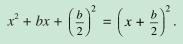


In general, the expression $x^2 + bx$ becomes a perfect square when $\left(\frac{b}{2}\right)^2$ is added to it.

Thus,



Solving a quadratic equation using this approach is called the completing the square method.

Example 7

Solve the equation $x^2 + 6x - 7 = 0$ by the completing the square method.

Move the constant term to the RHS.

Write the LHS as a perfect square. Take the square root of both sides.

Add $\left(\frac{6}{2}\right)^2$ to both sides.

Solution

$$x^{2} + 6x - 7 = 0$$

$$x^{2} + 6x = 7$$

$$x^{2} + 6x + \left(\frac{6}{2}\right)^{2} = 7 + \left(\frac{6}{2}\right)^{2}$$

$$x^{2} + 6x + 3^{2} = 7 + 3^{2}$$

$$(x + 3)^{2} = 16$$

$$\therefore \quad x + 3 = \pm \sqrt{16}$$

$$x + 3 = 4 \text{ or } x$$

$$\therefore \quad x = 1 \text{ or }$$

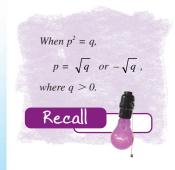
 $x^2 - x - 3 = 0$ $x^2 - x = 3$

 $\left(x - \frac{1}{2}\right)^2 = \frac{13}{4}$

...

...

 $x - \frac{1}{2} = \pm \sqrt{\frac{13}{4}}$



Note: The equation $x^2 + 6x - 7 = 0$ can be solved by the factorisation method.

Try It 7!

Solve the equation $x^2 + 2x - 15 = 0$ by the completing the square method.

x + 3 = -4

x = -7

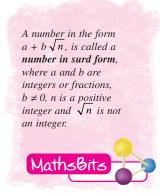
Example 8 Solve the equation $x^2 - x - 3 = 0$ by the completing the square method.

 $x^{2} - x = 3$ b = -1 $x^{2} - x + \left(-\frac{1}{2}\right)^{2} = 3 + \left(-\frac{1}{2}\right)^{2}$ $\left(\frac{b}{2}\right)^{2} = \left(-\frac{1}{2}\right)^{2}$

 $x - \frac{1}{2} = -\frac{\sqrt{13}}{2}$ or $x - \frac{1}{2} = \frac{\sqrt{13}}{2}$

Solution

Compare this solution with that in Example 4 for the same equation.





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Solve the equation $x^2 - 3x - 5 = 0$ by the completing the square method.

 $x = \frac{1}{2} - \frac{\sqrt{13}}{2}$ or $x = \frac{1}{2} + \frac{\sqrt{13}}{2}$ x = -1.30 or x = 2.30 (correct to 3 sig. fig.)