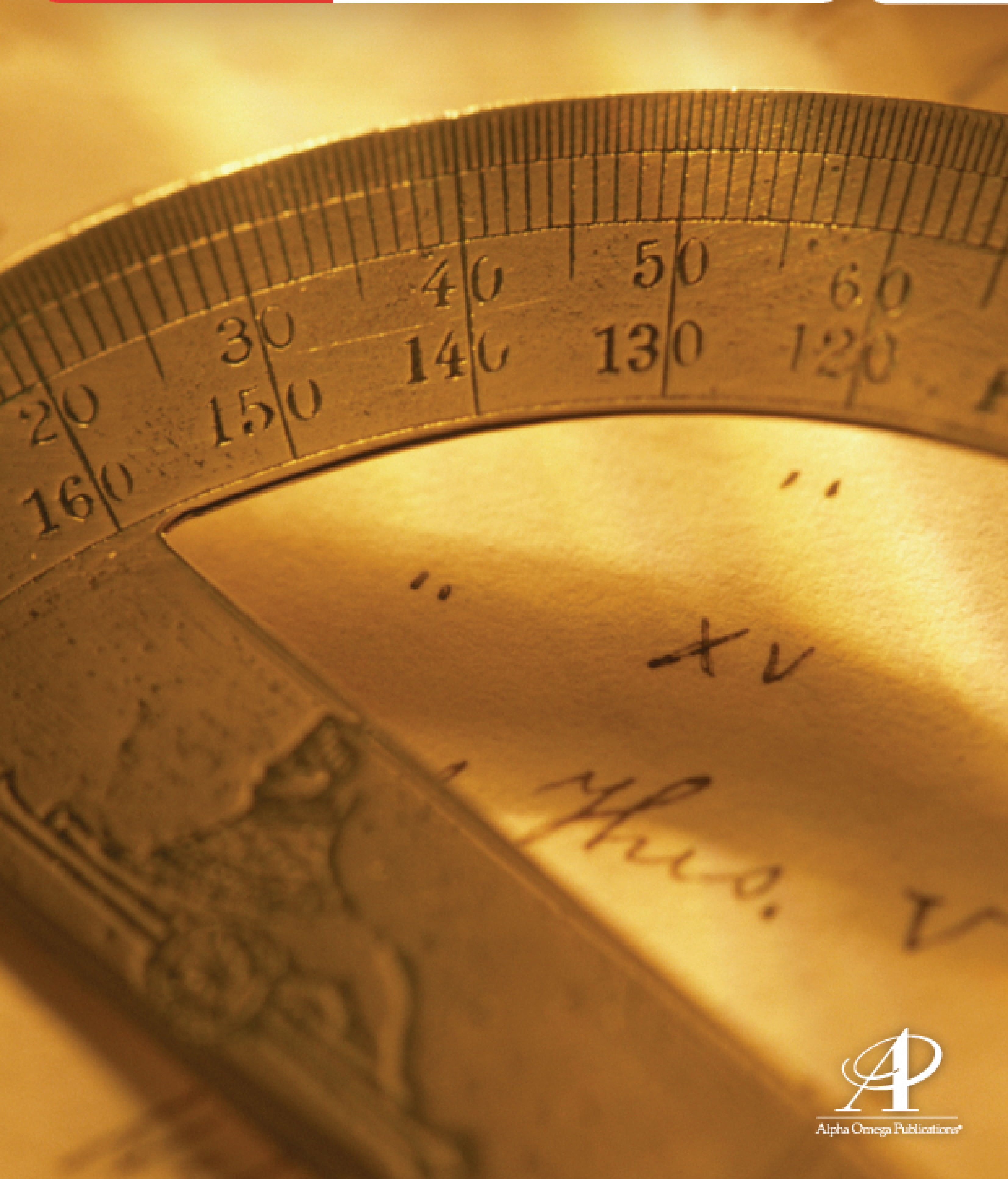
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1201
RELATIONS AND FUNCTIONS

CONTENTS

Author: **Donald E. Bade, M.A.**

Editor: Richard W. Wheeler, M.A.Ed.

Consulting Editor: Robert L. Zenor, M.A., M.S.

Illustrator: Thomas Rush

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804 N. 2nd Ave. E., Rock Rapids, IA 51246-1759

RELATIONS AND FUNCTIONS

Relations and functions are sets of ordered-pair numbers. These sets of numbers represent lines (the linear function), simple curves (the quadratic function), and curves of higher degree functions. You will soon study the exponential function, the logarithmic function, and the circular function (trigonometry). The study of functions becomes more necessary and important as you prepare for the study of calculus.

OBJECTIVES

Read these objectives. The objectives tell you what you should be able to do when you have successfully completed this LIFEPAC®.

When you have completed this LIFEPAC, you should be able to:

1. Specify the domain and range of a given relation.
2. Determine whether a given relation is also a function.
3. Identify and work with rules of correspondence for given relations.
4. Read and work with function notation.
5. Find values of a given function for specified elements of the domain.
6. Perform the arithmetic of functions.
7. Find the composition of two or more functions.
8. Graph the constant and identify functions.
9. Find the inverse of a function.

I. ORDERED-PAIR NUMBERS

An ordered-pair number is a pair of numbers that go together. The numbers are written within a set of parentheses and separated by a comma. (6, 2) is an ordered-pair number; the order is designated by the first element 6 and the second element 2. The pair (2, 6) is not the same as (6, 2) because of the different ordering. Sets of ordered-pair numbers can represent relations or functions. Some relations and functions are defined by rules of correspondence.

RELATIONS

This section discusses the concept of relations and gives its definition.

DEFINITION

A *relation* is any set of ordered-pair numbers.

Suppose that the weights of any number of students are recorded by the school nurse.

Student	1	2	3	4
Weight	150	130	100	160

The pairing of the student number and his corresponding weight is a relation and is written:

$$A = \{(1, 150), (2, 130), (3, 100), (4, 160)\}$$

These data are written as a set of ordered-pair numbers. Each element of the set is an ordered-pair.

The first element of each pair is the student number, and the set of all first elements is called the *domain* of the relation or data set.

The domain of $A = \{1, 2, 3, 4\}$

The second element of each pair is the weight, and the set of second elements is called the *range* of the relation or data set.

The range of $A = \{150, 130, 100, 160\}$

WRITE THE DOMAIN AND RANGE OF THE FOLLOWING RELATIONS IN LIST FORM.

1.1 $\{(5, 0), (6, 1), (7, 2)\}$

a. Domain ______________________

b. Range ______________________

1.2 $F = \{(6, \sqrt{2}), (7, \sqrt{3}), (8, \sqrt{4}), (9, \sqrt{5})\}$

a. Domain of F ______________________

b. Range of F ______________________

1.3 $G = \{(\frac{1}{2}, \frac{\pi}{6}), (\frac{1}{2}, \frac{\pi}{4}), (\frac{1}{2}, \frac{\pi}{3}), (\frac{1}{2}, \frac{\pi}{2})\}$

a. Domain of G ______________________

b. Range of G ______________________

1.4 $J = \{(6.2, 0.3), (7.3, 0.3), (8.4, 0.3), (9.5, 0.3)\}$

a. Domain of J ______________________

b. Range of J ______________________

WRITE THE REQUIRED RELATIONS.

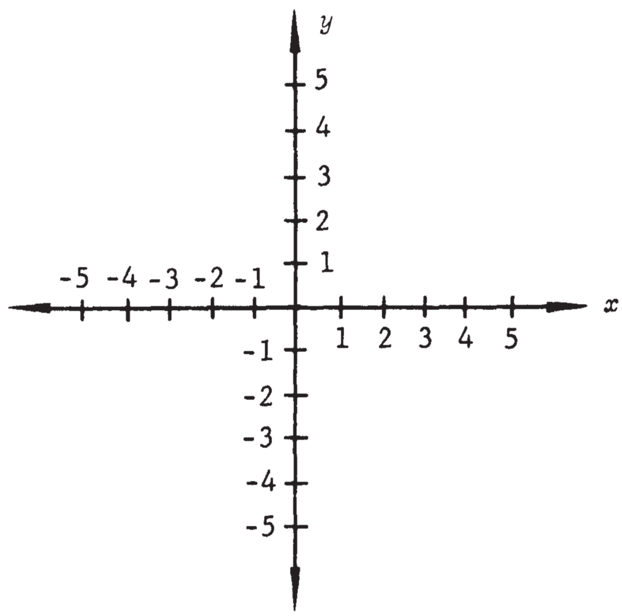
1.5 Write a relation in ordered-pair form for six different packages of fruit whose weights are the domain of the relation and are 10, 15, 20, 30, 60, and 90; and whose costs are the range of the relation and are \$2, \$3, \$4, \$6, \$8, and \$10.

Q = ______________________

1.6 A physics student, conducting an experiment of a falling object, drops a baseball from the top of a building and records the distance traveled for each second of elapsed time. If the distances recorded are 16 ft., 64 ft., 144 ft., 256 ft., and 400 ft. respectively for each second of time, write this relation in ordered-pair form.

F = ______________________

A relation that consists of ordered pairs of real numbers is a subset of $R \times R$. This set, read "R cross R," is made up of all the possible combinations of real numbers denoting ordered pairs. To visualize $R \times R$, think of a horizontal number line extending infinitely in both directions, crossed by a vertical number line extending infinitely in both directions.



Each positive and negative real number on the horizontal number line can be paired with each positive and negative real number on the vertical number line to give all possible combinations of two real numbers. Any relation that consists of ordered pairs of real numbers, therefore, is a subset of $R \times R$.

Some relations can be specified or defined by a rule enabling you to determine the element or elements of the range paired with each element of the domain. For example the solution set over $R \times R$ of an open sentence such as $x - y = 4$ is a relation L that you can specify:

$$L = \{(x, y): (x, y) \in R \times R \text{ and } x - y = 4\}$$

or

$$L = \{(x, y): y = x - 4, x \in R\}$$

The domain of the relation L is the set of real numbers.

domain of data = {Real numbers}

The range of the relation L is the set of numbers $x - 4$, where x is any real number.

range of data = $\{x - 4; x \in R\}$

Whenever an open sentence specifies a relation whose domain and range are not explicitly stated, we agree to include in the domain and range those real numbers and *only* those real numbers for which the open sentence is true.

STUDY THESE EXAMPLES:

Determine (a) the domain and (b) the range of

$$H = \{(x, y): y = \sqrt{1 - x^2}\}.$$

Solution: a. Since each value of y is to be a real number, each value of $1 - x^2$ must be a nonnegative number. Since $x^2 = |x|^2$, $1 - x^2 \geq 0$ means $1 - |x|^2 \geq 0$ or $|x|^2 \leq 1$. Then $|x| \leq 1$.

Therefore, the domain of H is $\{x: -1 \leq x \leq 1\}$.

b. $|x| \leq 1$ implies $0 \leq x^2 \leq 1$. Then $0 \leq 1 - x^2 \leq 1$.

Therefore, the range of H is $\{y: 0 \leq y \leq 1\}$.

State the domain of $F = \{(x, y): y = \frac{3x}{(x - 2)(x + 5)}\}$.

Since each value of y is to be a real number, $(x - 2)(x + 5) \neq 0$; since if $(x - 2)(x + 5) = 0$, then y would be undefined. $(x - 2)(x + 5) = 0$ when $x = 2$ or $x = -5$. Therefore, the domain of F consists of all the real numbers except 2 and -5.