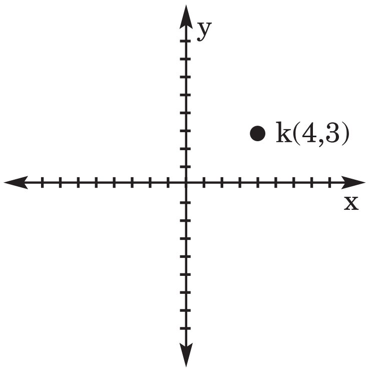
Model 2: Find the standard equation of a circle with center at the origin and passing through the point (4, 3).

Using the distance formula,

radius $= \sqrt{(4 - 0)^2 + (3 - 0)^2} = 5$.

Therefore, $x^2 + y^2 = r^2$ and the equation of the circle is

$$x^2 + y^2 = 25.$$



Note: If $r = 0$, $x^2 + y^2 = 0$. The only point belonging to this equation is (0,0). If $r < 0$, the circle does not exist.