LESSON 31 Equations with Parentheses

When equations contain parentheses, we begin by eliminating the parentheses. If the parentheses are preceded by a number, we use the distributive property. We multiply the number by every term inside the parentheses and discard the parentheses.

example 31.1 Solve: $2(3 - b) = b - 5$

solution As the first step, we will use the distributive property on the left-hand side to expand $2(3 - b)$. Then we will complete the solution.

$$\begin{array}{rcll}
2(3 - b) & = & b - 5 & \text{original equation} \\
6 - 2b & = & b - 5 & \text{used distributive property} \\
+ 2b & & + 2b & \text{added } 2b \text{ to both sides} \\
\hline
6 & = & 3b - 5 & \\
+5 & & +5 & \text{added 5 to both sides} \\
\hline
11 & = & 3b \longrightarrow b = \mathbf{\frac{11}{3}} & \text{divided both sides by 3}
\end{array}$$

example 31.2 Solve: $3(1 + 2x) + 7 = -4(x + 2)$

solution This equation has parentheses on both sides. Thus we begin by using the distributive property on the left-hand side and again on the right-hand side to eliminate both sets of parentheses.

$$\begin{array}{rcll}
3(1 + 2x) + 7 & = & -4(x + 2) & \text{original equation} \\
3 + 6x + 7 & = & -4x - 8 & \text{used distributive property} \\
10 + 6x & = & -4x - 8 & \text{added like terms} \\
+ 4x & & +4x & \text{added } 4x \text{ to both sides} \\
\hline
10 + 10x & = & -8 & \\
-10 & & -10 & \text{added } -10 \text{ to both sides} \\
\hline
10x & = & -18 & \\
x & = & -\frac{18}{10} \longrightarrow x = \mathbf{-\frac{9}{5}} & \text{divided both sides by 10 and simplified}
\end{array}$$

In this problem, we used all of the five steps that we will use to solve equations. Sometimes one of the steps is not necessary, as in example 31.1 above where addition of like terms was not required. If the variable is x, the five steps are:

1. **Eliminate parentheses.**
2. **Add like terms on both sides.**
3. **Eliminate x on one side or the other.**
4. **Eliminate the constant term.**
5. **Eliminate the coefficient of x.**

example 31.3 Solve: $15(4 - 5x) = 16(4 - 6x) + 10$

solution As the first step, we will use the distributive property as required on both sides of the equation.

$$\begin{array}{rcll}
15(4 - 5x) &=& 16(4 - 6x) + 10 & \text{original equation} \\
60 - 75x &=& 64 - 96x + 10 & \text{used distributive property} \\
60 - 75x &=& 74 - 96x & \text{added like terms} \\
+ 96x & & + 96x & \text{added } 96x \text{ to both sides} \\ \hline
60 + 21x &=& 74 & \\
-60 & & -60 & \text{added } -60 \text{ to both sides} \\ \hline
21x &=& 14 & \\
x &=& \dfrac{14}{21} \longrightarrow x = \mathbf{\dfrac{2}{3}} & \text{divided both sides by 21 and simplified}
\end{array}$$

In the preceding three examples we began by using the distributive property. To solve the next two problems, we need to have two rules for eliminating parentheses preceded by a plus sign or a minus sign. The rules are:

1. **When parentheses are preceded by a plus sign, both the parentheses and the sign may be discarded, as demonstrated here.**

$$+(-4 + 3x) = -4 + 3x$$

2. **When parentheses are preceded by a minus sign, both the minus sign and the parentheses may be discarded if the signs of all terms within the parentheses are changed. This rule is used because the minus sign indicates the negative of, or the opposite of, the quantity within the parentheses.**

$$-(x - 3y + 6 - k) = -x + 3y - 6 + k$$

example 31.4 Solve: $12 - (2x + 5) = -2 + (x - 3)$

solution As the first step we drop the parentheses, remembering that if the parentheses are preceded by a minus sign, we must change all signs inside the parentheses.

$$12 - 2x - 5 = -2 + x - 3$$

Now we simplify on both of sides of the equation

$$7 - 2x = x - 5$$

and solve for x

$$\begin{array}{rcll}
7 - 2x &=& x - 5 & \\
+5 + 2x & & +2x + 5 & \text{added } 5 + 2x \text{ to both sides} \\ \hline
12 &=& 3x & \\
\dfrac{12}{3} &=& \dfrac{3x}{3} \longrightarrow x = \mathbf{4} & \text{divided both sides by 3 and simplified}
\end{array}$$

example 31.5 Solve: $-(4y - 17) + (-y) = (2y - 1) - (-y)$

solution Again we remember that when we discard parentheses preceded by a minus sign, the signs of all terms within the parentheses are changed.

$$-4y + 17 + -y = 2y - 1 + y$$

First we add like terms and then we solve.

$$\begin{array}{rcl} -5y + 17 &=& 3y - 1 \\ +5y + 1 && +5y + 1 \\ \hline 18 &=& 8y \end{array}$$

added like terms
added $5y + 1$ to both sides

$$\frac{18}{8} = \frac{8y}{8} \longrightarrow y = \mathbf{\frac{9}{4}}$$

divided both sides by 8 and simplified

practice Solve:

a. $-3(2 - c) = c - 2$

b. $-(6c - 5) = 4(7c - 8) + 3$

c. $-(7 - 9)z - 6z = 8(-6 + 2)$

problem set 31

1. (21) Which of the following terms are like terms?
(a) abc^2 (b) $-2ab^2c$ (c) $-3c^2ba$ (d) $-3a^2bc$

2. (2) (a) Define a rectangle.
(b) Define a rhombus.
(c) Define a square.
(d) Is every square also a rectangle?

3. (4) Use two unit multipliers to convert 20 meters to inches.

4. (10) Use two unit multipliers to convert 1800 square meters to square kilometers.

5. (3,8) The perimeter of a square is 20 in. Find the area of the square.

Write the algebraic phrases that correspond to these word phrases.

6. (30) Seven times the sum of a number and –5.

7. (30) Seven less than twice the opposite of a number.

8. (30) The sum of 7 times a number and –51.

9. (30) A number is multiplied by 4 and this product decreased by 15.

10. (30) 0.21 of what number is 7.98?

11. (30) 0.32 of 62 is what number?

Simplify:

12. (29) 2^{-4}

13. (29) $\dfrac{1}{3^{-2}}$

14. (29) $(-4)^0$

15. (28) What fraction of 60 is 42?

16. (28) $5\frac{1}{3}$ of 120 is what number?

17. (28) If $f(x) = -2x + 3$, find $f(3)$.

Solve:

18. (25) $5k - 4 = -30$

19. (25) $3\dfrac{1}{3}x - \dfrac{1}{2} = 5$

20. (27) $0.002k + 0.02 = 2.06$

21. (31) $3(p - 2) = p + 7$

22. (31) $2(3x - 5) = 7x + 2$

23. (22) Is –1 or 4 a root of the equation $x^2 + 5x = -4$?

24. (21) Simplify by adding like terms: $xmp^{-2} - 4p^{-2}xm + 6p^{-2}mx - 5mx$

Expand by using the distributive property:

25. (29) $p^0x^{-1}(x - 2x^0)$

26. (29) $y^0x^{-4}(x^4 - 5y^4x^4)$

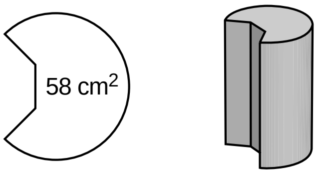
Evaluate:

27. (19) $-a^2 - 3a(a - b)$ if $a = -2$ and $b = -1$

28. (29) $-c(ac - a^0)$ if $a = -3$ and $c = 4$

29. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in feet.

30. (20) A base of the right solid has an area of 58 cm^2. The height of the right solid is 15 cm. Find the volume of the right solid.



LESSON 32 Word Problems

To solve word problems, we look for statements in the problems that describe equal quantities. Then we use algebraic phrases and equals signs to write equations that make the same statements of equality. We will begin by solving problems that contain only one statement of equality. These problems require that we write only one equation. Later, we will encounter problems that contain more than one statement of equality. These problems will require more than one equation for their solution.

We will avoid the use of the letters x and y in writing these equations. We will try to use variables whose meaning is easy to remember. The problems in this lesson discuss some unknown number. We will use the letter N to represent the unknown number.

example 32.1 The sum of twice a number and 13 is 75. Find the number.

solution We will use N to represent the unknown number. **The word *is* means "equal to." Thus, the sum of twice a number and 13 equals 75.**

$$2N + 13 = 75 \qquad \text{equation}$$

We can solve this equation by adding –13 to both sides and then dividing both sides by 2.

$$\begin{array}{rcrl} 2N + 13 & = & 75 & \quad \text{equation} \\ -\ 13 & & -13 & \quad \text{added } -13 \text{ to both sides} \\ \hline 2N & = & 62 & \\ N & = & \mathbf{31} & \quad \text{divided both sides by 2} \end{array}$$

Solutions to word problems should always be checked to see if they really do solve the problem.

$$2(31) + 13 = 75 \longrightarrow 62 + 13 = 75 \longrightarrow 75 = 75 \qquad \text{Check}$$