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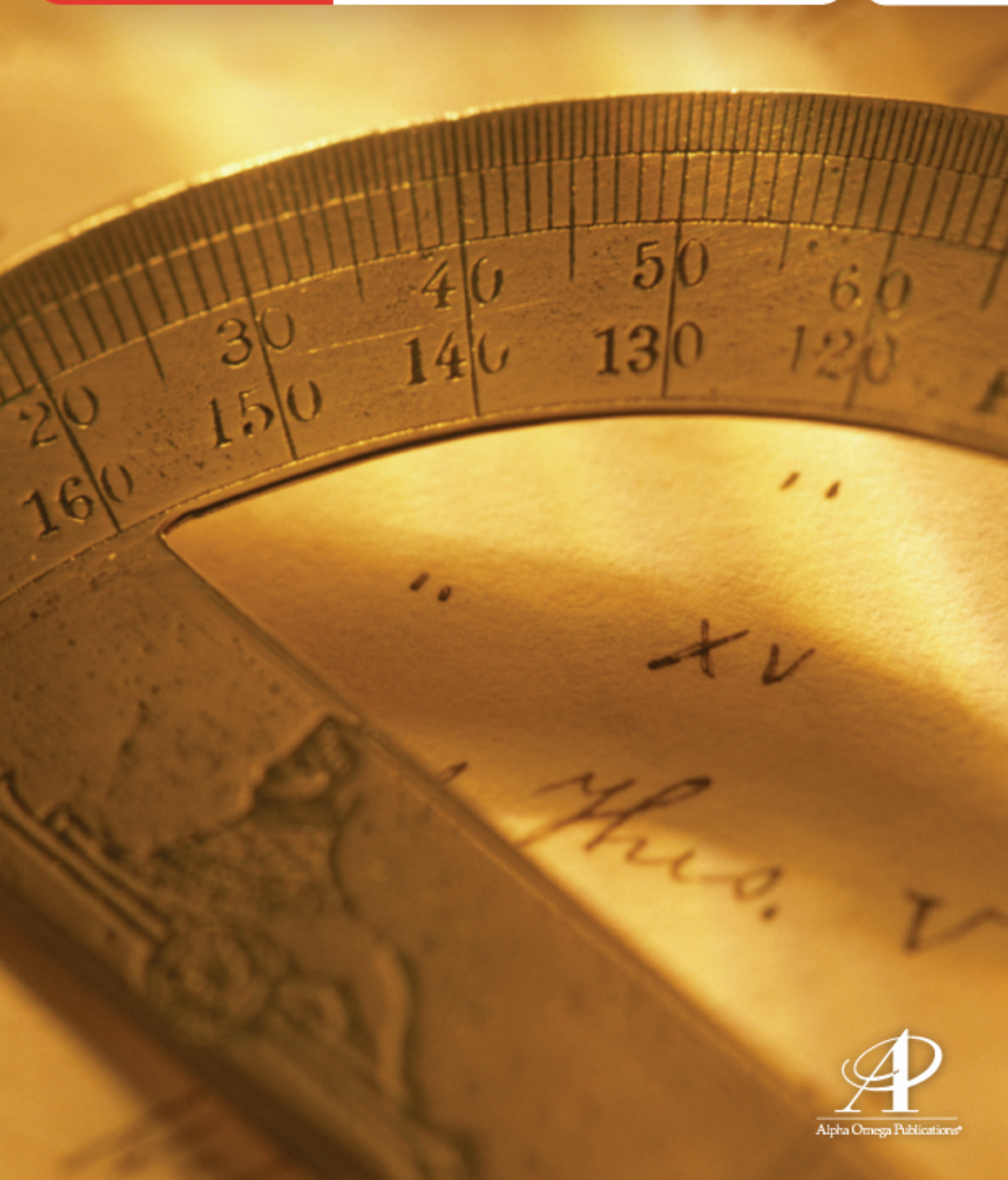

Alpha Omega Publications®

MATHEMATICS 1206
APPLICATION OF TRIGONOMETRIC FUNCTIONS

CONTENTS

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APPLICATION OF TRIGONOMETRIC FUNCTIONS

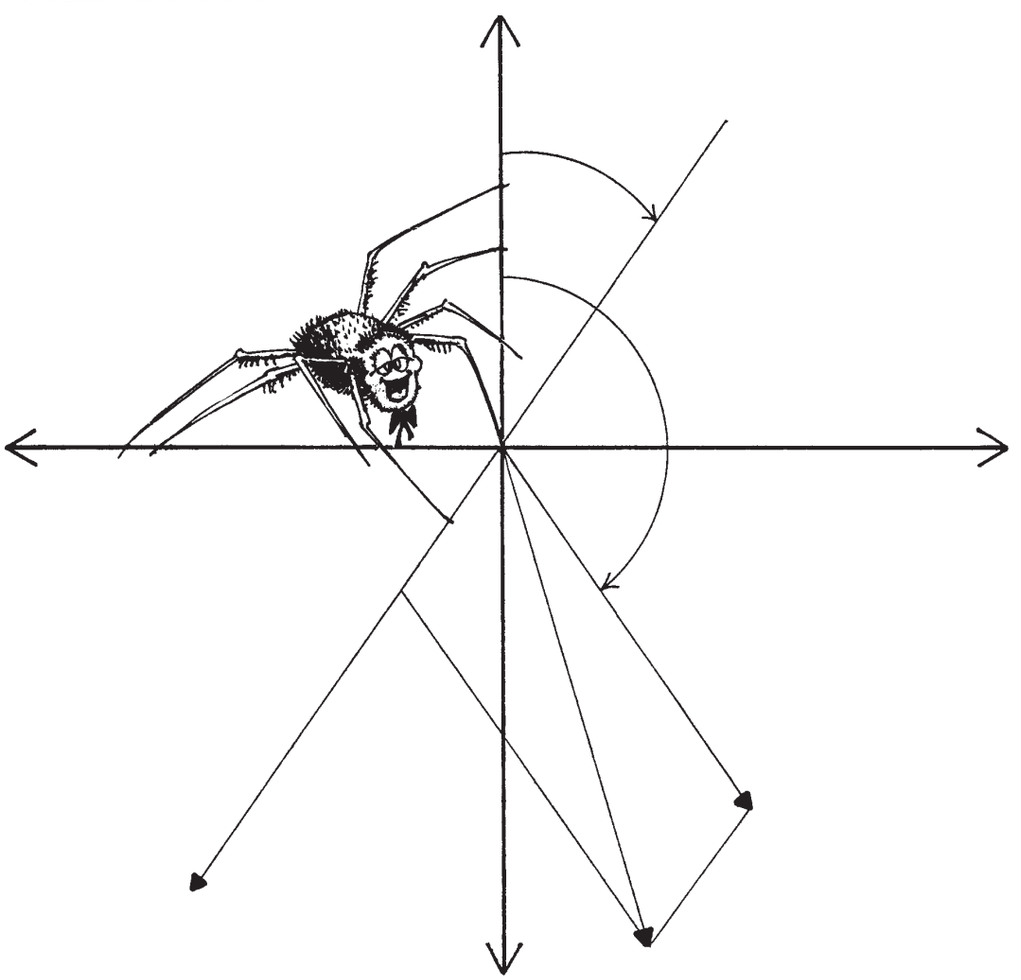
In this LIFEPAC® you will learn more about the trigonometric functions which were the subject of Mathematics LIFEPAC 1203. You will learn to find the trigonometric functions for any given angle and to work with scalar quantities, vectors, and resultants. You will also study the Law of Sines and the Law of Cosines and will use them to solve application problems. Mastery of this LIFEPAC will prepare you to move on to the inverses of the trigonometric functions.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have completed this LIFEPAC, you should be able to:

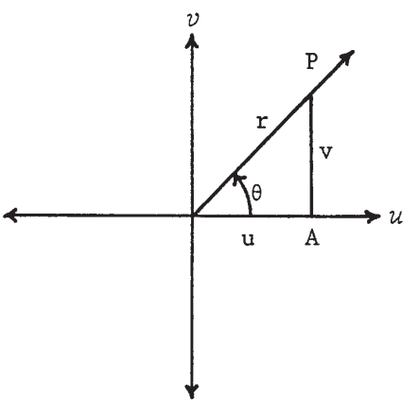
1. Define the trigonometric functions with reference to the right triangle.
2. Use the right triangle, trigonometric functions, and parallelogram of forces in applied problems to determine horizontal and vertical components, and the resultant.
3. Solve applied problems by using the Laws of Cosines and Sines.
4. Solve navigation problems using the trigonometric functions and the Laws Cosines and Sines.



I. TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

Let θ be any angle in standard position. P is any point not at the origin on the terminal side of θ, and $\overline{PA}$ is perpendicular to the u-axis. The functions are defined in the following illustrations:

QUADRANT I



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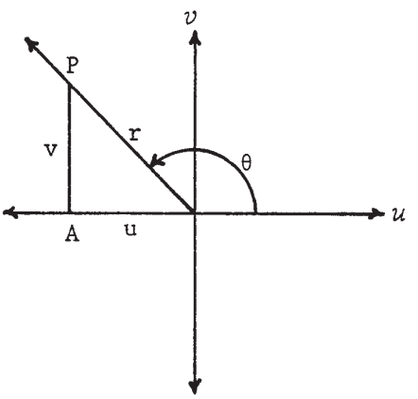
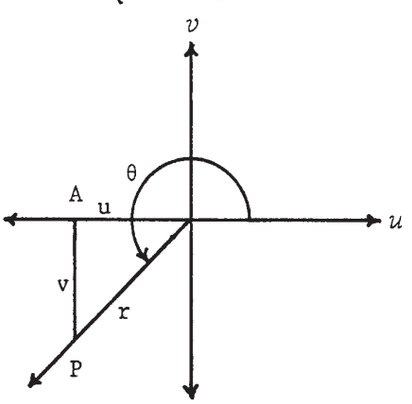
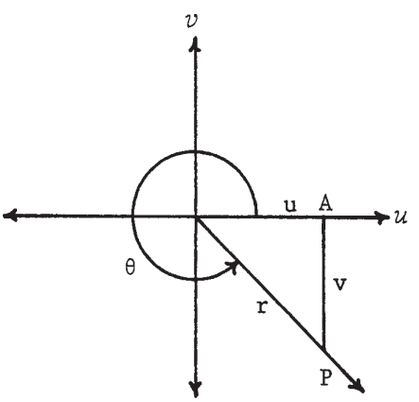


QUADRANT III



QUADRANT IV



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{v}{r} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{r}{v}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{u}{r} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{r}{u}$$

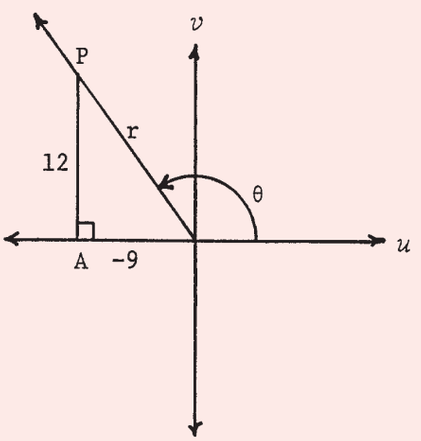
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{v}{u} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{u}{v}$$

These ratios are called functions of θ since they depend solely on the value of θ and are independent of the position of P on the terminal side of the angle.

STUDY THESE EXAMPLES:

Find the value of r for each of the following positions of P, and then find the trigonometric functions of angle θ. Leave answers in fractional form.

P = (-9, 12)



$r^2 = u^2 + v^2$

$r^2 = (-9)^2 + (12)^2$

$r^2 = 81 + 144$

$r^2 = 225$

$r = 15$

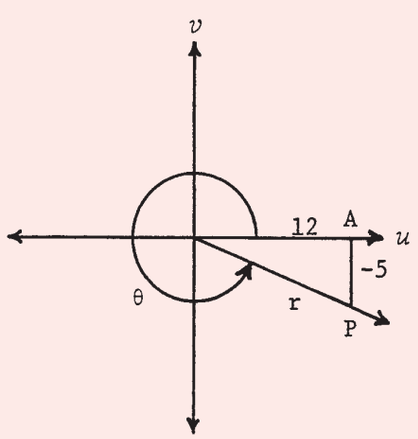
Using the definitions of the trigonometric functions, we have

$\sin\theta = \frac{12}{15} = \frac{4}{5}$ $\qquad$ $\csc\theta = \frac{15}{12} = \frac{5}{4}$

$\cos\theta = \frac{-9}{15} = \frac{-3}{5}$ $\qquad$ $\sec\theta = \frac{15}{-9} = \frac{5}{-3}$

$\tan\theta = \frac{12}{-9} = \frac{4}{-3}$ $\qquad$ $\cot\theta = \frac{-9}{12} = \frac{-3}{4}$

Given P(12, -5), determine the trigonometric functions of angle θ:



$r^2 = u^2 + v^2$

$r^2 = (12)^2 + (-5)^2$

$r^2 = 144 + 25$

$r^2 = 169$

$r = 13$

$\sin\theta = \frac{-5}{13}$ $\qquad$ $\csc\theta = \frac{13}{-5}$

$\cos\theta = \frac{12}{13}$ $\qquad$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{-5}{12}$ $\qquad$ $\cot\theta = \frac{12}{-5}$

COMPLETE THIS EXERCISE.

Given the ordered pair P, determine the sin, cos, and tan of angle θ.

		SIN	COS	TAN
1.1	P(4, 5)	a._____	b._____	c._____
1.2	P(-5, -3)	a._____	b._____	c._____
1.3	P(5, 5)	a._____	b._____	c._____
1.4	P(-4, 3)	a._____	b._____	c._____
1.5	P(6, -5)	a._____	b._____	c._____

The previous exercise made use of the right triangle in determining the trigonometric functions. Properties of the right triangle are listed for your review:

1. The square of the hypotenuse is equal to the sum of the squares of the other two sides ($r^2 = u^2 + v^2$).

2. The acute angles are complements of each other; that is, the sum of the two acute angles is 90° (A + B = 90°).

3. The hypotenuse is greater than either of the other sides and is less than their sum.

4. The greater angle is opposite the greater side, and the greater side is opposite the greater angle.

The remaining part of this section will deal with the use of the right triangle in applied problems. Every problem should be approached and solved in a planned and systematic manner. A planned and systematic approach develops habits of clear and ordered thinking, develops understanding of the principles involved, and reduces errors to a minimum. The following suggestions are listed as a guide:

1. Make a drawing of the triangle and mark the known (given) elements. This procedure shows the relation of the elements and helps in choosing the functions needed.

2. List what is to be found.

3. To find an unknown element, select a formula that contains two known elements and the required unknown element. Substitute the known elements in the formula and solve for the unknown.

4. Check your solution to see if your answer is reasonable.