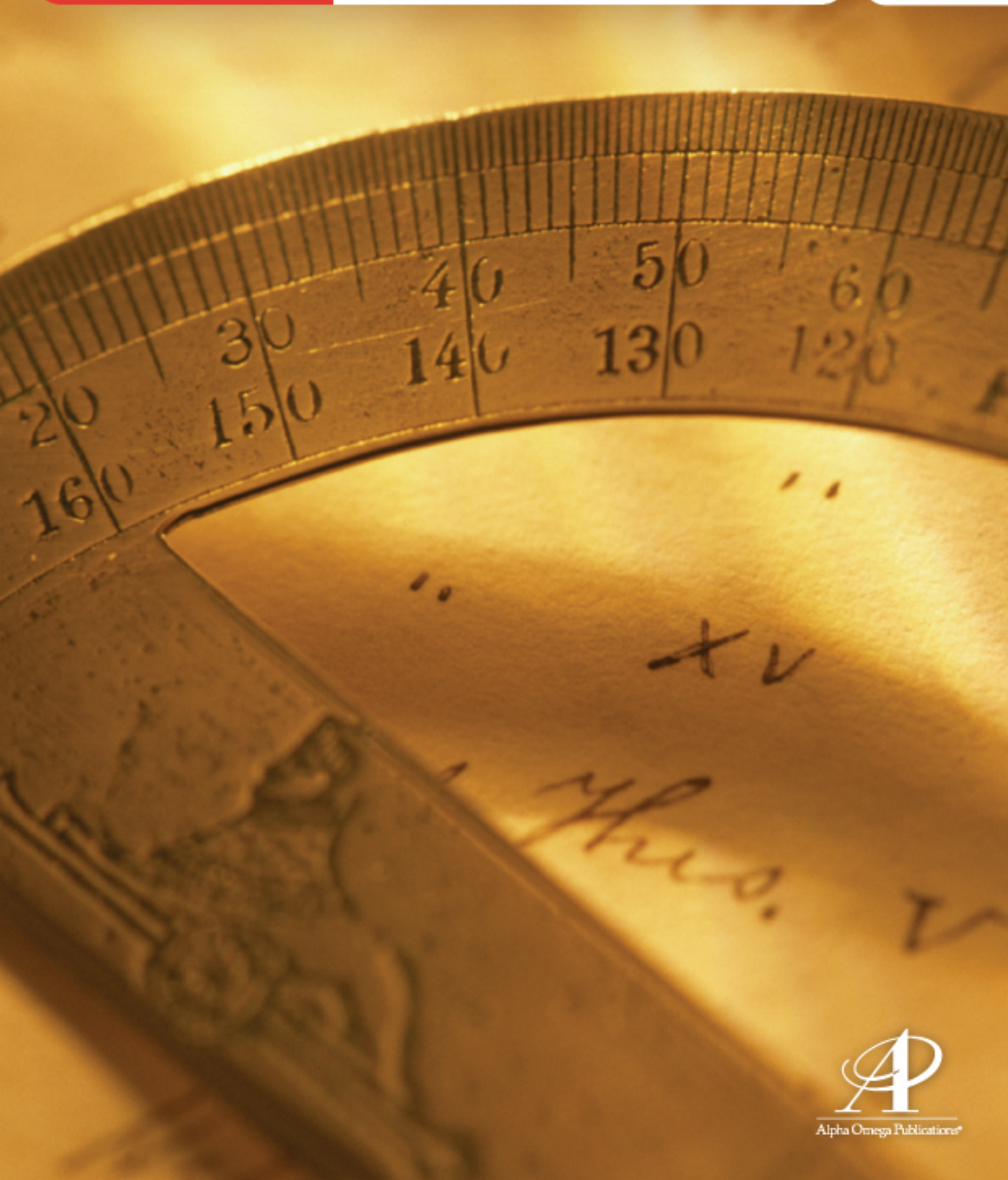
LIFEPAC® Math



Alpha Omega Publications®

MATHEMATICS 1105
ALGEBRAIC FRACTIONS

CONTENTS

Author: **James Coe, M.A.**
Editor-In-Chief: Richard W. Wheeler, M.A.Ed.
Editor: Robin Hintze Kreutzberg, M.B.A.
Consulting Editor: Robert L. Zenor, M.A., M.S.
Illustrator: Thomas Rush

Alpha Omega Publications®

804 N. 2nd Ave. E., Rock Rapids, IA 51246-1759

ALGEBRAIC FRACTIONS

In this LIFEPAC® you will study algebraic fractions. You have previously studied fractions in arithmetic and in your first year of algebra; therefore, you should already be acquainted with many of the ideas presented here.

Reducing fractions, along with adding, subtracting, multiplying, and dividing with algebraic fractions, lead to the solving of certain equations that involve fractions. In this LIFEPAC you will put the finishing touches on most of your previous work, and you will see how all the things you have learned fit together to help in solutions of word problems that you might have considered impossible before.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC.

When you have finished this LIFEPAC, you should be able to:

1. Use positive, negative, and zero integers as exponents.
2. Simplify algebraic fractions by reducing them to lowest terms.
3. Multiply and divide with algebraic fractions.
4. Use the lowest common denominator to find sums and differences of algebraic fractions.
5. Solve equations containing fractions and fractional equations.
6. Solve motion problems, mixture problems, and work problems.

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

I. MULTIPLYING AND DIVIDING WITH FRACTIONS

OBJECTIVES

1. Use positive, negative, and zero integers as exponents.
2. Simplify algegraic fractions by reducing them to lowest terms.
3. Multiply and divide with algebraic fractions.

Exponents may be negative integers or zero as well as the familiar positive integers. The laws of exponents are important when dealing with fractions.

(An exponent is a small digit written to the upper right of a number, indicating how many times the number is to be used as a factor.)

REMEMBER?

Algebraic fractions are fractions that contain variables in either the numerator or denominator or both. Basically algebraic fractions are treated in the same way that arithmetic fractions are. Of course, the expressions will be more complex and will require careful work.

DEFINITION

Algebraic fraction: a fraction with a variable in the numerator or the denominator.

ZERO AND NEGATIVE EXPONENTS

Originally exponents were used to tell us the number of times a factor occurs in certain expressions.

Model 1: $10^4 = 10 \cdot 10 \cdot 10 \cdot 10$

Certain laws of exponents were listed in Mathematics LIFEPAC 1104.

LAWS OF EXPONENTS

A. $a^m a^n = a^{m+n}$

B. $(a^m)^n = a^{mn}$

C. $(ab)^m = a^m b^m$

When dividing with numbers that have the same base, we simply subtract the exponent of the divisor (denominator) from the exponent of the dividend (numerator).

D. $\frac{a^m}{a^n} = a^{m-n}$

Model 2: $\frac{a^{16}}{a^{14}} = a^{16-14}$ or a^2

Sometimes we end up with a negative exponent or with zero as an exponent.

Model 3: $\frac{x^8}{x^{10}} = x^{8-10}$ or x^{-2}

Model 4: $\frac{x^4}{x^4} = x^{4-4}$ or x^0

Explaining the negative exponents and the 0 exponent as the number of factors occurring does not make sense. How could x^{-2} be a number with x as a factor -2 times? How could x^0 be a number with x as a factor 0 times? Negative and zero exponents are defined by the next law of exponents.

E. $a^m = \frac{1}{a^{-m}}$ when $m < 0$; and $a^0 = 1$.

If $m < 0$, then $\frac{1}{a^{-m}}$ will be a fraction with 1 as the numerator and with a positive exponent in the denominator.

Model 1: $x^{-2} = \frac{1}{x^{-(-2)}}$ or $\frac{1}{x^2}$

Model 2: $\frac{x^4}{x^4} = x^0$ or 1

In an algebraic expression, you may sometimes wish to replace the negative exponent by the reciprocal to obtain positive exponents only. A principle of quotients can be applied.

PRINCIPLE OF QUOTIENTS

$\frac{xy}{ab} = \frac{x}{a} \cdot \frac{y}{b} \qquad a \neq 0,\ b \neq 0$

Model 1: $\frac{ab^{-2}c^4de^0}{a^2b^4c^{-2}e^{-3}} = \frac{a}{a^2} \cdot \frac{b^{-2}}{b^4} \cdot \frac{c^4}{c^{-2}} \cdot d \cdot \frac{1}{e^{-3}}$

$= \frac{a}{a^2} \cdot \frac{1}{b^2b^4} \cdot \frac{c^4c^2}{1} \cdot \frac{d}{1} \cdot \frac{1 \cdot e^3}{1}$

$= \frac{c^6de^3}{ab^6}$

Exponents may be variables. If they are variables, they will be treated as any other exponents.

Model 2: $\frac{x^{3a}}{x^{-2a}} = x^{3a-(-2a)}$

$= x^{5a}$

Model 3: $\left(\frac{2x^ay^{-b}}{3y^{2b}}\right)^d = \left(\frac{2x^ay^{-3b}}{3}\right)^d$

$= \frac{2^dx^{ad}y^{-3bd}}{3^d}$

$= \frac{2^dx^{ad}}{3^dy^{3bd}}$

n+1= State the value of each expression.

1.1 7^0 _____

1.2 $5^0 \cdot 2$ _____

1.3 2^{-2} _____

1.4 $\frac{4}{2x^0}$ _____

1.5 $2^{-4} \cdot 18^0$ _____

1.6 $(12a)^0$ _____

1.7 $(2 \cdot 5)^{-2}$ _____

1.8 $2^{-2} + 5^{-2}$ _____

1.9 $3^0 \cdot 10^{-4}$ _____

1.10 $\frac{1}{2^0 + x^0}$ _____