



Logic

—
*A God-Centered
Approach to the
Foundation of
Western Thought*

—
Vern Sheridan
Poythress

“Poythress does a masterful job of showing how our ability to think rationally is grounded in the very nature of God himself. I heartily recommend this book to every student of the subject.”

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Logic: A God-Centered Approach to the Foundation of Western Thought

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Preface

In this book we explore elementary parts of logic and neighboring fields. Part I of the book lays the foundation. In parts II and III we look at further developments in the nineteenth and twentieth centuries. These later developments have affected people's views of logic as a whole. But not all readers will be equally interested in them. I have placed the discussion of Christian foundations for logic at an early point, in part I, so that people may access it without worrying about technical details. Some readers may content themselves just with part I. Parts I.A–I.C provide the basic discussion of Christian foundations. Part I.D illustrates how these Christian foundations influence our view of Aristotelian syllogisms, which were the earliest and longest lasting form of formal logic.

Logic can be studied without considering the history of philosophy and its interaction with logic. But, for those interested, I have included in appendices F1–F5 some indications of how the nature of logic affects philosophy.

Fully appreciating modern logic involves understanding its interfaces with neighboring fields of study: rhetoric, analytic philosophy, set theory, proof theory, computation theory, abstract algebra, model theory. These fields have experienced extensive development in the twentieth century. One book or even several books cannot begin to cover them. So we have made only a beginning. In addition, logic has a rich and fascinating history.¹ I regret that I can mention only a few pieces of history in passing.

I have received help from many sources, both direct and indirect. I thank the Lord, the almighty God, the Creator of heaven and earth, who has given me life and breath and every truth and insight that I have received. He has redeemed me from the pit through Christ my Savior, and set me on the path of eternal life. To him I give all the glory.

I owe a debt also to many human beings, living and dead, to whom God has given truth and insight through his common grace or special grace. I thank my wife, who has borne with the production of this book and has

¹“Logic, History of,” *Encyclopedia of Philosophy*, 2nd ed., ed. Donald M. Borchert, 10 vols. (Detroit/New York/San Francisco/ . . . : Thomson Gale, 2006), 5:397–484.

helped in editing it. I want to recognize Kenneth Pike, Edmund P. Clowney, and John Frame, whose insights gave me many of the tools that I have used in undertaking a Christian analysis of logic. I appreciate Cornelius Van Til, who boldly stressed the distinctiveness of a Christian approach to logic, and D. H. Th. Vollenhoven, who wrote about the necessity for a Christian logic.² These two men built upon Augustine, who understood the radical absoluteness of God; John Calvin, who vigorously articulated the Creator-creature distinction; and Abraham Kuyper, who proclaimed the lordship of Christ over every sector of life and over every field of academic study.

Then there are those who have worked on logic and neighboring areas: the Sophists, Socrates, Plato, Aristotle, Euclid; and in modern times János Bolyai, Nikolai Lobachevsky, George Boole, Gottlob Frege, Charles S. Peirce, Alfred North Whitehead, Bertrand Russell, Kurt Gödel, David Hilbert, Alan Turing, Alonzo Church, Stephen Kleene, and Alfred Tarski, to name a few. Among them I may also list Hilary Putnam and Saul Kripke, who were my teachers in logic, and Garrett Birkhoff, my advisor and mentor in abstract algebra. I have not always agreed with them, but the world has been blessed by the positive insights that they have contributed through common grace.

²Dirk Hendrik Theodoor Vollenhoven, *De noodzakelijkheid eener christelijke logica* (Amsterdam: H. J. Paris, 1932); see also Vollenhoven, "Hoofdlijnen der logica," *Philosophia Reformata* 13 (1948): 59–118.

Part I

Elementary Logic

We develop a Christian approach to logic. In part I, where we consider elementary logic, no special symbols are needed. Our discussion focuses on traditional classical logic, leaving until parts II and III developments in the nineteenth and twentieth centuries.

Part I.A

Introducing Logic and Argument

Chapter 1

Logic in Tension

In the original *Star Trek* TV series, the characters Spock and Leonard McCoy are opposites. Spock is logical; McCoy is passionate. Spock is cold; McCoy is hot. The contrast raises lots of questions. How does logic fit in with our humanity? Is logic opposite to emotion? What should we be like as human beings—logical or emotional or both?

Logic and Humanity

The *Star Trek* series gained popularity not only because it had entertaining plots but also because it laid out in narrative form some of the big questions about man and his relation to the cosmos. Who are we? What is the meaning of life? What is the cosmic purpose of humanity? Why do logic and emotion struggle within us?

Viewers' reactions to Spock reveal different attitudes toward logic. To some people, Spock's logic is an ideal. To others, he may be either admirable or pitiable, but he lacks something. The creators of the show make their own comment by revealing that, while McCoy is human, Spock is the offspring from a Vulcan father and a human mother. He is only half human. A deeper look at Spock reveals further complexity: though Spock endeavors to follow logic, he sometimes struggles with inner emotions because of his human side. Does this fictional portrayal hint that logic is not enough?

What about us? How do we relate to logic? Does it appeal to us? Or do we feel that by itself it is too "cold"?

Some people are more logical, some more emotional. Some people think that we have problems because we are not logical enough. Others think that we are much too logical. In their view, devotion to logic creates difficulties, and we ought to move beyond logic to something else—to nature or mysticism or art. Science, in the minds of some, is driven by logic and by a tightly defined, cold rationality. Human beings in their full personality are driven by warmth: they have desires and emotions and imagination, which are

aply expressed in the arts, in leisure, in entertainment, and in the humanities. Science, according to this view, is at odds with the humanities and with what is most precious to us.

So what is logic? Is it important? How do we understand its relation to emotion, intuition, and other aspects of human life? How do we use it? Does it have limits?

Christian Logic?

I believe that common conceptions about logic do not provide healthy answers to these questions. We need a new approach to the subject—we need a distinctively *Christian* approach.

Is there such a thing as a *Christian* view of logic? We would not be surprised to find a distinctively Christian approach to theology or ethics, because the Bible has much to say about God and ethics. But could there be a distinctively Christian approach to logic? Many people would say no. They would say that logic is what it is, irrespective of religious belief. I think that the reality is more complicated. There *is* a Christian view of logic. But it will take some time to see why.¹

Readers may, if they wish, treat this book as a general introduction to logic. Our discussion does not assume any previous acquaintance with the subject. We try to make the ideas accessible by including simple explanations with each new concept. But the discussion also has pertinence for experts, because we do not take a conventional approach. We develop a distinctively Christian approach. Human thinking about logic needs redeeming. As a result, it will take us some time to come to the point of discussing details that typically become the focus of logic textbooks.

¹I appreciate the inspiration I have received from Abraham Kuyper, *Lectures on Calvinism* (Grand Rapids, MI: Eerdmans, 1931), and from Cornelius Van Til, who continued Kuyper's legacy. Both men counsel us to think and act in all our lives as committed followers of Christ, and to bring our distinctive Christian commitments to bear on every area of life. In principle, Christian distinctiveness applies to logic. But Kuyper says at one point that logic does not need a distinctly Christian reading, and for this concession Cornelius Van Til rightly criticizes him (Abraham Kuyper, *Principles of Sacred Theology* [Grand Rapids, MI: Eerdmans, 1968], 159–160; Cornelius Van Til, *Common Grace and the Gospel* [n.l.: Presbyterian & Reformed, 1973], 42–44; Van Til, *The Defense of the Faith*, 2nd ed. [Philadelphia: Presbyterian & Reformed, 1963], 287–288).

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For Further Reflection

1. What makes the difference between Spock and McCoy so fascinating?
2. What different reactions are there to Spock as a character, and what do they say about people's views about logic?
3. When people think about an ideal for humanity, what role do they assign to logic?
4. How might human beings deal with the apparent tension between logic and emotion? What implications are there for the nature of our humanity?
5. Why might some people think that a distinctively Christian approach to logic makes no sense?

Chapter 2

Why Study Logic?

Why should we bother to study logic? Spock exemplifies one part of its importance. On the one hand, Spock's rational analysis gives the *Star Trek* crew valuable advice. On the other hand, we struggle with an apparent conflict between logic and emotion, or even between logic and humanness. We need a remedy.

We can find other reasons for studying logic. Some people find logic intrinsically interesting. For them, it is fun. Others study it for practical purposes. They hope that studying logic can help them sharpen their ability to reason carefully. Practice in logic can help us detect logical errors in reasoning, which have been called *logical fallacies*.

The Influence of Logic

Logic is important for another, historical reason. Logic has had a profound influence on the whole history of Western thought. In the Western world, the formal study of logic began largely with the Greek philosopher Aristotle—though Aristotle built to some extent on his philosophical predecessors, Socrates and Plato.¹ Plato and Aristotle hoped to find deep truths about the nature of the world by careful reasoning. Aristotle's study of logic tried to codify the most basic forms of reasoning. This codification could then serve as a solid foundation for philosophical investigations trying to answer the big questions about the nature of reality and the meaning of life.

Western philosophy ever since Aristotle's time has followed in the steps of Plato and Aristotle. Philosophers have *reasoned*. They have used logic. Up until the nineteenth century, with few exceptions, they built on the foundation of Aristotle's logic. The nineteenth and twentieth centuries have seen further, more technical developments in logic, which have gone well beyond what Aristotle achieved. But for the most part these developments have

¹Susanne Bobzien, "Logic, History of: Ancient Logic," *Encyclopedia of Philosophy*, 2nd ed., ed. Donald M. Borchert, 10 vols. (Detroit/New York/San Francisco/ . . . : Thomson Gale, 2006), 5:397–401.

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enhanced rather than overthrown the classical logic developed by Aristotle. (For more detailed discussion of logic and philosophy, see part IV.F, especially appendix F2. For some alternatives to classical logic, see chapters 63 and 64.)

Logic and philosophy have had a broad influence on intellectual culture in the West. Philosophy has directly influenced intellectual life, because it has seemed to many people to offer the most profound and far-reaching kind of knowledge. Science has taken a leading role in more recent times, but for centuries reasoning in intellectual centers was influenced and guided by ideas from philosophy.

In addition, logic has had indirect influence. People engage in reasoning in every area of serious study, not just in philosophy. In almost every sphere, universities today rely on reasoning—in natural sciences, medicine, historical studies, law, economics, political science, language study, literary analysis, mathematics. Academic work aspires to conduct its reasoning *rigorously*. And logic is a model for rigor. Reasoning in universities today still has underneath it the foundation for logic that Aristotle laid.

Though Aristotle's logic functions as a foundation for Western thought, we should not exaggerate its role. In both the past and the present, much influential reasoning takes the form of informal reasoning and does not explicitly invoke Aristotelian logic or any kind of formally organized logic. Appropriately, logicians themselves distinguish between the *formal logic* that Aristotle developed and the *informal logic* involved in more ordinary instances of reasoning.² Yet rigorous formal logic offers an ideal that can still influence what people expect and how people evaluate informal reasoning. Logic has an influence far wider than its core.

Logic has also influenced perceptions about the contrast between rationality on the one hand and emotion, desire, and imagination on the other. The historical movement called the Enlightenment championed *reason*. But soon people became restless. They sensed that reason was not enough. Reason gave us only half of humanity—or less. The Enlightenment stimulated a reaction, the Romantic movement, which depreciated reason and championed the imaginative, the spontaneous, the natural, and the pre-rational aspects of humanity. Like the opposition between sciences and humanities, the opposition between the Enlightenment and the Romantic movement expresses the contrast between logic and emotion, or between

²On the distinction between formal and informal logic, see chapter 4.

Spock and McCoy. Thus, the contrast between Spock and McCoy has analogues that play out in culture and history.

At the foundation of this cultural opposition lies logic. It feeds into the Enlightenment's conception of reason, and it shapes the Romantic opposition to the Enlightenment as well, because the opposition defines itself in reaction to reason.

This foundation for Western thought in logic needs to be redone. And that means that the whole of Western thought has to be redone. It is a most serious issue.

Arguments

What do we mean by logic? One textbook on logic defines it as “the analysis and appraisal of arguments.”³ When we hear the word *argument*, we may picture a situation where two people are having a dispute with each other—perhaps a bitter, heated dispute. They are fighting verbally, each person vigorously defending his own view. But the word *argument* can be used not only to describe quarrels but to describe any reasoning in support of a conclusion.

Arguments of this kind may crop up in friendly settings. An advertisement for a car may present arguments to persuade you to buy one. The advertisement tells you that its car gives you good gas mileage. It is durable. It has special computerized features to play your favorite songs. It has a luxurious interior. It looks cool. And so on. These are informal arguments in favor of buying the car.

We meet arguments not only when someone else is trying to lay out the desirable features of a product, but when we are quietly trying to decide something for ourselves. For example, Irene may be “arguing with herself” about which college to attend. College A is closer to home. College B has lower tuition. College A is reputed to have a better program in economics. College B has a beautiful rural campus. College A is right in the middle of exciting city life. College B has a larger student body. Irene formulates arguments in her own mind in favor of each of the options. Arguments are useful not only for small purchases, but also for major decisions like choosing a college or deciding what kind of job to pursue.

³Harold J. Gensler, *Introduction to Logic* (London/New York: Routledge, 2002), 1. This definition is picked up in Wikipedia: <http://en.wikipedia.org/wiki/Logic>, accessed September 11, 2010. Isaac Watts's book *Logic*, widely used in an older era, defines logic more broadly: “Logic is the art of using reason well in our enquiries after truth, and the communication of it to others” (*Logic; or, the Right Use of Reason in the Enquiry after Truth: With a Variety of Rules to Guard against Error, in the Affairs of Religion and Human Life, as Well as in the Sciences* [many editions] [London: Tegg, 1811], 1).

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We also meet arguments in academic settings. A university class may lay out reasoning to reach conclusions in chemistry or in the history of World War I. When a class considers disputed ideas, the class members may study arguments both for and against the ideas. Underneath the particular arguments lies a foundation in logic, which analyzes general principles of argument.

Arguments can help to lead us to a wise conclusion. But they can also lead us astray. For example, a student says, “Either you get an A in the course or you show that you are an idiot.” But might there be a third alternative? The presentation of two extreme alternatives as if they were the *only* alternatives is called the *fallacy of bifurcation*. There are other forms of fallacy as well. A fallacy is a kind of argument that may sound plausible but that uses tricks rather than solid reasoning.⁴ Logic includes the study of various kinds of fallacies. People hope that by studying fallacies they may more easily detect them in the future.

Arguments in the Bible

Arguments occur in the Bible. We should not be surprised, because the Bible describes human life in all its ups and downs. For example, a major argument takes place in 2 Samuel 17:1–14. Absalom, the son of David, has just mounted a rebellion against the kingship of his father David. He has forced David out of Jerusalem, the capital city. But as long as David is alive, Absalom’s own position in power remains in jeopardy. Absalom asks for advice from Ahithophel, who has a reputation for giving shrewd counsel (2 Sam. 16:23). Absalom also consults Hushai, who gives opposite advice. Ahithophel says Absalom should attack David right away with a small force of select troops (17:1). Hushai advises Absalom to wait in order to assemble a large army. Both Ahithophel and Hushai give supporting reasons in favor of their stratagems.

Absalom and his supporters think that Hushai’s advice is better. Hushai’s arguments are convincing; but they lead to disaster. Absalom is killed in the battle that eventually takes place (2 Sam. 18:15). Clearly an argument can be a major turning point in a person’s life, and even in the life of a whole kingdom—in this case, the kingdom of Israel.

The arguments from Ahithophel and Hushai are even more striking

⁴A fallacy is “an often plausible argument using false or invalid inference” (*Webster’s Ninth New Collegiate Dictionary*). See S. Morris Engel, *With Good Reason: An Introduction to Informal Fallacies* (New York: St. Martin’s, 1982).

because the reader of 2 Samuel receives some information that Absalom and Ahithophel did not know. Hushai is pretending to serve Absalom, but secretly he is loyal to David. In fact, David has earlier told Hushai to go to Absalom and to try to interfere by dissuading Absalom from following Ahithophel's advice (2 Sam. 15:34). Hushai appears to Absalom to give his advice sincerely, and the arguments that he offers are plausible and attractive. But the reader can infer that Hushai does not believe in these arguments himself. He is acting out a role. Hushai's arguments therefore have two layers: what he intends Absalom to understand and what he himself understands and intends. In fact, the arguments have a third layer, because God the Lord is active behind the scenes: "For the LORD had ordained to defeat the good counsel of Ahithophel, so that the LORD might bring harm upon Absalom" (17:14).

Arguments can be used to deceive and manipulate. But they can also become part of wise counsel. At one point David has decided to order his men to attack Nabal and kill him. Abigail, Nabal's wife, comes out and dissuades him with her arguments (1 Sam. 25:23–31). David is persuaded, and blesses Abigail for having kept him back from sin (v. 33). The story has a further happy ending because after Nabal dies—by God's act rather than David's—David and Abigail marry (v. 42). Abigail's arguments have steered David toward righteous action and away from sin.

We meet still further arguments within the Bible, including arguments that address all-important religious decisions. The serpent in Genesis 3 gives arguments to try to induce Adam and Eve to sin. Elijah in 1 Kings 18 gives arguments (and a demonstration) to try to turn the people of Israel away from worshipping Baal and toward worshipping the Lord, the true God of Israel. Since Elijah presents himself as a prophet of God, his arguments claim to be not merely human but also divine. Elijah claims that God is presenting the arguments to Israel through him.

The New Testament indicates that God continues to speak, and it includes arguments to call people to come to Christ for salvation. The apostle Peter presents arguments in his sermon in Acts 2:14–36. Since Peter is an apostle, commissioned by Christ, these arguments also present themselves as divine arguments. The apostle Paul presents arguments in his sermons here and there in Acts. Acts 13:16–41; 14:15–17; and 17:22–31 give examples. In addition, some of the summaries of Paul's preaching mention argument and reasoning:

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And Paul went in, *as was his custom*, and on three Sabbath days he *reasoned* with them from the Scriptures, explaining and *proving* that it was necessary for the Christ to suffer and to rise from the dead, and saying, “This Jesus, whom I proclaim to you, is the Christ.” (Acts 17:3)

So he [Paul] *reasoned* in the synagogue with the Jews and the devout persons, and in the marketplace every day with those who happened to be there. (Acts 17:17)

And he [Paul] *reasoned* in the synagogue every Sabbath, and tried to *persuade* Jews and Greeks. (Acts 18:4)

And he [Paul] entered the synagogue and for three months spoke boldly, *reasoning and persuading* them about the kingdom of God. (Acts 19:8)

We also hear of arguments within the church when controversies arose:

And after Paul and Barnabas had no small *dissension and debate* with them, Paul and Barnabas and some of the others were appointed to go up to Jerusalem to the apostles and the elders about this question. (Acts 15:2)

The apostles and the elders were gathered together to consider this matter. And after there had been *much debate*, Peter stood up . . . (Acts 15:6–7)

In 1 Corinthians 15 Paul presents an extended argument to try to correct wavering in the Corinthian church over the question of whether there will be a future resurrection of the body.

The Bible contains many other types of communication in addition to arguments. It has songs, historical reports, prophecies, and so on. But we can use the idea of argument and persuasion as a perspective on everything the Bible does. In a looser sense, we can say that the whole of the Bible functions as an argument to induce us to change ourselves, our beliefs, and our behavior.⁵

Clearly, arguments play an important role within the Bible. They also have important roles in modern life. Arguments are important, and so logic as the analysis of argument also has an important role.

⁵I owe to John Frame this idea of using argument as a perspective on the whole of the Bible.

For Further Reflection

1. Is logic important? Why or why not?
2. How has logic influenced Western thought?
3. How does logic function in universities?
4. What kinds of arguments take place in Genesis 18:23–33; 27:5–13; 41:33–40; Exodus 4:1–17; 18:13–27; 2 Samuel 12:1–15; 14:1–24; Job; Acts 2:14–36; 3:12–26; 4:8–12; 7:2–53; 13:16–41; 14:15–17; 15:6–21; 17:22–31; 1 Corinthians 15; Galatians; Colossians; Hebrews; James?
5. What do you think are the most crucial arguments for human well-being?
6. Why do good arguments sometimes fail to persuade people?

Chapter 3

What Do We Trust?

In the discussion above we have introduced the Bible into our thinking about argument. What status does the Bible have?

The Status of the Bible

People have different opinions about the Bible. I believe that the Bible is God's word, his own speech in written form. What the Bible says, God says. But not everyone agrees.

So on this question we have the opportunity to examine arguments and analyze them. Jesus himself testifies to the authority of the Old Testament (Matt. 5:17–18; John 10:35; Luke 24:44–47). Other parts of the Bible and evidence from outside of the Bible can be drawn into the arguments. The arguments about the Bible have already been presented many times in extended form.¹ We do not have space to repeat them here. Rather, we are going to use the Bible to try to understand more deeply the character of arguments and logic.

An approach using the Bible may leave many people uneasy. Why? People may have many reasons, but one reason is that, in the modern world, we are accustomed to examining all claims critically. We use reasoning to sift through claims, and we do not trust anything—including statements within the Bible—until they are sifted.

People have attempted to sift through the Bible in many ways, and as a result we have a lot of disagreement about ideas in the Bible. In the modern world, people do not agree about whether God exists. There are other questions as well, a whole list of them. Is Jesus Christ really the Messiah and Savior promised in the Old Testament? Did Jesus Christ really rise from the dead? Is he the only Savior? Does the Bible give us an accurate picture

¹There could be an extended list. As a beginning, we might mention John M. Frame, *The Doctrine of the Word of God* (Phillipsburg, NJ: Presbyterian & Reformed, 2010); and Benjamin B. Warfield, *The Inspiration and Authority of the Bible* (reprint; Philadelphia: Presbyterian & Reformed, 1967). For arguments about the truth of the Christian faith as a whole, see Timothy J. Keller, *The Reason for God: Belief in an Age of Skepticism* (New York: Dutton, 2008).

of who Jesus is and what he did? Does following him lead to trusting in the Bible? Is the Bible God's word?

Foundations: Divine Instruction versus Autonomy

These questions are all important, and they have led to books full of arguments, both pro and con. Any inquirer may examine them for himself. We could repeat some of these arguments, or add further arguments. But such arguments are for other books. In this book, we are focusing on logic. That is, we are focusing on the very process of analyzing arguments. When an inquirer undertakes to analyze a specific argument, whether about God or about some other issue, he inevitably has in the background of his thought some general principles or ideas about evaluating arguments. In effect, he is relying on logic, even if he is not consciously aware of it.

Now a difficulty arises. There are two radically different ways of understanding logic, not just one. There is the Christian way, and there is the usual modern way, which has also been the dominant way within the history of Western philosophy.² The Christian way is to listen submissively to the instruction of Jesus Christ, who is the Lord of the universe. The modern way is the way of *autonomy*, where we treat our own human powers as ultimate when we engage in the process of evaluation.

We can illustrate the difference using an incident from the philosopher Socrates, as recorded in Plato's dialogue *Euthyphro*. At a key point Socrates requests, "Tell me what holiness is, no matter whether it is loved by the gods or anything else happens to it."³ The gods in question are the Greek gods, each of whom is limited in relation to the others, and all of whom are finite. They quarrel with one another; they are not reliable. Given that context, it seems eminently reasonable for Socrates to try to find out the real nature of holiness, independent of what the gods may say. He will *reason* it through. In the context of later philosophical developments in the Western world, Socrates becomes an emblem for using one's mind and one's reasoning powers *autonomously*. The word *autonomy* in its etymology means "self-law." Autonomy means making human judgment and human standards for judgment an ultimate touchstone in one's life.

In contrast to the way of autonomy we have the way of submitting to

²See John Frame, "Greeks Bearing Gifts," in *Revolutions in Worldview: Understanding the Flow of Western Thought*, ed. W. Andrew Hoffercker (Phillipsburg, NJ: Presbyterian & Reformed, 2007), 6–7.

³Plato, *Euthyphro*, trans. Harold N. Fowler (London: Heinemann; Cambridge, MA: Harvard University Press, 1966), 13B. See the further discussion in appendix F2.

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divine revelation. But is this way really open to us? The situation with the Greek gods shows the difficulty. So-called revelations from so-called gods may be unreliable. They may be worse—they may be manipulative. Human beings may falsely claim to have revelations in order to gain power and prestige. According to the Bible, evil spirits may come to people and give them deceitful “revelations” (Acts 16:16–18; 2 Thess. 2:9–12).

The reality of such counterfeit revelations does not show that genuine revelation is impossible. The counterfeit is the counterfeit of the genuine. The Bible’s claim is precisely that it is the genuine revelation from the one true God. Is that claim true?

Each person has to decide. He has to decide what he thinks about God, about Jesus Christ, about the resurrection of Jesus Christ, and about the status in the Bible. He may find himself weighing arguments pro and con.

Each person has his own personal history. But in some cases, people start with the account of Jesus Christ given in the Bible in the four Gospels—Matthew, Mark, Luke, John. They find out who Jesus is. They read about what he has done. They see the evidence within the Bible for believing that he rose from the dead. Maybe they hear arguments from others. Somewhere along the line, they may become convinced that Jesus really did rise from the dead, and that this miracle proves his claims. They may also become convicted concerning their own rebellion against God and their need for Christ to save them. They commit themselves to become followers or disciples of Christ.

As part of this process, they see that Christ testifies to the divine authority of the Old Testament, and indirectly to the New Testament, because Christ authoritatively commissioned the apostles as witnesses (Acts 1:8). So their view of the Bible changes. They begin to use the Bible’s instruction rather than autonomous judgment as their ultimate guide. Whether the process is long or short, we can see a marked difference between the beginning and the end: they were formerly in rebellion, and now they have been reconciled to God through Christ.

But according to the Bible no one is neutral in the process. We are all by nature rebels against God and we do not want to submit. The Bible itself indicates that the heart of the difficulty is not in the alleged doubtful character of the evidence presented in the Bible (the evidence for the resurrection of Christ is particularly pertinent), but in the doubtful or rather sinful character of us who read it. Moreover, our sinfulness infects our reasoning, so that

we come to the evidence with corrupted standards for judging it. Even if the Bible is genuine, we want to judge it rather than submit to God. We want to remain in charge of our life (autonomy), including the life of reasoning. Our desire for autonomy, and the conception of reasoning that goes with it, need changing. We need to be redeemed by God from our rebellion.

“But,” someone may ask, “if an unbeliever is interacting with the Bible and with the evidence for the resurrection of Christ, is he not engaging in autonomous reasoning? Are you not endorsing autonomy at the beginning, when an unbeliever starts his investigation, only to move beyond it at the end?” No, we are not endorsing autonomous reasoning, either at the beginning or at the end. The Bible makes it clear that such reasoning constitutes a form of rebellion against God. It is sinful.

The Bible indicates that God comes to sinners and *changes them*, through the power of Christ and the power of his resurrection. Christ was raised to new life physically. People who come to Christ receive new life *spiritually*. They are “born again,” to use the expression in John 3. Such is the only way to overcome sinful rebellion: “Truly, truly I say to you, unless one is *born again* he cannot see the kingdom of God” (John 3:3).⁴

This new birth from God is mysterious, because it happens inside people, and no human being is fully aware of all that is going on (John 3:8). On the level of spiritual reality, any particular individual is either for God or against him. But on the level of conscious perception, the situation can often appear to be mixed. People may find themselves attracted to Jesus and yet unwilling to believe his claims or submit to him. God uses his own word in the process of change (1 Pet. 1:23). God’s power and God’s truth in Jesus overcome and change the autonomous dispositions in a person’s heart. A positive result comes about in spite of autonomous desires, not because of them.

If our thinking about reasoning needs redeeming, we are not going to be able confidently to use reasoning in the way that it has often been understood in the Western tradition. We must have a more reliable foundation. God himself is that foundation. We come to know God through Christ. God instructs us about his ways in the Bible. By loving him and absorbing his instruction, we have hope of coming to a sound understanding of reasoning and logic.

⁴The underlying expression in Greek can mean either “born again” or “born from above.” Both meanings are probably intended. New spiritual life is new, like being born a second time, and it is from above, that is, from God.

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But immediately we confront objections to this kind of approach. Objectors might say that they do not accept the Bible as a trustworthy source of truth. They might present arguments. And we in turn may respond with further arguments. But in this process, we differ not only in the conclusions but in our means for evaluating arguments, because there is more than one possible understanding of reasoning and logic.

Reasoning in a Circle?

Are we engaged in circular reasoning? We are already relying on a particular conception of reasoning and logic when we use arguments to establish our conception of logic. But there is no other way of arguing when the nature of logic itself is at stake. We start with instruction in the Bible, and we use it in order to reform logic. And after our reform, we find that logic is in harmony with the God who is described in the Bible. So what have we really accomplished?

The process is really a spiral rather than a circle, because, by the grace of God, we can learn in the process. But it is also worthwhile to point out that when we come to consider the ultimate foundations for thought and the ultimate foundations for human life, everyone is moving in a circle of some kind.⁵

Autonomy is a circle. Socratic reasoning assumes autonomy at the beginning, and in the end it will develop an autonomously shaped idea of holiness—or justice or goodness or whatever else is the topic of discussion. The typical university program of instruction assumes autonomy at the beginning, and naturally it ends there as well. It appeals to autonomy to establish autonomy. But autonomy is a fruitless circle. In actuality, we are human beings and not gods. We have to rely on other people and on a lot of assumptions, but we typically do not notice it. We do not worry about it.

Should we worry? If we were all naturally good and naturally healthy in our reasoning and in our assumptions, we might conclude that we have no cause for worry. We might also conclude that we can confidently accept the common assumptions made by the people around us, and we can confidently accept what they take to be true. Thoughtful people know better. Why do we grow suspicious?

Are we naturally good? Are we naturally rational in a healthy way? The Bible says we are not. We are corrupted by sin and by sinful desires:

⁵On circularity, see John M. Frame, *The Doctrine of the Knowledge of God* (Phillipsburg, NJ: Presbyterian & Reformed, 1987), 130–133.

. . . you must no longer walk as the Gentiles do, in the futility of their minds. They are darkened in their understanding, alienated from the life of God because of the ignorance that is in them, due to their hardness of heart. They have become callous and have given themselves up to sensuality, greedy to practice every kind of impurity (Eph. 4:17–19).

None is righteous, no, not one;
no one understands;
no one seeks for God.

All have turned aside; together they have become worthless;
no one does good,
not even one. (Rom. 3:10–12)

People could present arguments back and forth, arguing for and against the proposition that human beings are naturally good. But when we undertake to evaluate the arguments, we already have implicit assumptions or presuppositions about whether we are naturally good and sound in our ability to evaluate. The dispositions of our hearts, whether toward sin or toward righteousness, affect our evaluations.⁶

As we shall see, we covertly rely on God all along, but we suppress the truth about our reliance. The modern university aspires to be radically critical, but it is not at all critical of the widespread assumption of autonomy, nor is it critical of its own rational foundations.

Biblical Teaching

As a background for our work, we need to take into account the overall message of the Bible. The Bible says that there is one God. This God created the whole world and human beings within it (Gen. 1:1–31). Originally, as it came from God’s hand, this world was good (Gen. 1:31). The human beings whom God created were good, and enjoyed his love and his presence. But human beings rebelled against God—they sinned. Ever since, the human race has suffered under the reign of sin, and human sin has had indirect effects on the rest of the world, which human beings were appointed to care for and rule over (Gen. 1:28–30).

God sent the definitive and perfect remedy for sin in the person of his Son, Jesus Christ, who died for our sins and rose to make us right with God. We are to “believe in him who raised from the dead Jesus our Lord,

⁶See chapter 8, and the discussion of reason in K. Scott Oliphint, *Reasons for Faith: Philosophy in the Service of Theology* (Phillipsburg, NJ: Presbyterian & Reformed, 2006), chapters 1 and 2.

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who was delivered up for our trespasses and raised for our justification” (Rom. 4:24–25). By believing in him, we are saved (John 3:16; Acts 10:43; 13:38–39; 16:31; Rom. 10:9–10). Christ was raised from the dead, and now reigns over the whole universe (Eph. 1:20–22). We wait for the full restoration of human beings and the cosmos when Christ returns (Rom. 8:18–23). Thus we have a sequence of core events: creation, fall into sin, redemption through Christ, and future consummation.

What do these events have to do with arguments? The coming of sin contaminates and distorts arguments. Absalom sinned in trying to murder his father David. Hushai used his arguments deceitfully to try to block the consequences of Absalom’s sin. Abigail had to try to persuade David with arguments because David had undertaken a sinful course of action. The arguments in the sermons in Acts try to persuade people to turn from their sins. Arguments can be used for good, but they can also be used for ill. In Acts 13:8, “Elymas the magician . . . opposed them [Paul and Barnabas], seeking to turn the proconsul *away from* the faith.”

Christ came to redeem us from sin, not merely to redeem us from bad arguments. The overall picture of the effects of sin is large in scope and deep in its implications. But bad arguments are clearly part of the picture. So the central realities of redemption, and the hope for the consummation of redemption in the future, are pertinent to our understanding of argument.

Moreover, Christ’s reign over the universe implies that he is Lord and judge over all, including being Lord over arguments and over logic. It is beneficial for us to submit to him, because he is infinitely wise with the wisdom of God himself (Col. 2:3). But we also have an ethical obligation to submit to him. Our submission should be thorough, and so it should include submitting our thoughts to him in the area of logic. The apostle Paul talks about taking “every thought captive to obey Christ” (2 Cor. 10:5), and in principle this includes thoughts about logic. But does allegiance to Christ actually make a difference in logic, and if so, what difference? That is the remaining question.

Apologetics

The Bible indicates that in our time the human race is divided in two. We all have sinned (Rom. 3:23); we have all rebelled against God. But some people—not all—have had their rebellious hearts changed and renewed,

because God has worked in them to save them. They have trusted in Christ to save them from their sin, and have been united to him as their Savior. Within this life, they are not totally free from sin, but in their hearts they have turned to God and have begun to follow Christ (1 Thess. 1:9). Their minds are being renewed (Rom. 12:1–2).

As a result of this renewing work of God, there are two modes of thinking among human beings. There is rebellious thinking, and there is thinking in communion with God, that is, thinking that endeavors to have fellowship with God, to listen to him, and to submit to his instruction, relying on the power of the Holy Spirit. We might call these two kinds of thinking non-Christian thinking and Christian thinking. But the word *Christian* needs attention. Many people today may think of themselves as Christian because their parents were, or because they have feelings of admiration for Jesus, or because they attend services in a church building whose roots were Christian. All this is merely superficial. If Christianity is nothing more than this, it is fake Christianity. True Christianity is a matter of the heart, not a matter of a name.

We should also say that, historically, much evil has been done by people who claimed to be Christians. Some of them were only fake Christians. Others were genuine Christians but they nevertheless acted in accordance with sin that was still in them. Christians are not necessarily morally better than anyone else. In fact, they may be worse. But through the Holy Spirit they have recognized that they are worse and that they need help. They have come to Christ, and they have begun to change. But they may still have a long way to go. They may still commit terrible sins. Following the way of Christ does not imply that we condone evil deeds done in his name.

In short, even genuine Christians are not perfect in their deeds. Likewise, they are not perfect in their thoughts. Nevertheless, in principle there are two kinds of thinking, the Christian way and the non-Christian way. In terms of fundamental assumptions and commitments, these two ways are at odds with each other. They are *antithetical* to each other.

Because there are two kinds of thinking, rather than one, communication is a challenge. It is a challenge even when we study logic, because there are two ways of studying logic, the Christian way and the non-Christian way. The Christian way submits to God's instruction through Christ. We can receive Christ's instruction because God has caused it to be written down in the Bible. The non-Christian way follows other standards. Those

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standards may be the standards within some other religion. But most commonly they are standards of *autonomy*. Everyone simply judges for himself.⁷

As a result, we need to reckon with people's allegiances and heart commitments. Within the twentieth century, some Christians have grappled with this difficulty, and *presuppositional apologetics* has arisen as a result.⁸ Because presuppositional apologetics aspires to be based on the Bible's teaching, it disclaims any independent authority. It intends that its ideas and principles be based on the Bible. Presuppositional apologetics articulates how Christians may be fully loyal to Christ and to the Bible's teaching when they engage in dialogue with non-Christians. We cannot expound presuppositional apologetics at length, so we will be content to summarize.

Simply put, we who are followers of Christ must be consistent with our basic commitment to him. We submit to his instruction in the Bible. We sift human ideas using God as our standard. We know that God is the source of all truth. We know that even those in rebellion against him know him (Rom. 1:20–21) and rely on him (Acts 14:17). We can communicate with them because they are created in the image of God and live in his world. We can talk about any subject we choose, because every area of life reflects God's presence in the world. We may speak about what the Bible says, because the Bible as God's word has spiritual power to convict listeners, even when they do not yet agree that it is God's word (1 Cor. 2:1–5).⁹

But in our communication with non-Christians we try to make it clear that we do not agree with their fundamental assumptions and fundamental commitments against God. We have *presuppositions* different from theirs.

⁷We have phrased the issue of autonomy in black and white terms, because it is more easily explained if we present the opposing alternatives in their purest form. But many people, Christian and non-Christian alike, have endeavored to find compromise routes. For example, some people have tried having autonomy in some respects, and listening to God's revelation in some respects. Or they have fallen into inconsistencies. The compromises, as well as "purer" forms of intellectual life, are also part of the history of the West. Since the Enlightenment the Western intelligentsia has become more and more secularized. But prior to the Enlightenment Christian faith had a major influence. What about this history of Christian thought? Without minimizing its influence, we may still observe that theologians in the ancient church were influenced by Platonism, and that medieval theology critically sifted Aristotle's legacy without achieving a fundamental revision of Aristotle's logic or his system of categories. Christian thought in the West, like non-Christian thought, is a mixed legacy. If we revise Aristotle's logic, we must also reinspect the history of Christian thought, with an awareness of how Aristotle's logic may have had a damaging influence (as well as positive influence because of insights from common grace).

⁸For a simple introduction, see Richard L. Pratt, *Every Thought Captive: A Study Manual for the Defense of Christian Truth* (Phillipsburg, NJ: Presbyterian & Reformed, 1979); more advanced are Cornelius Van Til, *Christian Apologetics* (Phillipsburg, NJ: Presbyterian & Reformed, 2003); John M. Frame, *Apologetics to the Glory of God: An Introduction* (Phillipsburg, NJ: Presbyterian & Reformed, 1994); Cornelius Van Til, *Defense of the Faith*, 2nd ed. (Philadelphia: Presbyterian & Reformed, 1963); John M. Frame, *Cornelius Van Til: An Analysis of His Thought* (Phillipsburg, NJ: Presbyterian & Reformed, 1995).

⁹See K. Scott Oliphint, *The Battle Belongs to the Lord: The Power of Scripture for Defending Our Faith* (Phillipsburg, NJ: Presbyterian & Reformed, 2003).

Because of God's mercy, non-Christians can know and do know many bits of truth. In fact, they know God (Rom. 1:20–21). But Christians and non-Christians see truth differently, because non-Christians suppress the fact that they are receiving truth from God, and that what they know is found first of all in the mind of God.

These principles apply to the study of logic. We will try to study logic as followers of Christ. In the process, we need to acknowledge that our thinking is distinct from the thinking of non-Christians. We may still invite non-Christians to listen to our thinking. But the issues are clearer if they are aware that Christians and non-Christians have differing presuppositions.

Are Arguments Unspiritual?

Some Christians have imagined that engaging in argument is innately unspiritual. Argument is indeed unspiritual when it is carried out in an unloving or contentious spirit (2 Tim. 2:24–26; Titus 3:10–11). But what about the apostles' sermons in Acts or the argument that the Bible presents in 1 Corinthians 15? Are they unspiritual? We cannot draw that conclusion without criticizing the apostles and by implication criticizing God himself. No, in these cases argument clearly has a positive, spiritual role. When Peter and Paul spoke as apostles, they acted out of love for God, zeal for God's truth, and out of love for people who did not know the truth or who were in danger of rejecting it. In addition, God himself empowered them to speak, so that what they spoke was God's own word (1 Thess. 2:13; 1 Cor. 2:13; 2 Pet. 1:21).

We should note, however, that if people are stubborn, they may reject an argument even when they should be convinced. Even when an apostle is the preacher, it takes the Holy Spirit to soften the hearts of the hearers (Acts 16:14; 2 Cor. 2:15–16; 2 Cor. 4:4–6).

The resistance to God's message is serious. It is not merely that people may defiantly stop their ears to argument (Acts 7:57). First Corinthians 1:18–31 indicates that the gospel seems "folly" to those who consider themselves wise. How can a criminal death on a cross, which looks like an ignominious defeat, really mean salvation? People also consider the Christian message foolish because it threatens their pride and position. They already have their own standards for evaluating claims; they have their own views of what is wrong with the world and what a reasonable remedy would be like. They have their autonomous standards for evaluating the Christian message.

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In sum, argument has an important role not only in human communication but in God's own speech to us through agents like the apostles Peter and Paul. God himself uses arguments in religious persuasion. But God is also present through the Holy Spirit to bring about inward readiness in a person's heart, and to bring subjective conviction in response to arguments and other explanations of the truth. Until God changes people's hearts, they resist the truth of the gospel.¹⁰

For Further Reflection

1. What are the two antithetical basic stances in human life and in human reasoning?
2. Does the antithesis between two stances in human life make argument between the two impossible? Why or why not?
3. What answer can be given to the objection that the Bible must first be sifted before it can be used in support of an argument?
4. How is the Bible's message of redemption relevant for the treatment of arguments?
5. How can people come to know the truth when they are resisting the message of the Bible?
6. How could you reply to someone who claims that all arguments are unspiritual?

¹⁰On the work of the Holy Spirit in bringing conviction concerning God's word, see John Murray, "The Attestation of Scripture," in *The Infallible Word: A Symposium by the Members of the Faculty of Westminster Theological Seminary*, 3rd rev. printing (Philadelphia: Presbyterian & Reformed, 1967), 42–54.

Chapter 4

Formal Logic

The arguments we encountered in chapter 2 were arguments from everyday life. They were what we might call *informal* arguments. But we can also consider more *formal* arguments. Formal arguments have been carefully pruned and arranged to follow one step after another. Consider an example:

Premise 1: All men¹ are mortal.

Premise 2: Socrates is a man.

Conclusion: Therefore, Socrates is mortal.

The first two lines are called *premises*. The argument treats them as assumptions that are supplied at the start. The final line is called a *conclusion* because it is not given beforehand. Rather, it follows from what is given in the premises. Each of the three sentences on the three lines is called a *proposition*. A proposition is simply the content of a declarative statement.² The three propositions together form what has been called a *syllogism*. (The term *syllogism* is derived from a Greek word for *reasoning* or *inference*.) Ever since Aristotle inaugurated the study of syllogisms, they have been an important part of logic.³

Formal and Informal Logic

In a broad sense, logic includes the study of both the *informal* arguments in chapter 2 and *formal* arguments like Aristotle's syllogisms. We can therefore divide logic into two parts. *Informal logic* studies informal arguments; *formal logic* studies formal arguments. In the nature of the case, informal logic

¹This piece of reasoning has often been used as a classic example within the literature on logic and syllogisms. "Men" and "a man" were used in this example before the rise of gender issues in the last half of the twentieth century.

²More precisely, a *proposition* in the context of logic is usually viewed as the content of a statement, *independent of the language* used to express the content. For more on propositions, see chapter 21. The focus on content depends on the distinction between form and meaning (since "meaning" is similar to "content"; see chapter 20).

³Susanne Bobzien, "Logic, History of: Ancient Logic," *Encyclopedia of Philosophy*, 2nd ed., ed. Donald M. Borchert, 10 vols. (Detroit/New York/San Francisco/ . . . : Thomson Gale, 2006), 5:398–401.

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is a fairly diffuse field, because informal arguments come in many forms and have many purposes. Informal logic overlaps with the study of rhetoric and persuasion, and more broadly with the field of communication and even the whole field of human knowledge.

Formal logic is more focused. It has devoted special attention to the general conditions for valid argumentation, conditions that hold no matter what is the subject matter. Informal logic may have a more significant role in ordinary life. But formal logic has greater prestige, because of its greater precision. Historically, it has been viewed as an ideal to which we ought to aspire, and informal arguments have been evaluated by how well they approximate to the rigor of formal logic.

The Influence of God in Informal Arguments

What difference does God make in arguments and in logic? First, consider informal arguments. Informal arguments are obviously shaped by human motivations, human beliefs, and heart commitments. For example, Absalom's desires, his pride, and his fears had a role in swaying him to prefer the advice of Hushai to the advice of Ahithophel. According to the Bible, the most fundamental issue of all is the commitment of the heart. Are you for God or against him (see Josh. 24:14–15; 2 Cor. 2:15–17; 4:1–6)? Do you rely on Christ for salvation or do you direct your hopes in other directions (Acts 4:12)? Do you follow Christ as Lord or do you serve some other lord—perhaps the lordship of some strong desire? Our hearts clearly influence which arguments we will be inclined to accept.

The Bible indicates that unbelievers who reject the message of salvation in the gospel, the good news about Christ, have their eyes blinded by Satan:

And even if our gospel is veiled, it is veiled to those who are perishing. In their case the god of this world [Satan] has *blinded the minds* of the unbelievers, to keep them from seeing the light of the gospel of the glory of Christ, who is the image of God. (2 Cor. 4:3–4)

This statement about the reception of the gospel indicates that the gospel deserves to be welcomed and believed. There is plenty of evidence, including the divine witness of the Holy Spirit that accompanies it (1 Thess. 1:5). But unbelievers are not persuaded by this evidence. They are blind to it. At some crucial points they do not accept the arguments, no matter how convincing these arguments may be in their own right.

Our hearts can easily deceive us. Even if we pride ourselves on our ability to criticize bad arguments, we are more likely to undertake a critique if our heart inclines us to dislike the argument's conclusion or to be suspicious of it. On the other hand, in practice we are more likely to neglect the task of critical analysis and to swallow a flawed or fallacious argument if our hearts tell us that the conclusion is pleasing.

God is involved in our use of informal arguments in everyday affairs and in the big issues of life. He is of course especially involved when it comes to the proclamation of the gospel and human response to the proclamation. But he is involved also in more mundane affairs. He can allow people's sinful desires to trap them into foolish beliefs and foolish decisions, as happened in the case of Absalom (2 Sam. 17:14). He can also give wisdom to people (Prov. 2:6; Job 32:8; 28:20–28). Then they become skilled in evaluating arguments.

We can already begin to see ways in which logic and emotion relate to each other. On the one hand, emotion or commitments in the heart can sway people to reject good arguments, arguments in accord with sound logic. On the other hand, God can change human hearts, and he can give wisdom to people who love him. Rightly understood, love and logic go together. We will explore the connection more fully later on.

The Influence of God in Formal Arguments

What about formal arguments and formal logic? We will focus a lot of our attention on this narrower arena, because many people have thought that formalization and rigor in logic eliminate the need for God. Formal arguments appear to people to lead to conclusions *in and of themselves*, independent of any religious interference. Apparently, God is absent. Or if he is present, in some vague sense, people think that his presence makes no difference. Formal logic, according to this view, is cold, impersonal, and Spockian.

Consider again the syllogism about Socrates.

Premise 1: All men are mortal.

Premise 2: Socrates is a man.

Conclusion: Therefore, Socrates is mortal.

It is a *valid* argument. The conclusion, namely, that Socrates is mortal, is true if both premises are true. The same pattern of reasoning remains true if we use different content:

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All cats are carnivores.
Felix is a cat.
Therefore, Felix is a carnivore.

Aristotle studied general patterns of reasoning like these, which display valid argumentation.

There are several kinds of syllogisms. For simplicity we confine ourselves in part I of this book to *categorical syllogisms*, the most well-known form. A categorical syllogism consists of two premises and a conclusion, such that both premises and conclusion are simple propositions about categories or classes. “All cats are carnivores” is a simple proposition relating the category of cats to the category of carnivores. By contrast, the proposition “All cats are carnivores or some cats are not carnivores” is a compound proposition.

(Technically, “Felix is a cat” is called a *singular* proposition because it makes a statement about a single individual, Felix, in relation to a single class, “cats,” rather than making a statement about two classes [“All cats are carnivores”; “No dogs are cats”]. For convenience we have included such cases along with cases that make assertions about two classes, such as the class of cats and the class of carnivores.)

We can illustrate a syllogism with a more fanciful case:

All horses are green.
George is a horse.
Therefore, George is green.

In this case, if George in fact is a human being, both premises are false. But the form or argumentation is still *valid*. “Validity,” in a technical sense, has to do with whether the conclusion follows from the premises, not with whether the premises are true. Whether the premises are true must often be determined from various sources of information about the world. By contrast, the *validity* of the overall argument does not depend on observations about horses or about George, but only on the logical form. Valid reasoning always results in true conclusions whenever the premises are true.⁴ But it does not explicitly *claim* that the premises are true; it only claims that the conclusion follows *if* the premises are true.

Logicians have tried to distinguish consistently among three concepts:

⁴Part I.C indicates how summaries about formal logic must be qualified.

truth, validity, and soundness. A *valid* argument is an argument whose conclusion follows from the premises. But in using the word *valid* we do not indicate whether the premises are actually true. A *sound* argument is an argument that is valid and whose premises are all true. The argument about the mortality of Socrates is both valid and sound.⁵ The argument that George is green is valid but unsound, because neither of its premises are true. We can also give examples of arguments that are invalid but whose premises and conclusions are true:

All mammals are animals.
 All cats are animals.
 Therefore, all cats are mammals.

The argument is *invalid* because the conclusion, though true, does not follow from the premises. We can see the invalidity of this form of argument by providing another case of the same form that results in an untrue conclusion:

Premise 1: All dogs are animals.
 Premise 2: All cats are animals.
 Conclusion: Therefore, all cats are dogs.

Whether or not various premises are true usually depends on particular facts about the world. Logic focuses not on whether the premises are true, but on whether a conclusion follows from the premises. It focuses, in other words, on the question of which arguments are *valid*.

⁵There are complexities about being “mortal.” Human beings in the new heaven and the new earth are free from the threat of death. Human beings within the present world are mortal, that is, capable of dying, but Enoch and Elijah did not die; they were taken up to heaven alive (Gen. 5:24; 2 Kings 2:11; Heb. 11:5). After his death and resurrection Jesus was no longer subject to death (Rom. 6:9) and was taken up to heaven (Acts 1:9).

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For Further Reflection

1. What is the difference between an *informal* and a *formal* argument?
2. What is a *premise*? What is a *conclusion*?
3. Identify which of the following arguments are valid, sound, and/or have true premises and conclusions.

Everything made of green cheese is edible.
The moon is made of green cheese.
Therefore the moon is edible.

No dogs are fish.
All collies are dogs.
Therefore no collies are fish.

All fish are vertebrates.
All salmon are vertebrates.
Therefore all salmon are fish.

No books are intelligible.
All ads are intelligible.
Therefore no ads are books.

If Socrates is a Greek, Socrates speaks the Greek language.
Socrates is Greek.
Therefore, Socrates speaks the Greek language.

If Socrates is a Greek, Socrates speaks the Greek language.
Socrates speaks the Greek language.
Therefore, Socrates is a Greek.

All dogs are invisible.
All cats are dogs.
Therefore all cats are invisible.

Chapter 5

Inductive Logic

Syllogisms are a form of *deductive* argument, because the conclusion is *deduced* from the premises. We may also say that the conclusion *follows* from the premises, or that the conclusion is *implied* by the premises, or that the conclusion is *inferred* from the premises. All these formulations are meant to be equivalent. If we know that the premises are true, we can also be certain that the conclusion is true. The valid formal arguments in the previous chapter are all examples of deductive argument.

Examples of Inductive Argument

A second kind of argument, called an *inductive* argument, generalizes from individual cases.¹ Consider the following reasoning:

Premise 1: Swan #1 is white.
Premise 2: Swan #2 is white.
Premise 3: Swan #3 is white.
...
Premise 1001: Swan #1001 is white.
Conclusion: Therefore, all swans are white.

Is the conclusion valid? If the premises are true, do we know that the conclusion is true? We might still entertain a nagging doubt, that sometime, somewhere, we might find a swan that turns out not to be white.

Inductive arguments are used all the time in scientific experiments. For example, a scientist may drop a ball one, two, three, or a hundred times, and each time measure the time it takes to fall to the ground. He then concludes that the ball *always* drops at the rate that he has measured.

¹Some authors use the word *inductive* in a broader sense, such that it includes several kinds of arguments by analogy (Maria Carla Galavotti, *Philosophical Introduction to Probability* [Stanford, CA: Center for the Study of Language and Information, 2005], 29–31). For simplicity, we confine ourselves to the most typical form of induction.

Differences between Deductive and Inductive Arguments

What is the difference between a deductive argument and an inductive argument? They differ with respect to the certainty of the conclusion. A deductive argument implies its conclusion with certainty; an inductive argument does not.

When we consider a valid deductive argument, if we know that the premises are true, we know for certain that the conclusion is true. In particular, if “All cats are mammals” and “Felix is a cat,” we know for certain that “Felix is a mammal.”

By contrast, inductive reasoning results in conclusions that are merely possible or probable, given the truth of the premises. The conclusion does not follow without fail merely from the truth of the premises. After a scientist performs his experiments with balls, he still does not know for certain that the next drop of the ball will have the same result. For example, unknown to him, some trickster may have concealed a piece of iron inside the ball. As the scientist is preparing for the next drop, the trickster turns on an electromagnet in the vicinity, and the magnetic force affects the ball. All of the scientist’s previous experiments cannot absolutely guarantee that the next experiment will involve nothing new or unexpected. Thus, in the case of inductive arguments, the conclusions always fall short of certainty.

A second difference lies in the kinds of propositions used in the premises. A deductive argument moves from general propositions in the premises to a general proposition in the conclusion. Or it may use a general proposition plus a singular proposition to deduce a second singular proposition:

Premise 1: All cats are mammals. [general proposition]

Premise 2: Felix is a cat. [singular proposition]

Conclusion: Felix is a mammal. [singular proposition]

By contrast, an inductive argument moves from singular propositions like “Swan #1 is white” in the premises to a general proposition like “All swans are white” in the conclusion.

God’s Involvement in Inductive Reasoning

We have observed that even after repeating an experiment many times, a scientist cannot draw a generalizing conclusion with complete certainty. But we may explore another type of question, namely, why the scientist has

any degree of confidence at all. Why should a scientist have *any* positive expectation that the ball will behave as it did before? If we lived in a world of complete chaos, nothing would be in the least predictable. It would hardly be a world at all. And we ourselves could not count on the reliability of our memories or the regularity of our heartbeat, or anything else. Why, most of the time, do we find that regularities that we have observed continue to be observed?

Philosophers have puzzled over these conundrums, but they have not reached a consensus. If we listen to what the Bible says, we have the beginning of an answer. God has created a world that has regularities in it. He has created a world using his wisdom, and he has made a world suitable for human habitation. Inductive arguments about balls work because God has seen to it that balls behave in a regular fashion. In addition, he has made human beings in his image, so that our minds are in some ways in tune with his.² So our expectations about regularities frequently (though not always) match God's own plans for the world. Inductive arguments work because God made the world with regularities that harmonize with our sense of what to expect.

On the other hand, God is God and is superior to us. So we should not be shocked if sometimes our inductive conclusions turn out to have exceptions. Our uncertainty about conclusions is an expression of our finite knowledge and our dependence on God.

Retroduction

Students of logic have also identified another form of logical argument, called *retroduction*. In retroduction, an analyst infers a probable cause or causes or a deeper explanation on the basis of observable phenomena. For example, in a jury trial the jury tries to determine, on the basis of the evidence, who committed the crime. In chemistry, even before individual atoms had ever been observed, chemists inferred that chemical reactions could be explained by recombinations of atoms.

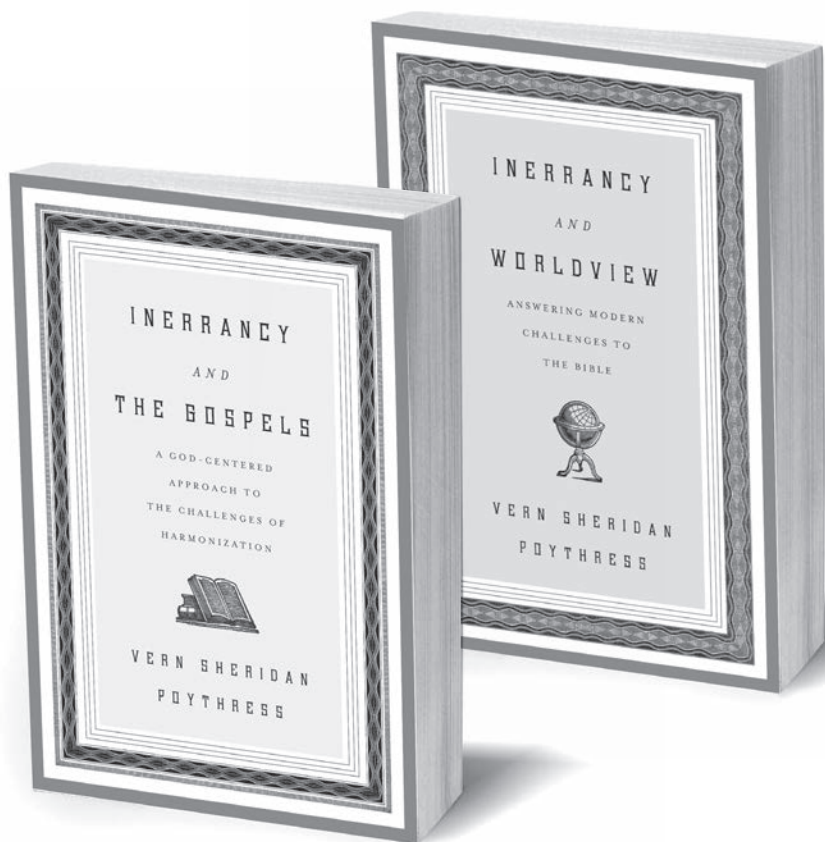
Retroduction is similar to induction, in that the inferences are not completely certain. Usually the inferences involve more complicated assumptions and an understanding of circumstances, so that the reasoning is not fully formalized. Thus, typical cases of retroduction belong to *informal logic*.

²For further discussion of regularity as a presupposition for science, see Vern S. Poythress, *Redeeming Science: A God-Centered Approach* (Wheaton, IL: Crossway, 2006), especially chapter 1.

For Further Reflection

1. What are the differences between deductive and inductive arguments?
2. Why do inductive arguments tend to “work” a good deal of the time?
3. What do the limitations in inductive arguments imply about the nature of science?
4. Classify the following arguments as deductive or inductive.
 - a. All fish are vertebrates.
All trout are fish.
Therefore, all trout are vertebrates.
 - b. All 100 trout that I have caught in this stream are fish.
Therefore, all trout are fish.
 - c. Galileo’s experiments with falling objects show that the rate of fall does not depend on the weight of the object.
 - d. The measurement of deflection of starlight by the sun conforms to Einstein’s general theory of relativity. Therefore, Einstein’s general theory of relativity is true.
 - e. Biochemical analysis shows that all living cells contain DNA.
 - f. No birds are fish.
All robins are birds.
Therefore, no robins are fish.

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