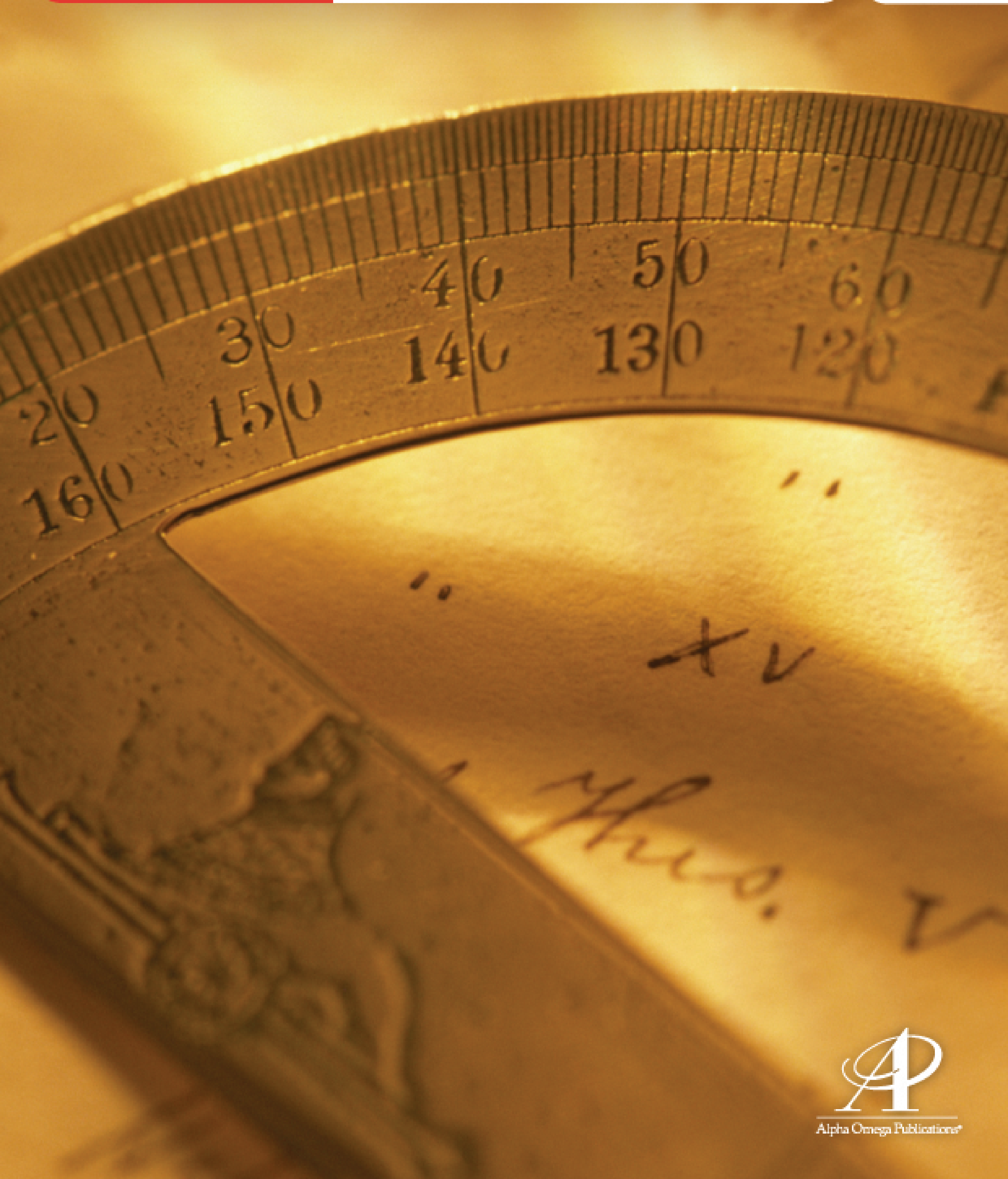
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1204
THE CIRCULAR FUNCTIONS AND THEIR GRAPHS

CONTENTS

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THE CIRCULAR FUNCTIONS AND THEIR GRAPHS

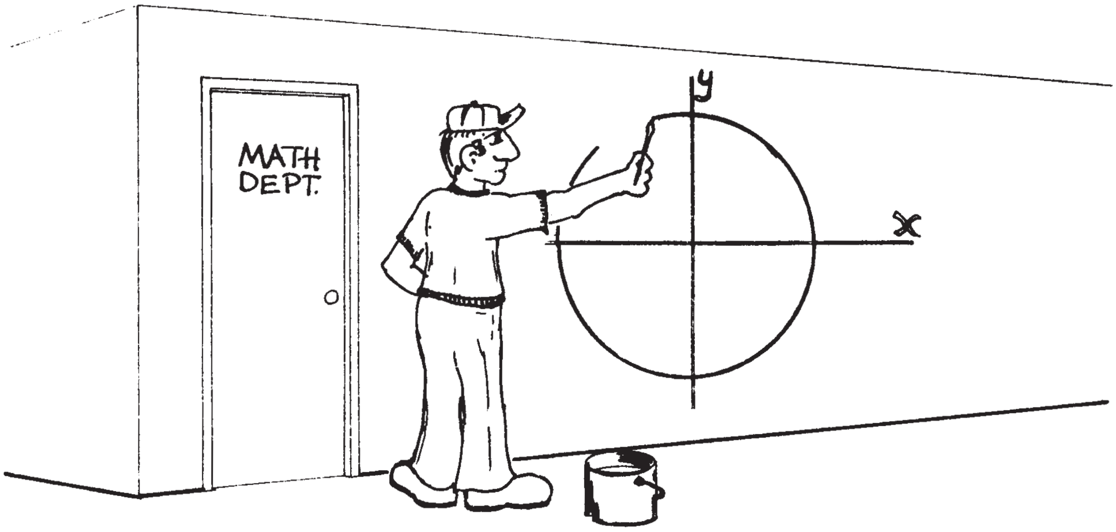
The trigonometric functions have been defined and developed by means of the (u, v) coordinate axis. This method involves the use of a central angle in degrees. The u-axis and v-axis were of different units of measure. For the functions and their graphs, we must use the same unit of measure. For example, in the equation $y = \sin x$, x and y must be of the same unit of measure; therefore, we shall use the radian unit of measure for x that was defined and developed in the previous LIFEPAC.® In graphing the trigonometric functions, we shall also discuss the various constants A, B, and C that describe the more general form of the function, such as $y = A \sin (Bx + C)$.

OBJECTIVES

Read these objectives. The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC.

When you have completed this LIFEPAC, you should be able to:

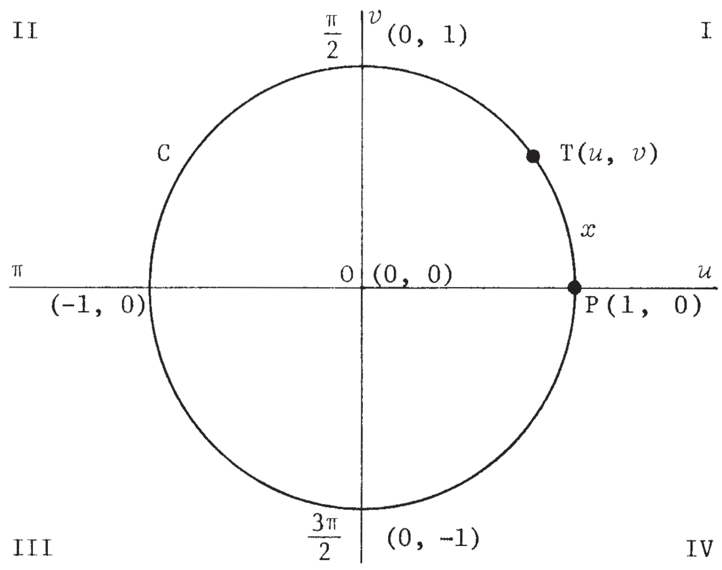
1. Convert radian measure to angle measure and, conversely, convert angle measure to radian measure.
2. Graph the six trigonometric functions.
3. Identify the amplitude of a trigonometric function, and graph the functions with various amplitudes.
4. Identify the period of a trigonometric function, and graph the functions with various periods.
5. Identify the phase shift of a trigonometric function, and graph the functions with various phase shifts.



I. THE CIRCULAR FUNCTIONS

> DEFINITION
>
> A unit circle is a circle whose radius is one.

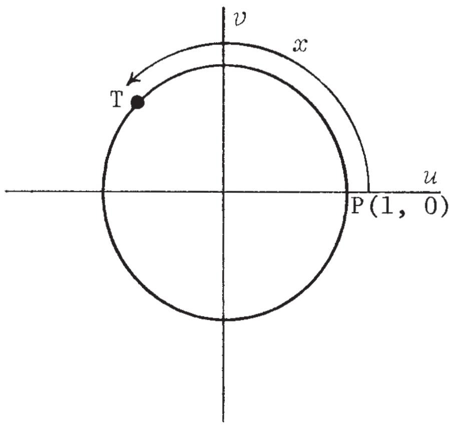
The circumference C of the unit circle is given by the formula $2\pi r$; hence, $C = 2\pi$. Let C be the unit circle with its center at 0(0, 0) of a rectangular coordinate system. Choose T(u, v), *any* point on C.



The arc length from P(1, 0) to T will be designated by x. When x measures the distance on C from P(1, 0) to a point, which lies within a given quadrant (I, II, III, IV), x is said to be in that quadrant.

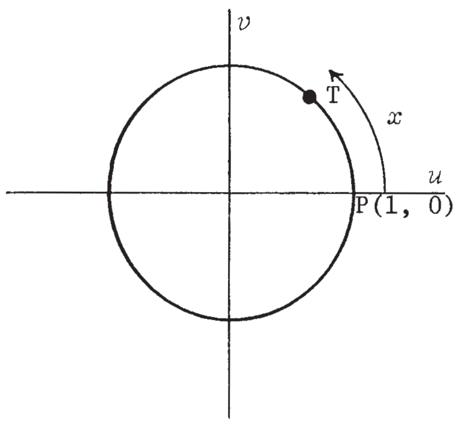
IN THE FOLLOWING EXAMPLES, IDENTIFY THE QUADRANT IN WHICH x IS LOCATED:

(a)



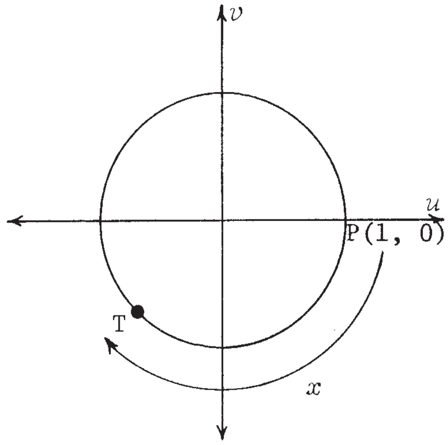
x is located in the second quadrant

(b)



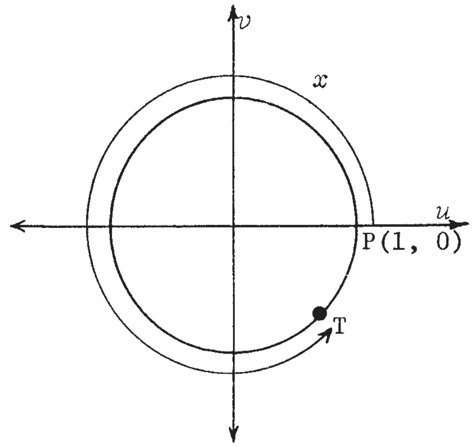
x is located in the first quadrant

(c)



$-x$ is located in the third quadrant

(d)



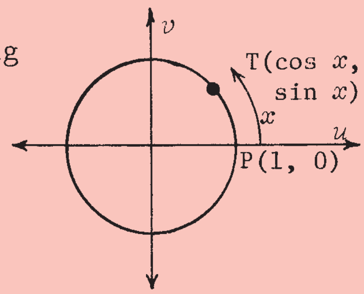
x is located in the fourth quadrant

DEFINITION

If C is the circle with equation $u^2 + v^2 = 1$ and x is the distance along C from P(1, 0) to T(u, v), then:

$$\text{cosine} = \{(x, u) : u = \cos x\}$$

$$\text{sine} = \{(x, u) : v = \sin x\}$$



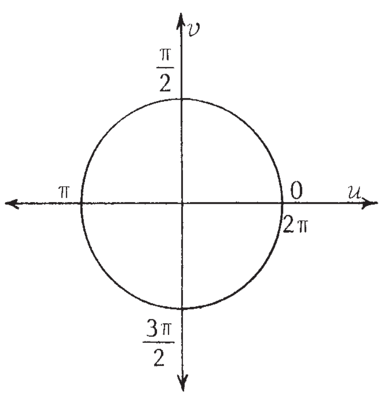


Figure 1

As the unit circle is involved in these definitions, the sine and cosine are referred to as *circular functions*.

The domain x is the set of all arc lengths of the unit circle. Each arc length is represented by a real number. Hence, the domain is the set of real numbers.

Figure 1 defines the positive arc length in each of the four quadrants. The values of x that terminate on the coordinate axis are $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π.

STUDY THESE EXAMPLES:

$\frac{\pi}{4}$ — $\frac{\pi}{4}$ lies between 0 and $\frac{\pi}{2}$. Hence, $\frac{\pi}{4}$ is located in the first quadrant.

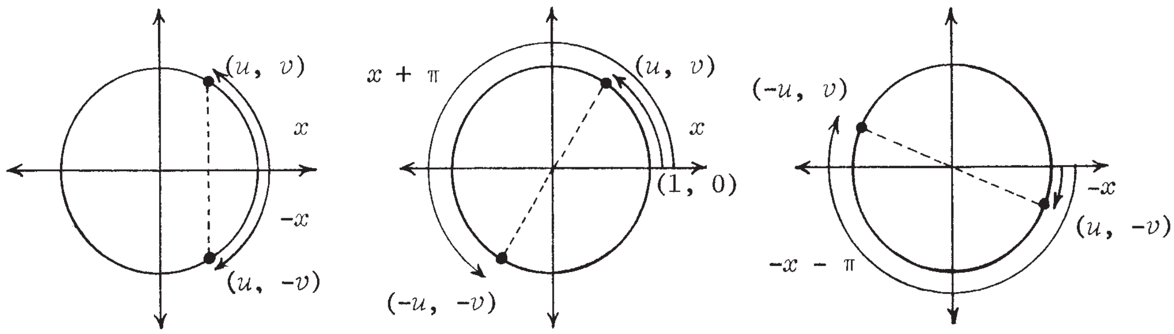
$\frac{5\pi}{6}$	$\frac{5\pi}{6}$ lies between $\frac{\pi}{2}$ and π. Hence, $\frac{5\pi}{6}$ is located in the second quadrant.

IF T IS THE POINT AT THE GIVEN DISTANCE ON THE UNIT CIRCLE C FROM P(1, 0), DETERMINE THE QUADRANT IN WHICH T LIES:

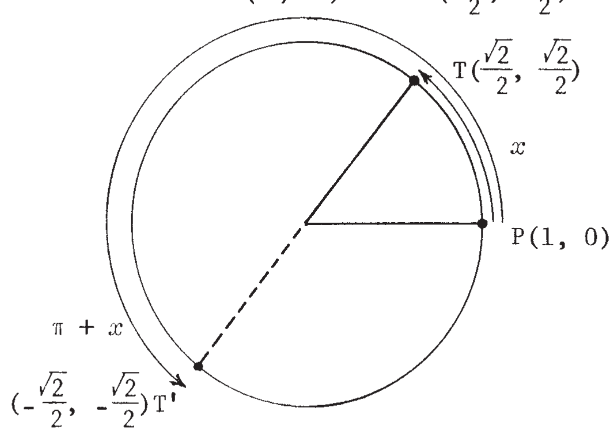
1.1 $\frac{\pi}{6}$ ______ 1.3 $\frac{5\pi}{6}$ ______ 1.5 $\frac{7\pi}{4}$ ______ 1.7 $\frac{4\pi}{3}$ ______ 1.9 $\frac{5\pi}{4}$ ______

1.2 $\frac{3\pi}{4}$ ______ 1.4 $\frac{2\pi}{3}$ ______ 1.6 $\frac{\pi}{3}$ ______ 1.8 $\frac{11\pi}{6}$ ______ 1.10 $\frac{13\pi}{12}$ ______

The following diagrams show the symmetry of C with respect to the u, v axes.



In the following example, determine the coordinates of $\pi + x$ if x is the distance from P(1, 0) to T$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$:



Answer: T'$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

IF ON C THE DISTANCE FROM P(1, 0) TO THE POINT T IS x, DETERMINE THE COORDINATES OF THE POINTS AT THE INDICATED DISTANCE ON C FROM P:

1.11 T$(\frac{3}{5}, \frac{4}{5})$

a. $\pi + x$ ______ c. $\pi - x$ ______

b. $-x$ ______ d. $2\pi + x$ ______

1.12 T$(\frac{12}{13}, \frac{5}{13})$

a. $\pi + x$ ______ c. $\pi - x$ ______

b. $-x$ ______ d. $x - \pi$ ______

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.