LESSON 2 *Negative exponents • Product and power theorems for exponents • Circle relationships*

2.A negative exponents

Negative exponents cannot be "understood" because they are the result of a definition, and thus there is nothing to understand. We define 2 to the third power as follows:

$$2^3 = 2 \cdot 2 \cdot 2$$

We have agreed that 2^3 means 2 times 2 times 2. In a similar fashion, we define 2 to the negative third power to mean 1 over 2 to the third power.

$$2^{-3} = \frac{1}{2^3}$$

Thus, we have two ways to write the same thing. We give the formal definition of negative exponents as follows:

> DEFINITION OF x^{-n}
>
> If n is any real number and x is any real number that is not zero,
>
> $$x^{-n} = \frac{1}{x^n}$$

This definition tells us that when we write an exponential expression in reciprocal form, the sign of the exponent must be changed. If the exponent is negative, it is positive in reciprocal form; and if it is positive, it is negative in reciprocal form. In the definition we say that x cannot be zero because division by zero is undefined.

example 2.1 Simplify: (a) $\frac{1}{3^{-2}}$ (b) 3^{-3} (c) -3^{-2} (d) $(-3)^{-2}$ (e) $-(-3)^{-3}$

solution (a) $\frac{1}{3^{-2}} = 3^2 = \mathbf{9}$ (b) $3^{-3} = \frac{1}{3^3} = \mathbf{\frac{1}{27}}$

(c) Negative signs and negative exponents in the same expression can lead to confusion. If the negative sign is not "protected" by parentheses, a good ploy is to cover the negative sign with a finger. Then simplify the resulting expression and remove the finger as the last step.

-3^{-2} problem

 3^{-2} covered minus sign

 $\frac{1}{3^2}$ equivalent expression

 $\frac{1}{9}$ simplified

$-\mathbf{\frac{1}{9}}$ removed finger

(d) When we try to slide our finger over the minus sign in (d), we find that we cannot because the minus sign is "protected" by the parentheses.

$(-3)^{-2}$ problem